

7ci fgY⁻Cj Yfj]Yk

'GhJZZ'7cbhJWh'8YhJ]`g'

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
9f]b [·] 6fUbb][Ub	Y"VfUbb][Ub4 i bgk "YXi "Ui	&!' da 'H\i fgXUmg	F "%% 'K YVghYf 6i]`X]b[- ' ,) '+,) (

'@YWhi fYfg'

BUa Y	9a U]`	5j U]`UV]`]hm	@cWUh]cb	D∖cbY
>c\b'5hf]X[Y	^'Uhff]X[Y4 i bgk "YXi "Ui			

GWXcc`7cbHJWhi±bZcfa Uhjcb

Fcca " & ž`Yj Y`" FcVYfhK YVghYf'6i]`X]b[

D\cbY. 'f\$&Ł'-',) '(,) *

9a U]`.'gUa 4 i bgk "YXi "Ui

K YVg]hY. \hdg.##gUa "Ufhg"i bgk "YXi "Ui

 $H\Y`GVXcc``cZh\Y`5fhg`UbX`A YX]U`k ci `X``] \underline{Y`hc`FYgdYWM} ``m5W_bck `YX[Y`h\Y`HfUX]h]cbU``7i ghcX]Ubgžh\Y`6YXY[U``f? Ybg]b[hcb`WJa di głž; UX][U``f?]hmUbX`5fh/ `8Yg][b`7 Ua di gYgŁUbX`h\Y`B[i bbUk U`$

GWYbWrg. \http://dcfa_Uhcb#dfchcWc\g![i]XY\]bYg#

.

5WUXYa]W±bZcfaUh]cb

: cf`YggYbhJU``ghi XYbh']bZcfa UhJcb`fY`UhJb['hc.`fYei Yghg`Zcf`Yl hYbg]cb/fYj]Yk 'cZa Uf_g/cWW dUhJcbU``\YU`h\
UbX`gUZYhm'Yl Ua]bUhJcb'dfcWYxi fYg/gdYWJU``Wcbg]XYfUhJcb`]b'h\Y'Yj YbhcZ]``bYgg'cf`a]gUXj Ybhi fY/
ghi XYbhYei]hmUbX`X]gUV]`]hm'UbX`ch\Yf`YggYbhJU``UWUXYa]W]bZcfa UhJcbž
qYY`\hdg.#k k k "Ufhq"i bak "YXi "Ui #W ffYbhqhi XYbhq#UWUXYa]W]bZcfa UhJcb#dfchcWc`a![i]XY`]bYa#

7ci fgY⁻8YHJ][^]g

7 fYX]h'Dc]bhg '* '

Gi a a UfmcZ'h\Y'7ci fgY

 $\label{eq:GivYWhi5fYU.FZWSfdWS`VBWbAd_S`UWEfgV[WeH\]gWb15fYU.FZWSfdWS`VBWbAd_S`UWEfgV[WeH\]gWb15fYU.FZWSfdWS`VBWbAd_S`UWEfgV[WeH\]gWb15fYU.FZWSfdWS`VBWbAd_S`UWEfgV[WeH]cb.5dWSf[hWIdff]YBWb15fYU.FZWSfdWS`VBWBAd_S`UWEfgV[WeH]cb.5dWSf[hWIdff]YBWb15fYU.FZWSfdWS`VBWBAd_S`UWEfgV[WeH]cb.5dWSf[hWIdff]YBWb15fYU.FZWSfdWS`VBWBAd_S`UWEfgV[WeH]cb.5dWSf[hWIdff]YBWb15fYU.FZWSfdWS`VBWBAd_S`UWEfgV[WeH]cb.5dWSf[hWIdff]YBWb15fYU.FZWSfdWS`VBWBAd_S`UWEfgV[WeH]cb.5dWSf[hWIdff]YBWb15fYU.FZWSfdWS`VBWBAd_S`UWEfgV[WeH]cb.5dWSf[hWIdff]YBWb15fYU.FZWSfdWS`VBWBAd_S`UWEfgV[WeH]cb.5dWSf[hWIdff]YBWb15fYU.FZWSfdWS`VBWBAd_S`UWEfgV[WeH]cb.5dWSf[hWIdff]YBWb15fYU.FZWSfdWSYBWBAd_S`UWEfgV[WeH]cb.5dWSf[hWIdff]YBWB15fYU.FZWSfdWSf[hWIdff]YBWB15fYU.FZWSfdWSf[hWIdff]YBWB15fYU.FZWSfdWSf[hWIdff]YBWB15fYU.FZWSff[hWIdff]YBWB15fYU.FZWSff[hWIdff]YBWB15fYU.FZWSff[hWIdff]YBWB15fYU.FZWSff[hWIdff]YBWB15fYU.FZWSff[hWIdff]YBWB15fYU.FZWSff[hWIdff]YBWB15fYU.FZWSff[hWIdff]YBWB15fYWB15fYU.FZWSff[hWIdff]YBWB15fYWB15fWWB15fYWB15fYWB15fYWB15fYWB15fYWB15fYWB15fYWB15f$

