

# **ARTS2096**

A YX]U'F][\hgž'A YX]U'K fcb[g

GYa YghYf'Hk c''#'&\$%

# 7ci fgY Cj Yfj ]Yk

## GHJZZ7cbHJWh8YHJ]`g`

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D\cbY
<yuh\yf': cfx<="" th=""><th></th><th>Æca % &amp;da cb Û H∖i fgXUmXi f]b[</th><th>Fcca '&amp;' %Hž FcVYfh KYVghYf 6i]'X]b[</th><th></th></yuh\yf':>		Æca % &da cb Û H∖i fgXUmXi f]b[	Fcca '&' %Hž FcVYfh KYVghYf 6i]'X]b[	

### GWkcc~7cbhJWh~abZcfa Uhlcb

Fcca '\ %\z\ \ Y \ FcVYfhK \ YVghYf\ 6i \]\ \ [ \]

D\cbY. f\$&£'-',) '(,) \*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVq]hY. '\hdq.##qUa "Ufhq"i bqk "YXi "Ui

### 5 hmy b X Ub WY F Yei ] f Ya Yb hg

5 'ghi XYbhi]g'Yl dYWYX'hc'UhhYbX'U```W`Ugg'WcbhUWi\ci fg'Zcf'U'ZUWY!hc!ZUWYfl &: Ł'cf'V`YbXYX'Wci fgY'UbX Wca d`YhY'U```UWfj ]hjYg'Zcf'U'V`YbXYX'cf'Z ``micb`]bY'Wci fgY''

5 'ghi XYbhik \c'Uff]j Ygʻa cfYʻh\Ubʻ%) 'a ]bi hYgʻ`UhYʻa UmVYʻdYbU`]gYXʻZcfʻbcb! UhhYbXUbVW'''-⊋gi W\`U'dYbU`hm ]gʻ]a dcgYXž'h\Yʻghi XYbha i ghiVYʻ]bZcfa YXʻj YfVU``mUhih\YʻYbXʻcZWUggʻUbXʻUXj ]gYXʻ]bʻk f]h]b[ˈk ]h\]bʻ&(\ci fg"

⊋Uʻgh XYbhYl dYf]YbW'gʻ]`bYggža ]gUXj Ybh fYʻcfʻch\YfʻcVW ffYbW h\Uha U\_YgʻUVgYbW Zfca ʻU

# 5WUXYa]W±bZcfaUh]cb

: cf`YggYbhJU``ghi XYbh`]bZcfa UhJcb`fY`UhJb[ 'hc. 'fYei Yghg`Zcf`Yl hYbg]cb/fYj ]Yk 'cZa Uf\_g/cWW dUhJcbU``\YU`h\UbX`gUZYhm'Yl Ua ]bUhJcb'dfcWYXi fYg/gdYWJU``Wcbg]XYfUhJcb`]b'h\Y'Yj YbhcZ]``bYgg'cf'a ]gUXj Ybhi fY/ghi XYbhYei ]hmUbX`X]gUV]`]hm'UbX`ch\Yf`YggYbhJU``UWUXYa ]W]bZcfa UhJcbž gYY`\hhdg.##k k k "Ufhg"i bgk "YXi "Ui #W ffYbhghi XYbhg#JWUXYa ]W]bZcfa UhJcb#dfchcWc`g![i ]XY`]bYg#

# 7ci fgY 8YHJ] g

7 fYX]hDc]bhg \*

Gi a a UfmcZh\Y 7ci fgY

Gi VYWn5fYU. ? WW[Sł 5g/fgdWS` V FWUZ` a 'aYk

# 5 hm Y WcbWi g]cb cZm ]g Wci fgY h Y ghi XYbhk ] ``VY UV Y hc

- %" '8 YgWf]VY 'UbX'X]gWi gg`gca Y`cZh\Y`WYblfU``XYVUlYg`fY`Uh]b[ 'hc`a YX]UžYh\]Wg`UbX`\i a Ub`f][ \hg" &" 5dd`mWcbWYdhg`WYblfU``hc`a YX]U`Yh\]Wg`UbX`\i a Ub`f][ \hg`UbX`Wf]h]WU``mUbU`mgY`h\Y`Yh\]WD``UbX dc`]h]WU`]a d`]WUh]cbg`cZa YX]U`dfUWf]WYg`UbX`fYdfYgYbhUh]cbg"
- " ": UW]]hUhY 'U f][ cfci g mYh]bWi g]j Y [ fci d X]gW gg]cb"
- ("7 fUZhU WYUf UbX dYfgi Ug]j Y k f]hYb Uf[i a Ybh'

# HYUWX]b[ GhfUhY[]Yg

FYUX]b[gžj]XYcgʻUbXʻZUWY!bc!ZUWYʻYWifYgʻk]``dfcj]XYʻ\]ghcf]WU``WcbhYI hUbX`Wf]hJWU``hYfa]bc`c[mZcfibXYfghUbX]b[fY`Uh]cbgʻVYhkYYbfaYX]UžYh\]WgžUbX`\iaUbf][\hg":Hihcf]U`gʻk]```giddcfhghiXYbhYXYb[U[YaYbhk]h\]ggiYgʻUbX`WUgYʻghiX]Yg":H\]gʻk]```YUX`hckUfXgʻh\Y]fʻZ]bU``Ugg][baYbhžUfaUfcfYggUmžh\UhUbU`mgYgʻU'WffYbh]ggiYcfʻgWUbXU``]bfaYX]UžYh\]WgʻUbX`\iaUbf][\hg"

