

# **ARTS3123**

Gc`c'DYfZcfa UbWY'A U\_]b[

GYa YghYf'Hk c''#'&\$%

## 7ci fgY Cj Yfj ]Yk

#### GHJZZ7cbHJWh8YHJ]`q`

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D\cbY
6fmcb] HfYn]gY	V"lfYn]gY4 i bgk "YXi "Ui	D`YUgY'Ya U]`'Z[fgh	K YVghYf	- ' , ) ( ) %'
		Zcf'Ub'Uddc]bha Ybh	6i ]`X]b[ <sup>·</sup> %%+	

#### GW.cc`7cbhJWh =bZcfa Uhlcb

Fcca " %&ž"Yj Y" FcVYfhK YVqhYf'6i ]X]b[

D\cbY. f\$&Ł'-',) '(,)\*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVq]hY. \hada.#qUa "Ufhq"i bgk "YXi "Ui\_

H\Y'GW\cc``cZh\Y'5fhg'UbX'A YX]U'k ci `X'`]\_Y'hc'F YgdYWMZ ``m'5W\_bck `YX[Y'h\Y'HfUX]h]cbU``7i qhcX]Ubqž h\Y'6YXY[U`fPYbq]b[hcb'WJa di qŁž; UX][U`f7]hmUbX'5fh/ '8Yq][b'7Ua di qYqŁ'UbX'h\Y'B[i bbUk U` dYcdYffuEcZh\Y "UbXgk\YfYYUW\ WJa digcZl BGK ]g `cWJhYX"

## 5 h Y b X U b W F Y e i ] f Y a Y b h g

5 qh XYbh]q Yl dYWYX hc UhYbX U``WUqq WcbhUWi\ci fq Zcf U ZUW!hc! ZUW! fl & L'cf V YbXYX Wci fqY UbX Wca d`YhY'U```UWhjj]hjYg'Zcf'U'V`YbXYX'cf'Zl ``micb`]bY'Wci fgY"

5 'gh XYbhk\c'Uff]j Yg'a cfY'h\Ub'%) 'a ]bi hYg'`UhY'a UmVY'dYbU`]gYX'Zcf'bcb!UhhYbXUbWY"'-Zgi W\'U'dYbU`hm ]g^]a dcgYXžh\Y^gh XYbha i ghVY^]bZcfa YX^j YfVU``mUhh\Y^YbX`cZWUgg^UbX`UXj ]gYX^]b^k f]h|b[^k]h\]b^&( \ci fg"

ZUʻqh XYbhYl dYf[YbWYq']`bYqqža ]qUXj Ybh fYʻcfʻch\YfʻcVW ffYbW h\Uha U\_YqʻUVqYbW Zfca ʻU Wugg#JWnjj ]mmi buj c]XUV`Yžcf`Yl dYWng hc VY`UVgYbhZfca `U`Zcfh\Wca ]b[ `Wugg#JWnjj ]mmzh\Ymg\ci `X`gYY\_ dYfa ]gg]cb Zfca 'h\Y'7 ci fgY'5i h\cf]mž'UbX'k \YfY'Udd`]WUV`Yž'h\Y]f'fYei Yghg\ci `X'VY'UWWca dUb]YX'VmUb cf][]bU`cf`WfflZYX`WcdmcZU'a YX]WU`WYfhlZWJhY`cf`ch\Yf`Zcfa 'cZUddfcdf]UhY`Yj ]XYbWY"

5 7 ci fqY 5 i h\cf|lma UmYl W qY U qhi XYbh2fca WUqqYq cf UWlij |h|Yq Zcf i d hc cbY a cbh\" < ck Y j Yfz h\Ym a Um'Ugg][b'UXX]hcbU`UbX#cf'U`hYfbUh]j Y'hUg\_g'hc'Ybgi fY'Wca d`]UbWY"'5'7 ci fgY'5i h\cf]miWcbg]XYf]b['h\Y [fubh]b[cZuvgYbWraighvYguh]gZjYX'u'gh XYbhk]```gh]```vY'uv`Y'hc'a YYhh\Y'WcifgY'g``Yufb]b[cihWcaYg UbX#cf'j c'i a Y'cZ'YUfb]b["'5 'gh XYbhgYY\_]b['Uddfcj U'hc'VY'UVgYbhZcf'a cfY'h\Ub'cbY'a cbh\'a i qhUdd`m ]b'k f]h]b['hc'h\Y'8YUb'UbX'dfcj]XY'U```cf][]bU``cf'Wfh]ZYX'gi ddcfh]b['XcW a YbhUh]cb"

: cf'a cfY']bZcfa Uf]cb'UVci hih\Y'UffYbXUbWY'dfcfcWc`q']b'h\Y': UW'\micZ5ffq'UbX'GcWJU\ GWYbWrg. \httq.#k k k "Uffig"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa ]W]bZcfa Uhjcb#dfchcWc`g![i ]XY`]bYg#