 $FYj]Yk]b["h\ Y"5fhg"Yl d`cfYg"U"j Uf]YhmcZk f]h]b["a cXYg"]b"Ufhg"ci fbU`]ga \ \check{z}"]bWl X]b["]bhYfj]Yk g\check{z}"dfYj]Yk g$

5ggYgga Ybh

5 'ghi XYbhik \c'UhhYbXg``Ygg'h\Ub'Y][\hmdYf'WYbhicZh\Y'WUggYg'k]h\]b'U'Wci fgY'a Um'VY'fYZ gYX'Z]bU`UggYgga Ybh'H\Y'Z]bU``UggYgga Ybh'H\Y'Z]bU``UggYgga Ybh'H\g_']g'h\Y'dcfhZc`]c"

7`UggYg`gi V^YWihc`UHhYbXUbWI`fYei]fYa Ybhg`UfY`hi hcf]U`g`UbX``YWi fYg`UbX`UHhYbXUbWI`k]```VY`hU_Yb`Uh Vch\"

H\Y'a]b]a i a 'UhhYbXUbVW' fYei]fYa Ybh]g', \$1 'gc'gh XYbhg'WUb'a]gg'bc'a cfY'h\Ub') 'hi hcf]U`g#YWi fYg"

9 I d`UbUhjcbg`cZUVgYbWrg`Zfca `WUggYg`cf`fYei Yghg`Zcf`dYfa]gg]cb`hc`VY`UVgYbh'Zfca `Zcfh\Wca]b[`WUggYg g\ci `X`VY`UXXfYggYX`hc`h\Y`7 ci fgY`5 i h\cf]hm]b`k f]hjb[`UbXžk \YfY`Udd`]\WV`Yžg\ci `X`VY`U\Wca dUb]YX VmUddfcdf]UhY`XcW a YbhUhjcb`fY"["`a YX]\W\``\Wfh]Zj\WhYt"

K CF? GI 6A ≠H98.

5```WJrUrjcbg'a i ghi]bWi XY'dU[Y'fYZYfYbWrg"'Y['fbi h\cf'gi fbUa Y'MYUfz'dU[Y'bc"Ł'fGa]h\ &\$\$\$z') Ł

=b!hYl hfYZYfYbWho[cf ZcchochYg UfY UWWYdhUV`Y"

: i ``V]V`]c[fUd\mUhh\Y`YbX`cZmciffYgYUfW\dUdYfg`]g`fYei]fYX"

I bfY`]UV`Y k YVdU[Y fYZYfYbWg UfY bchUWWdHUV`YžY["K]_]dYX]U"`

: cbhg]nY'%&'d`YUgY'!'a m'YmYg'UfY'[Yhf]b['C@8"

5ggYgga YbhHUg_g

5ggYgga Ybh'HJg_	K Y][\h	8i Y'8UhY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
K cf_g\cd^5\Mj]hjYg^	&\$ı	±0.7.`Ugg.'KYY_g.''ž').'UbX'+	%₹
Dcfh7c`]c'cZFYj]Ykg'	(\$1	&*#%\$#&\$% `\$).\$\$`DA	%ž&
F YgYUfW. 'DUdYf'	(\$1	%#%\$#&\$% `\$).\$\$`DA	%Ž

5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% K cf_g\cd 5Whjj]hjYg

GhUfh'XUhY.

@Yb[h\."B#5

5XX]h]cbU`XYhJ]`g.

5ggYgga Ybh7f]hYf]U

.

i bXYfgHJbX]b['cZh\Y'a cXYgžW\UfUWYf'UbX'Y`Ya Ybhg'cZj Uf]ci g'fYj]Yk]b['ghm`Yg

UV]]|mhc]|XYbh]Zmh\Y'j Uf]ci g'Y`Ya Ybhg'UbX'ghfUhY[]Yg'cZj Uf]ci g'fYj]Yk]b['ghmYg

gY`ZfYZYW¶cb`UbX`UggYgga Ybh'

YI dfYgg]cb

cf][]bU`]mi]b`h\Y`ZfUa]b[`cZfYgdcbgYg`hc`h\Y`hUg_g"`

Gi Va]gg]cb bchYg. < UbXYX]b]b! WUgg

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh& DcfhZc`]c cZFYj]Yk g

Ghufh Xuhy.