# 5ggYgga Ybh

Mci 'UfY'YI dYWYX'hc'UtHYbX'U```WUggYg'Vi hmci 'a i ghUtHYbX'U'a ]b]a i a 'cZ%\$'k YY\_g'cZWUggYg'flYWfi fYg UbX'hi hcf]U'gŁ'hc'VY'fY[ UfXYX'Ug'\Uj ]b[ 'Z 'Z]``YX'h\Y'fYei ]fYa Ybhg'cZh\Y'Wci fgY"'5 'fc```k ]```VY'hU\_Yb']b 'YWfi fYg'UbX'mci f'gi Va ]gg]cbg'hc'h\Y'k YY\_`mhi hcf]U'g'k ]```VY'i gYX'hc'fYWcfX'mci f'hi hcf]U`'UtHYbXUbW''

# 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'tUg_	K Y][ \h	8iY'8UHY	Gh XYbh@YUfb]b[ CihWcaYg5ggYggYX
Hi hcf]U`: UWj^]hUh]cb'	&\$ı	8 i Y 'VYZcfY' h\Y 'ghUfhcZh\Y fY 'Yj Ubhhi hcf]U`	%2&2
A U⁄cf <sup>°</sup> 9ggUm	) \$1	' \$#\$-#&\$%	%ž&ž(
Dcgh/f <sup>-</sup>	' \$1	%(#%\$#&\$% `%%) - `DA	&Ž

## 5ggYgga Ybh8YHJ]`g

5ggYgga Ybh% Hi hcf]U`: UWj`]hUhjcb

GhUfhXUhY. Bch5dd`]WUV`Y

8 YHJ]`g. `Ga U```[fci dg`cZghi XYbhg`k]```ZUW[]hUhY`X]gW gg]cb`cZh\Y`Wci fgY`a UhYf]U`g`UbX`h\Ya Yg`]b`U XYg][bUhYX`k YY\_`UZhYf`gi Va]hi]b[`U`k f]hhYb`X]gW gg]cb`dUdYf`cZUddfcl]a UhY`m'%\$\$`hc`\&\$\$`k cfXg": UW]`]hUh]cbg``UghZcf`Uddfcl]a UhY`m'%) `hc`\&\$`a]bi hYg`UbX`ghi XYbhg`k]```fYWY]j Y`ZYYXVUW\_jj]U`h\Y`Wci fgY @A G"`

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tab$ 

5ggYgga Ybh& A U^cf 9ggUm

#### GhUfh XUhY.

Hi fb]h]b gYh]b[. `H\]g Ugg][ ba Ybh]g gi Va ]hhYX h\fci [\`Hi fb]h]b UbX gh XYbhg Xc bchgYY Hi fb]h]b g]a ]`Uf]hmfYdcfhg"

5ggYgga Ybh' DcghYf

GhUfhXUhY. Bch5dd`]WUV`Y

8YHU]`g. `Gh XYbhg`k ]```k cf\_'hc[Yh\Yf`]b`ga U```[fci dg`cZ'!) `gh XYbhg`hc`dfcXi WY`U`dcghYf`fY`Uh]b[`hc`a YX]U

Hi fb]h]b gYh]b[. H\]g ]g bchU Hi fb]h]b Ugg][ ba Ybh

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Gh XYbhg UfY Yl dYWYX hc di hh\Y]f bUa Yg UbX gh XYbhbi a VYfg cb Yj YfmdU[Y cZh\Y]f Ugg][ ba Ybhg"

## Hi fb]h]b Gi Va ]gg]cb

#mci YbWci bhYf'U'dfcV'Ya k \Yb'UhYa dhJo['hc'gi Va ]hmci f'Ugg][ ba Ybhh\fci [\'Hi fb]h]bž'd'YUgY
hY'Yd\cbY'9| hYfbU'Gi ddcfhcb'-', ) "''' %cf'Ya U]'h\Ya 'cb'<u>Yl hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui"</u>"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'''#mci
UfY'i bUV'Y'hc'gi Va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Yl hYbg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9| hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW'a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbWY'hc'gi ddcfhmci f'Yl hYbg]cb'Udd']WUh]cb'''#mci 'Ya U]''9| hYfbU''Gi ddcfhmci 'k]''
Ui hca UhJWJ'`mfYWY']j Y'U'HJW\_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'''bYYX'hc'gdYWJZJWJ'`mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'gfUh' g'cb'Hk]hYf"

; YbYfU`mž'UggYgga YbhitUg\_g'a i ghVY'gi Va ]hhYX'Y`YWfcb]WU``mj ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bghUbWfg'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'ghUhYX'cb'mci f'Wci fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYhU]`g"