5 WUXYa ]W = b Zcfa Uhich

: cf`YggYbhJU``ghi XYbh`]bZcfa UhJcb`fY`UhJb[`hc.`fYei Yghg`Zcf`Yl hYbg]cb/fYj ]Yk `cZa Uf\_g/cWW dUhJcbU``\YU`h\UbX`gUZ/hm/Yl Ua ]bUhJcb'dfcWXi fYg/gdYVJU``Wcbg]XYfUhJcb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a ]gUXj Ybhi fY/ghi XYbhYei ]hm'UbX`X]gUV]`]hm'UbX`ch\Yf`YggYbhJU``UWUXYa ]W]bZcfa UhJcbž gYY`\hdg.#k k k "Ufhg"i bgk "YXi "Ui #W ffYbhghi XYbhg#JWUXYa ]W]bZcfa UhJcb#dfchcWc`g![i ]XY`]bYg#

# 7ci fgY 8YHJ] g

7 fYX]hDc]bhg\*\*

Gi a a UfmcZh\Y 7ci fgY

Gi VYWh5fYU. FZWSfdWS` V BWdAd\_S`UWEfgV[We

7 U``]b['U```dYfZcfa UbW'`a U\_Yfg``Gc`c`DYfZcfa UbW'`]g'U'dfUWJWY!VUgYX`Wci fgY'h\Uhih\Uhi[]j Yg'mci 'h\Y cddcfhi b]mhc`XYj Y`cd'U') 'a ]bi hY 'gc`c'dYfZcfa UbW'`Zcf'di V`]WdfYgYbhUhJcb"'9UW\`k YY\_`mk cf\_g\cd'k ]``dfcj ]XY 'mci 'k ]h\ 'hYW\b]ei Yg'Zcf'WYUhJb['mci f'ck b'dYfZcfa UbW'`k cf\_'k \]'Y 'XfUk ]b['i dcb'WcbhYa dcfUfm hfUX]hJcbg']b' dYfZcfa UbW'`UfhUbX'h\Y'\]ghcf]WU``Uj UbhI[UfXY"'Nci 'k]``VY'YI dcgYX'nc'U'k ]XY 'fUb[Y'cZ UfhJgh]WdYfZcfa UbW'`dfUWJW'gž'WYUhJj Y'dfcW'ggYg'UbX'\]ghcf]WU``UbX'h\YcfYhJWD``WcbhYI hg'Zcf'h\Y'a U\_]b[cZmci f'ck b'k cf\_"'Nci 'k]``VY']b'WcbghUbhdfUWJWD``a cXYžk\]W\Zk\Yb'mci 'UfY'bcha U\_]b['mci fgY'Zk']``ffU]b'mci ']b'h\Y'fYgdcbg]j Y'g\_]``g'cZXfUa Uhi f[]WD'`ZYYXVUW\_"'Gc`c'DYfZcfa UbW''cZYfg'U'a cfY'ZcW' gYX'UbX dfUWJWD``m]bhYbg]j Y'a cXY'h\Ub 'ZfghUbX'gYwbX'mYUf'Wci fgYg''Nci 'k]```ZJbX'YI WJhJb['Wcggcj Yfg'Yhk YYbh\Y'dfUWJWD``n bXYf[cbY']b'h\]g'Wci fgYz'UbX'h\Y']XYUg'UbX'h\Ycf]Yg'UddfcUw\XYX']b'fY'UhX'Wci fgYg']b'h\Y H\YUhfY'UbX'DYfZcfa UbW''Ghi X]Yg'a U'cf"'H\]g'Wci fgY']g'Yei U``mXYg][bYX'Zcf'h\cgY'k]g\]b['hc'YI hYbX'h\Y]f'dYfZcfa UbW'''g\_]``gz'hc'XYj Y'cd'U'WJfYYf'Ug'Ub'UfhJghicf'hc'i gY'Ya VcX]YX``YUfb]b['hc'h\]b\_'h\fci [\'UbX'YI dfYgg'Wf]hJWD''ei YghJcbg'UbX'wca d'YI']XYUg'''H\]g'Wci fgY']g'dUfhcZh\Y'H\YUhfY'UbX'DYfZcfa UbW''Ghi X]Yg'g'hfYUa'''-