@Yb[h\.'&)\$\$k

8 YHJ]`g. 'H\Y'ghi XYbhg`k]```WfYUHY`U'dcfhZc`]c`cZfYj]Yk g`UbX`ch\Yf`UggcWJUhYX`k f]hJb[`hUg_g`f]bhYfj]Yk gž dfYj]Yk gŁ'i bXYfhU_Yb`Xi f]b['h\Y`k cf_g\cd#gYa]bUfg`UbX`Ug`gYh\ca Yk cf_'UbX`cb`]bY`V`c[g"'H\YgY`k]`` VY`k cf_g\cddYX`]b`WUgg`hJa Y`UbX`ZYYXVUW_'k]``VY`cb[c]b[`j]U'Vch\``YWi fYfg`UbX`dYYfg"'&!(`gUa d`Y fYj]Yk g`k]``VY`W\cgYb`Vmf\Y`ghi XYbhg`Yei U`]b['Ufci bX`&) \$\$k`]b`hchU`ž`]bWi X]b['d]YWYg`cZj UfmJb['Yb[h\g`UbX`a cXYg"'H\YgY`k]``VY`gi Va]hYX`Zcfa Uf_]b['hck UfXg`h\Y`YbX`cZh\Y`Wci fgY": YYXVUW_ik]``VY cb[c]b['UbX`U`Z]bU``UggYgga Ybhk]```VY`dfcj]XYX`a Uf_]b['h\Y'dcfhZc`]c`U[U]bghU`[f]X``]ghJb['h\Y'UggYgga YbhWf]hYf]U"

5XX]h]cbU`XYhU]`g.

5ggYgga Ybhi8YgWf]dh]cb. 'H\Y'ghi XYbhg'k]```WfYUHY'U'dcfhZc']c'cZfYj]Yk g'UbX'ch\Yf'UggcW]UHYX'k f]h]b[HUg_g'f]bHYfj]Yk gž'dfYj]Yk gž'cd]b]cb'd]YWygž'WfYUh]j Y'k f]h]b['HUg_gŁ'i bXYfHU_Yb'Xi f]b['h\Y k cf_g\cd#gYa]bUfg'UbX'Ug'gYh\ca Yk cf_'UbX'cb']bY'V'c[g"'H\YgY'k]``'VY'k cf_g\cddYX']b'WUgg'h]a Y'UbX ZYYXVUW_'k]``'VY'cb[c]b['j]U'Vch\''YWi fYfg'UbX'dYYfg"'GUa d'Y'fYj]Yk g'k]``'VY'W\cgYb'Vmh\Y'ghi XYbhg Yei U``]b['Ufci bX'&) \$\$k ']b'rchU'ž']bWi X]b['d]YWYg'cZj Ufm]b[''Yb[h\g'UbX'a cXYg"'D`YUgY']bWi XY df]a Uf]`mUffg'fY`UhYX'k f]h]b['k]h\'cbY'ch\Yf'mdY'Y["'I bfY`UhYX'cd]b]cb'd]YWY'cf'WfYUh]j Y'k f]h]b["8c'bch']bWi XY'k cf_'U'fYUXmUggYggYXžVi h'mci 'WUb'i gY'UggYgga YbhifUg_g'hc'k f]hY'bYk 'fYj]Yk g" D`YUgY'gYY'k cf_g\cdg'k YY_!Vmlk YY_'Zcf']b!WUgg'k f]h]b['HUg_g"'H\YgY'k]``VY'gi Va]hYX'Zcf UggYgga Ybh''

 $: YYXVUW_k] ``VY`cb[c]b['UbX`U`Z]bU``UggYgga Ybhk] ``VY`dfcj] XYX`a Uf_]b['h\Y'dcfhZc`]c`U[U]bghU'[f]X `]gh]b['h\Y'UggYgga YbhWf]hYf]U''$

.

a Uf_ <8	Wf]hYf]U

 $\label{thm:condition} $$ \Upsilon_{g}(x) = \Upsilon_{g}(x) + \Upsilon_{g}($

5XX]h]cbU``XYhJ]`g.

7f]hYf]U'Zcf'UggYgga Ybh

.

a Uf_ <8	W f] W f]U
< 8	

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gh XYbhgʻUfYʻYl dYWYXʻhcʻdi hh\Y]fʻbUa YgʻUbXʻgh XYbhbi a VYfgʻcbʻYj YfmdU[YʻcZh\Y]fʻUgg][ba Ybhgʻ

Hi fb]h]b Gi Va]gg]cb

#mci YbWci bhYf'U'dfcV`Ya k \Yb'UhYa dh]b['rc'gi Va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]h]bž'd`YUgY
hY`Yd\cbY'9I hYfbU`Gi ddcfhcb'-',) "''' %cf'Ya U]`'h\Ya 'cb'<u>YI hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui"</u>"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft!" #mci
UfY'i bUV`Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi `hk]h\ 'Hi fb]h]b'mci 'a UmUdd`m'Zcf'Ub'YI hYbg]cbž'Vi hmci
a i ghfYhU]b'mci f'h]W_Yhbi a VYf'Zfca '9I hYfbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fY'Yj UbhXcW a Ybhgt'hc
]dU XYb