## @UhY 5ggYgga Ybh'DYbU`h]Yg

Gh XYbhgʻUfYʻfYgdcbg]V`YʻZcfʻh\Yʻgi Va ]gg]cbʻcZUggYgga YbhhUg\_gʻVmh\YʻfYei ]fYX`XUhYgʻUbXʻhJa Yg" 8YdYbX]b[ˈcZh\Y`YI hYbhcZXY`Um]bʻh\Yʻgi Va ]gg]cbʻcZUbʻUggYgga YbhhUg\_ʻdUghh\Y`Xi Y`XUhY`UbXʻhJa Yž

- K cf\_'gi Va ]hh'X`%\$`hc`% 'XUmg`UZhYf`h\Y`gh]di `UhYX`XYUX`]bY`k]```VY`UggYggYX`UbX`ZYYXVUW\_dfcj]XYX`Vi h'U`a Uf\_'cZnYfc`k]```VY`fYWcfXYX"`\Zh\Y`k cf\_'k ci `X`\Uj Y`fYWf]j YX`U'dUgg`a Uf\_'Vi h'Zcf h\Y``UhYbYgg`UbX`h\Y`k cf\_']g`U`Wca di `gcfm'Wci fgY`Wca dcbYbh'f\i fX`Y`fYei ]fYa Ybh\z`U'gh XYbhk]``VY`XYYa YX`hc`\Uj Y`a Yhh\Uh'fYei ]fYa Ybh\
- K cf\_'gi Va ]hhYX`&\$`cf`a cfY`XUmg`UZhYf`h\Y`gh]di `UhYX`XYUX`]bY`k ]```bchVY`UWWdhYX`Zcf UggYgga YbhUbX`k ]```fYWY]j Y`bc`ZYYXVUW\_ža Uf\_'cf`[ fUXY"'=Zh\Y`UggYgga YbhHUg\_']g`U`Wca di `gcfm Wca dcbYbhcZh\Y`Wci fgY`U'ghi XYbhik ]```fYWY]j Y`Ub`l bgUh]gZJWcfmi: U]``fl : Ł`[ fUXY`Ug`U`fYgi `hcZ i bgUh]gZJWcfmidYfZcfa UbWY`]b`Ub`YggYbh]U``Wca dcbYbhcZh\Y`Wci fgY"

H\]g^]bZcfa Uh]cb^]g^U^gc^Uj U]^\$

**‰**fm

**‰** ‰

‰

# 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"'\lefi\Wb'hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UVW]XYbhU`mWcdm]b[ 'Zfca `U`gci f\W' k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidgdu[]Uf]ga ]bhch\YZc``ck]b[ WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b[ 'U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b[ 'h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k ]h\ci hUW\_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci hWfYX]hUbX'hc d]YW]b[ 'hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b[ "

 $7c^{i}$  g]cb. 'k cf\_]b[ 'k ]th. 'cth. Yfg'Vi h'dUgg]b[ 'c\Zth. Y'k cf\_'Ug'U'dYfgcb'g']bX]j ]Xi U''k cf\_"'7c''i g]cb'U'gc

 $i\ bXYfgfUbX`mci\ f`f][\ \ \ bX`fYgdcbg]V]`]fJYg`Ug`U`gfi\ XYbfiUfi]\ BGK$ 

# FYgci fWyg

# DfYgWf]VYX F Ygci fWYg

9IdYWYX`fYgcifWyg'UjU]`UV`Y`cb`h\Y`WcifgY`@AG"

# FYWca a YbXYX FYgci fWYg

FYWca a YbXYX fYgci fWrg 'Uj U] 'UV Y 'cb 'h\Y 'Wci fgY '@A G"

# 7ci fgY 9j U'i Uhjcb UbX 8Yj Y'cda Ybh

 $H\[g']g'h\Y'Z\]fghih\]a\Y'h\Uhih\]g'Wci\ fgY']g'VY]b[\ 'cZYfYX'''H\YfY'k\ ]``VY'cddcfhi\ b]h\]Yg'Zcf'ghi\ XYbhg'hc'dfcj\ ]XY \] U'i\ UV'Y'ZYYXVUW_']b'cfXYf'hc']a\ dfcj\ Y'h\Y'`YUfb]b[\ 'YI\ dYf]YbW'''$ 

# ⇒a U[Y'7fYX]h

H\Y`]a U[Y`cb'h\Y`ZfcbhdU[Y`cZh\]g`XcW a Ybh]g``]WYbgYX`Vmh\Y`9`YWfcb]W: fcbh]Yf': ci bXUh]cb'i bXYf'U`\\ \frac{7 fYUh]j Y`7 ca a cbg`5hff]Vi h]cb'l G'' "\$``]WYbgY`k\]W\`YbUV`Yg`ZfYY`Wcdm]b[`UbX`g\Uf]b[`k]h\`Uhff]Vi h]cb"`Hc\\\]bX`ci ha cfYzij ]g]h\\\hdg.#k k k "YZ"cf[\#\\cdotdm][\L''

### 7F = 7CG

7F = 7CG Dfc| ]XYf 7cXY. \$\$\$-,;