#### 5 hm Y WcbWi g]cb cZm ]g Wci fgY m Y ghi XYbh k ] ``VY UV Y hc

- %" gmbh\Yg]gY`Wca d`YI`a UhYf]U`g`Zfca `fYUX]b[gž`WUgg`YIYfW]gYgž`dYYfg`UbX`gY`ZIXYjY`cda Ybh`]bhc`U Wc\Yg]jY`)`a ]bi hY`gY`ZIXYj]gYX`dYfZcfa UbWY`Zcf`di V`]WdfYgYbhUh]cb
- &" i bXYfgHJbX`UbX`UfhJWi`UhYfVch\`]b`k f]hJb[`UbX`]b`dYfZcfa UbWYŁ'h\Y`bUhi fY`cZXfUa Uhi f[]WJ```Ub[i U[Y`k\]W]]g`]b`h\Y`dfcWYgg`cZVY]b[`XYj Y`cdYX`Zcf`di V`]Wg\ck]b[
- '"YZZYWNjiY`mWf]njeiY`n\Y`kcf\_`cZdYYfg`Ug`U`kUmcZcZZYf]b[`XfUaUhif[]WU``giddcfh
- ("ibXYfgHJbX'h\Y'bYWYgg]hmcZWcbh]biYX'UhhYadhgžWcadcg]h]cbU``ghfUhY[]Yg'UbX'fi]jYfidYfZcfaUbWYdfUhY[]Yg'UbX'fi]jYfidYfZcfaUbWY`Ub[iU[Y'cZcbYfgickb
- ) " fYZYWii dcb 'UbX 'Uff]W `UhY 'UVci hih\Y 'fY`Uh]cbg\]d 'VYlk YYb 'dfcWYgg 'UbX 'h\YcfYh]WU`ž'\]ghcf]WU``cf WcbhYl hi U``]XYUg 'UbX 'h\Y'dYfZcfa UbWY 'hYl h]hgY`Z
- \*"g\ck:\WJdUWJmiZcf\Z\fh\Yf\XYj\Y\cda\YbhcZh\Y]f\dYfZcfa\Ub\W\k\cf\_z\cf\cZh\Y]f\ck\b\g\_]\g\ub\cf\_Yfg\k\jh\jb\h\Y\dYfZcfa\Ub\W\Z\X

#### HYUWX]b[ GlfUlY[]Yg

HYUWX]b[ 'GhfUhY[]Yq

±0 'VY]b[ 'df]a Uf]`mdfUWf)WU`žth.]g'Wci fgY YbWci fU[ Yg'ghi XYbhg'hc'Ugg]a ]`UhY 'Wca d`YI 'XfUa Uhi f[ ]WU` UddfcUW.Ygžth.YcfYh]WU``dcg]h]cbg'UbX'UYgh.Yh]WXYW]g]cbg']bhc'3⁄23'UbX'h.fci [ \'3⁄23'U'`]j Y'dYfZcfa UbW k cf\_"'H\Y'Wci fgY'cZYfg'Ya VcX]YXžYI dYf]Ybh]U```YUfb]b[ 'Wca dcbYbhg'h\UhfYZYWh'U'h\YcfYh]WU``dcg]h]cb"'5h

 $\label{thm:condition} $$H^{g''}W' = G'' - G''$ 

# 5ggYgga Ybh

D`YUgY`gYY`Wci fgY`A ccX`Y`Zcf`XYHJ]`YX`]bZcfa Uh]cb`cb`UggYgga Ybhg"

## 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'hUg_	K Y][ \h	8i Y'8UHY	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX
7`Ugg'7f]hjei Yg'	&\$ı	%\$#\$-#&\$% `\$).\$\$`DA	&Ž Ž( Ž)
8YjY`cd]b['dfUWjVV'	&) ı	K YY_g`&!+ž'-	%ž&ž( ž)
A U'cf'dYfZcfa UbW	' \$ı	&*#%\$#&\$% `\$+.\$\$`DA	%ž&ž( ž*
FYZYW¶jY'5bU`mg]g'	&) ı	\$) #%#&\$% `\$) .\$\$`DA	&ž( ž) ž*

## 5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh'% '7`Ugg'7f]hjei Yg

GhUfhXUhY. Bch5dd`]WUV`Y

**8YHJ]**`g. '%\$\$\$`k cfXg"K f]hhYb`Wca a Ybhg`UbX`fi Vf]Wk ]```VY`dfcj ]XYX`cb`Hi fb]hJb"

5ggYgga Ybh& 8Yj Y`cd]b[ dfUWf]WY

GhUfh'XUhY.

 $8YHJ)^*g. "!) "a"] bi hY "fYgdcbgYg" hc" k YY_`mHJg_g" UbX "dYfZcfa UbW" YI YfW]gYg/cb[c]b[ "k cf_"6f]YZk f] hY b Wca a Ybhg" UbX "Zcfa Uh]j Y "[ fUXY k ]`` VY "dfcj ]XYX "VmhYUWX Yfg" cb" U "k YY_`mVUg]g" "Gi a a Uh]j Y "[ fUXY Uj YfU[ Y "cZVYghH\fYY "dfcj ]XYX"$ 

5ggYgga Ybh' . A U'cf'dYfZcfa UbWY

GHJfh:XUhY.