- K cf_'gi Va]hh'X`%\$`hc`% 'XUmg`UZhYf`h\Y`gh]di `UhYX`XYUX`]bY`k]```VY`UggYggYX`UbX`ZYYXVUW_dfcj]XYX`Vi h'U`a Uf_'cZnYfc`k]```VY`fYWcfXYX"`\Zh\Y`k cf_'k ci `X`\Uj Y`fYWf]j YX`U'dUgg`a Uf_'Vi h'Zcf h\Y``UhYbYgg`UbX`h\Y`k cf_']g`U`Wca di `gcfm'Wci fgY`Wca dcbYbh'f\i fX`Y`fYei]fYa Ybh\ž`U'gh XYbhk]``VY`XYYa YX`hc`\Uj Y`a Yhh\Uh'fYei]fYa Ybh\
- K cf_'gi Va]hhYX`&\$`cf`a cfY`XUmg`UZhYf`h\Y`gh]di `UhYX`XYUX`]bY`k]```bchVY`UWWdhYX`Zcf UggYgga YbhUbX`k]```fYWY]j Y`bc`ZYYXVUW_ža Uf_'cf`[fUXY"'=Zh\Y`UggYgga YbhHUg_']g`U`Wca di `gcfm Wca dcbYbhcZh\Y`Wci fgY`U'ghi XYbhik]```fYWY]j Y`Ub`l bgUh]gZJWcfmi: U]``fl : Ł`[fUXY`Ug`U`fYgi `hcZ i bgUh]gZJWcfmidYfZcfa UbWY`]b`Ub`YggYbh]U``Wca dcbYbhcZh\Y`Wci fgY"

H\]g^]bZcfa Uh]cb^]g^U^gc^Uj U]^\$

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5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'=hWJb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhU`mWcdm]b['Zfca `U`gci fWY'k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b[WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk'k cfXg'UbX'd\fUgYg'k\]Y 'a cghmfYhU]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU`žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'UbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk'k \c`Yžk]h\ci h'Uddfcdf]UhY'fYZYfYbW]b['"

7 c``i g]cb. k cf_jb[k]h ch\Yfg'Vi hdUgg]b[cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['Uh'Ubmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_žghYU`]b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d'Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[ˈgci fWYg'k\]WX`\Uj Y`bchVYYb`fYUXžk]h\ci hUW_bck`YX[]b[`h\Y`″gYWcbXUfm″gci fWY`Zfca 'k\]WX`_bck`YX[Y`cZh\Ya`\Ug'VYYb`cVhU]bYX"

8 i d`]\WUh]cb`ffgY`Zd`U[]\Uf]ga "Ł gi Va]\h]b[`mci f`ck b`k cf_ž]b`k \c`Y`cf`]b`dUfhžk \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\hYX`Zcf`Ubch\Yf`UggYgga Ybhcf\Wci fgY`UhI BGK `cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWhfYZYfYbWb[dfUWhWyg.

1		7 f]hjWjga "UbX"A U^cf"5Ygh\YhjWG\]Zhg
	GYa]bUf	Mci f'X]gW]d`]bY`UbX`h\Y`fc`Y`cZh\Y`Wf]h]W
6fYU_,`&(`GYdhYa VYf'! '\$`GYdhYa VYf	FYUX]b[
K YY_'%\$.`%`C W1cVYf'!`+ C W1cVYf	@YWhifY	@YWhifYf. '9f]b '6fUbb][Ub
		D\]`cgcd\]Yg`cZ5Ygh\Yh]Wg
	GYa]bUf	<]ghcf]\W)``a ca Ybhg`cZ]a dcfhUb\W``UbX`h\Y`fc`Y`cZ \Vf]h]\Wga
K YY_"%%", "C WcVYf"!"%(@YWn fY	C b 'h\ Y 'Ufh]gh Wf]h]W
C Whc VY f	GYa]bUf	K f]h]b['Zcf'U'dYYf
K YY_'%&.'%) 'CWncVYf'! &%'CWncVYf	@YWhifY	@YWi fYf. 9f]b 6fUbb][Ub
		C b 'h\ Y ']bhYfj]Yk #dfYj]Yk
	GYa]bUf	DfYj]Yk 'F YgYUfW\
K YY_'%'.'&&'CWncVYf'! &, 'CWncVYf	GYa]bUf	C bY!cb!cbY'dcfhZc`]c'Wcbgi `HJh]cb