

ARTS2034

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GYa YghYf'Hk c''#'&\$%

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UbX`gUZYhm'YI Ua]bUh]cb`dfcWXi fYg/gdYW]U``Wcbg]XYfUh]cb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybhi fY/
ghi XYbhYei]hmUbX`X]gUV]`]hm'UbX`ch\Yf`YggYbh]U``UWUXYa]W]bZcfa Uh]cbž
gYY`\hdg.#k k k "Ufhg"i bgk "YXi "Ui #W ffYbhghi XYbhg#UWUXYa]W]bZcfa Uh]cb#dfchcWc`g![i]XY`]bYg#

7ci fgY⁻8YHJ]`g

7fYX]hDc]bhg**

Gi a a UfmcZh\Y'7ci fgY

Gi VYWh5fYU. 7`Y~[eZ

 $\\ \text{H\Y'Wc'i fgY']} \\ \text{g'Ub'cddcfh' b]} \\ \text{Im} \\ \text{Zcf'mci'hc'Yb'cm'UbX'i bXYfgHUbX'G\U_YgdYUfY'g'HYa YbXci g'UVX]} \\ \text{Yj Ya Ybh'cm'UbX'i bXYfgHUbX'G\U_YgdYUfY'g'HYa YbXci g'UVX]} \\ \text{Yj Ya Ybh'cm'UbX'i bXYfgHUbX'G} \\ \text{Yb'cm'UbX'i bXYfgHUbX'i b$

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8YHJ]`g. '%| < ci f`h/gh\Y`X`Xi f]b[`cbY`fY[i `Uf``YWh fY`h]a Y"``Gh XYbhg`UfY`Ug_YX`hc`]XYbh]Zmh\Y`a Yf]hg`cZhk c`dUggU[Yg`Zfca `cbY`cZh\Y`\]ghcfmd`Umg`UbX`hc`UWVci bhZcf`h\cgY`a Yf]hg`Vmi g]b[`U`hYW\b]WU` j cWUVi `Ufm"| + bghfi Whcf`dfcj]XYg`k f]hhYb`ZYYXVUW_cb`UggYgga Ybhg\YYhk\]W\`k]```]bWi XY`fUb_]b[g`]b fY`Uh]cb`hc`UggYgga YbhWf]hYf]Už[fUXY`Ug`U`dYfWYbhU[Yž'UbX`gca Y`Vf]YZWca a Ybhg`cb`h\Y`gh XYbhfig dYfZcfa UbW

5ggYgga Ybh& 9ggUm

GhUfh'XUhY.

8YHJ]`g. '%) \$\$!k cfX`YggUmcb`cbY`cZgYj YfU``hcd]Wg`dfcj]XYX`hc`h\Y`ghi XYbhg"±bghi Wcf`dfcj]XYg`ZYYXVUW_j]U`UggYgga Ybhg\YYhk\]W\`k]```]bWi XY`fUh]b[g`]b`fY`Uh]cb`hc`UggYgga YbhWf]hYf]U`UbX`Wca a Ybhg`cb`h\Y ghfYb[h\g`UbX`k YU_bYggYg`cZh\Y`YggUm

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ h' \end{tabular} $$ h$

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GhUfhXUhY. Bch5dd`]WUV`Y

 $8YHU]`g. `\&! < ci f`Y| Ua]bUh]cb`Xi f]b[`th`Y`i b]j Yfg]lmY| Ua]bUh]cb`dYf]cX" ``H`]g`]g`th`Y`Z]bU``UggYgga YbhZcf th`Y`Wci fgY"+bglfi Wfcf'dfcj]XYg`a Uf_cb'th`Y`Y| Ua `]b`fYgdcbgY`fc`Zcfa U``fYei YghZfca `ghi XYbhgž'k \c`a Um U`gc`WU`W``UhY`th`Y]f`[fUXY`cb'th`Y`VUg]g`cZh`Y]f`Z]bU``[fUXY`Zcf'th`Y`Wci fgY`UbX'th`Y`[fUXYg'th`YmfYWY]j YX`cb th`Y`ch`Yf'tk c`d]YWYg`cZUggYggYX`k cf_"$

Hi fb]h]b gYh]b[. H\]g]g bchU Hi fb]h]b Ugg][ba Ybh

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gh XYbhg UfY Yl dYWYX hc di hh\Y]f bUa Yg UbX gh XYbhbi a VYfg cb Yj YfmdU[Y cZh\Y]f Ugg][ba Ybhg"