8YHJ]`g. ') 'a ]bi hYg"K f]hYb Wca a Ybhg UbX fi Vf]Wk ]``VY dfcj ]XYX cb Hi fb]hJb"

5ggYgga Ybh(. FYZYWhjj Y 5bU`mg]g

GHJfh'XUhY.

**8YHJ]**`g. '%\$\$'k cfXg"'H\]g']g'h\Y'Z|bU``UggYgga Ybh'HUg\_"Fi Vf]Wk]``VY'dfcj ]XYX'cb'Hi fb]hJb"

#### Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Gh XYbhg UfY Yl dYWYX hc di hh\Y]f bUa Yg UbX gh XYbhbi a VYfg cb Yj YfmdU[Y cZh\Y]f Ugg][ ba Ybhg"

#### Hi fb]h]b Gi Va ]gg]cb

#mci YbWci bhYf'U'dfcV'Ya k \Yb'UhYa dhJo['hc'gi Va ]hmci f'Ugg][ ba Ybhh\fci [\'Hi fb]h]bž'd'YUgY
hY'Yd\cbY'9l hYfbU'Gi ddcfhcb'-', ) "''' %cf'Ya U]'h\Ya 'cb'<u>Yl hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui"</u>"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUfŁ"'#mci
UfY'i bUV'Y'hc'gi Va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Yl hYbg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9l hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW a YbhgŁ'hc
]bWi XY'Ug'Yj ]XYbW'hc'gi ddcfhmci f'Yl hYbg]cb'Udd']WUh]cb"'#mci 'Ya U]''9l hYfbU''Gi ddcfhmci 'k]''
Ui hca Uh]WU''mfYWf']j Y'U'h]W\_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'''bYYX'hc'gdYW]ZJWU''mUg\_'Zcf'cbY"
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhitUg\_g'a i ghVY'gi Va ]hhYX'Y`YWfcb]WU``mj ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh'' =b']bghUbWfg'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'ghUhYX'cb'mci f'Wci fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYhU]`g''

@UhY 5 ggYgga YbhDYbU`hJYg

- K cf\_'gi Va ]hh'X`%\$`hc`% 'XUmg`UZhYf`h\Y`gh]di `UhYX`XYUX`]bY`k]```VY`UggYggYX`UbX`ZYYXVUW\_dfcj]XYX`Vi h'U`a Uf\_'cZnYfc`k]```VY`fYWcfXYX"`\Zh\Y`k cf\_'k ci `X`\Uj Y`fYWf]j YX`U'dUgg`a Uf\_'Vi h'Zcf h\Y``UhYbYgg`UbX`h\Y`k cf\_']g`U`Wca di `gcfm'Wci fgY`Wca dcbYbh'f\i fX`Y`fYei ]fYa Ybh\z`U'gh XYbhk]``VY`XYYa YX`hc`\Uj Y`a Yhih\Uh'fYei ]fYa Ybh\
- K cf\_'gi Va ]hhYX`&\$`cf`a cfY`XUmg`UZhYf`h\Y`gh]di `UhYX`XYUX`]bY`k ]```bchVY`UWWdhYX`Zcf UggYgga YbhUbX`k ]```fYWY]j Y`bc`ZYYXVUW\_ža Uf\_'cf`[ fUXY"'=Zh\Y`UggYgga YbhHUg\_']g`U`Wca di `gcfm Wca dcbYbhcZh\Y`Wci fgY`U'ghi XYbhik ]```fYWY]j Y`Ub`l bgUh]gZJWcfmi: U]``fl : Ł`[ fUXY`Ug`U`fYgi `hcZ i bgUh]gZJWcfmidYfZcfa UbWY`]b`Ub`YggYbh]U``Wca dcbYbhcZh\Y`Wci fgY"

H\]g^]bZcfa Uh]cb^]g^U^gc^Uj U]^\$

**‰**fm

**‰** ‰

‰

## 5 WUXYa ]W<cbYghmUbX D`U[ ]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"'\lefi\Wb'hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UVW]XYbhU`mWcdm]b[ 'Zfca `U`gci f\W' k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidgdu[]Uf]ga ]bhch\YZc``ck]b[ WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b[ 'U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b[ 'h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k ]h\ci hUW\_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci hWfYX]hUbX'hc d]YW]b[ 'hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b[ "

 $7c^{i}$  g]cb. 'k cf\_]b[ 'k ]th. 'cth. Yfg'Vi h'dUgg]b[ 'c\Zth. Y'k cf\_'Ug'U'dYfgcb'g']bX]j ]Xi U''k cf\_"'7c''i g]cb'U'gc

 $i\ bXYfgfUbX`mci\ f`f][\ \ \ bX`fYgdcbg]V]`]fJYg`Ug`U`gfi\ XYbfiUfi]\ BGK$ 

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1		