Hi fb]h]b Gi Va]gg]cb

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hY'Yd\cbY'9l hYfbU'Gi ddcfhcb'-',) "''' %cf'Ya U]'h\Ya 'cb'<u>Yl hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui"</u>"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUfŁ"'#mci
UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Yl hYbg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9l hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW'a YbhgŁ'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Yl hYbg]cb'Udd']WUh]cb"'#mci 'Ya U]''9l hYfbU''Gi ddcfhmci 'k]''
Ui hca Uh]WU''mfYWf']j Y'U'h]W_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'''bYYX'hc'gdYW]ZWU''mUg_'Zcf'cbY"
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; YbYfU``mž'UggYgga YbhitUg_g'a i ghVY'gi Va]hhYX'Y`YWfcb]WU``mj]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bghUbWfg'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'ghUhYX'cb'mci f'Wci fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYhU]`g''

@UhY 5 ggYgga YbhDYbU`hJYg

- K cf_'gi Va]hh'X`%\$`hc`% 'XUmg`UZhYf`h\Y`gh]di `UhYX`XYUX`]bY`k]```VY`UggYggYX`UbX`ZYYXVUW_dfcj]XYX`Vi h'U`a Uf_'cZnYfc`k]```VY`fYWcfXYX"`\Zh\Y`k cf_'k ci `X`\Uj Y`fYWf]j YX`U'dUgg`a Uf_'Vi h'Zcf h\Y``UhYbYgg`UbX`h\Y`k cf_']g`U`Wca di `gcfm'Wci fgY`Wca dcbYbh'f\i fX`Y`fYei]fYa Ybh\z`U'gh XYbhk]``VY`XYYa YX`hc`\Uj Y`a Yhih\Uh'fYei]fYa Ybh\
- K cf_'gi Va]hhYX`&\$`cf`a cfY`XUmg`UZhYf`h\Y`gh]di `UhYX`XYUX`]bY`k]```bchVY`UWWdhYX`Zcf UggYgga YbhUbX`k]```fYWY]j Y`bc`ZYYXVUW_ža Uf_'cf`[fUXY"'=Zh\Y`UggYgga YbhHUg_']g`U`Wca di `gcfm Wca dcbYbhcZh\Y`Wci fgY`U'ghi XYbhik]```fYWY]j Y`Ub`l bgUh]gZJWcfmi: U]``fl : Ł`[fUXY`Ug`U`fYgi `hcZ i bgUh]gZJWcfmidYfZcfa UbWY`]b`Ub`YggYbh]U``Wca dcbYbhcZh\Y`Wci fgY"

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IBGK [fcidgdu[]Uf]ga]bhch\YZc``ck]b[WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

 $7c^{i}$ g]cb. 'k cf_]b['k]th. 'cth. Yfg'Vi h'dUgg]b['c\Zth. Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U''k cf_"'7c''i g]cb'U'gc

- i bXYfglUbX mci f'f][\hg'UbX'fYgdcbg]V]]hJYg'Ug'U'gh XYbh'Uhil BGK
- VY 'Uk UfY'cZd`U[]Uf]ga ž'Wcdmf][\hž'l BGK 'Ghi XYbhi7cXY'cZ7cbXi WhUbX'5VWWdhUV`Y'l gY'cZ l BGK '=7 H'F Ygci fWYg'Dc`]Wh
- VY'Uk UfY'cZh\Y'ghUbXUfXg'cZVY\Uj]cif'YldYWhYX'cZYjYfmcbY']b'h\Y'l BGK 'Wcaaib]hm
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Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY'UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U_Y'h\Y'a cghcZmci f'gh X]Yg'UhI BGK "

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		″I bXYf'h\Y'[fYYbk ccX'hfYY″'f&") "%, と
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		″±ik UgʻUʻcj YfʻUbXʻ\]gʻ`Ugg"f) "' "%(!' %±
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K YY_`*.`&+`5i[igh!`& GYdhYa VYf	@YWhi fY	7 ca]W6 [S`a[S`]b'G\U_YgdYUfYUb'8 fUa U'fl W ok ;Hl bSdf # 'UbX'3e Kag >[] W;fL'fbK Ł
	@YWh fY	G\U_YgdYUfYfgj"HfUX]hjcbg"cZHfU[YXmfbK Ł
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		7 ca YXm]gʻgi ddcgYXʻhcʻa U_Yʻi gʻ`Ui [\žVi hG\U_YgdYUfYʻk Ubhgʻ\]gʻWca YXmhcʻa U_Yʻi gʻXc a cfYʻh\Ubʻh\Uhʻʻ< ck ʻXcYgʻBgʻK⁄agʻ®¶_W;fia U_Yʻmci ZYY`žYgdYWJU``mUhh\YʻYbX3```K\Uh']gʻ]hUVci h'h\Y d`chž'VXUfUWYfž'h\ci [\hž'X]WfJcbž'gcb[ž'UbX gdYWfUWYʻh\Uha U_Yʻmci ʻZYY``h\]gʻk Um3
K YY_`+.`' `GYdhYa VYf`!`-	@YWhi fY	G\U_YgdYUfY`UbX`5bWJYbhF ca Y`f6K Ł
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