

MDIA2002

5bU`mg]b['A YX]U'7ca a i b]WUh]cb

GYa YghYf'Hk c''#'&\$%

7ci fgY'Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g`

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUhjcb	D\cbY
GmYX'NU]X]	UVVUg"nU]X]4 i bgk "YXi "Ui	B <i>#</i> 5	KYVghYf" ‰	B#5

'Hi hcfg'

B Ua Y	9a U]`	5j U]`UV]`] m	(ar V/V/Inch	D\cbY
A UfmUa '5`Uj]'B]U	UVVUg"nU]X]4 i bgk "YXi "Ui	B#5	B#5	B <i>#</i> 5

GW.cc`7cbhJWh =bZcfa Uhlcb

Fcca " %&ž~Yj Y~" FcVYfhK YVghYf6i]X]b[

D\cbY. f\$&Ł'-',) '(,) *

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVq]hY. '\hda.#qUa "Ufha"i bak "YXi "Ui

H\Y`GW.cc``cZh\Y`5ffg`UbX`A YX]U`k ci`X``]_Y`fc`F YgdYWfI```m5W_bck``YX[Y`f\Y`HfUX]fjcbU``7i gfcX]Ubgž h\Y`6YXY[U``ff\Ybg]b[fcb`WJa di głž; UX][U``ff\]hmUbX`5fh\Z``8Yg][b`7 Ua di gYgŁ`UbX`f\Y`B[i bbUk U` dYcd`Y`fbi gffU`]Ub`8YZYbW\': cf\W`5\WJXYa m]b`7 UbVYffUŁ'cZh\Y``UbXg`k\YfY`YUW\`WJa di g`cZl BGK `]g`c\WJhYX"

5 hhyb XUb WY F Yei] fYa Ybhg

5 'ghi XYbh']g'Yl dYWYX'hc'UhhYbX'U```WUgg'WcbhUWi\ci fg'Zcf'U'ZUWY!hc!ZUWY'fl &: Ł'cf'V`YbXYX'Wci fgY'UbX Wca d`YhY'U```UWYj]h]Yg'Zcf'U'VYbXYX'cf'Z ``mcb`]bY'Wci fgY"

5 'ghi XYbhik \c'Uff]j Yg'a cfY'h\Ub'%) 'a]bi hYg'``UhY'a UmVY'dYbU`]gYX'Zcf'bcb!UhhYbXUbWY'''=Zgi W\'U'dYbU`hm]g']a dcgYXž'h\Y'ghi XYbha i gh'VY']bZcfa YX'j YfVU``mUh'h\Y'YbX'cZWUgg'UbX'UXj]gYX']b'k f]h]b['k]h\]b'&(\ci fg"

₹Uʻghi XYbh'Yl dYf]YbWʻgʻ]``bYggža]gUXj Ybhi fYʻcfʻch\YfʻcVW ffYbW h\Uha U_YgʻUVgYbWʻZfca ʻU WUgg#UWfj]hmi bUj c]XUV`YžcfʻYl dYWgʻhcʻVYʻUVgYbhZfca ʻUʻZcfh\Wca]b[ˈWUgg#UWfj]hmžh\Ymg\ci `XʻgYY_ dYfa]gg]cbʻZfca ʻh\Yʻ7 ci fgYʻ5i h\cf]hmžʻUbXʻk \YfYʻUdd`]WUV`Yžh\Y]fʻfYei Yghg\ci `X'VYʻUVWca dUb]YXʻVmUb cf][]bUʻcfʻWffjZJYX`WcdmcZUʻa YX]WU``WYffjZJWUhYʻcfʻch\YfʻZcfa ʻcZUddfcdf]UhYʻYj]XYbWY"

5 '7 ci fgY '5 i h\cf]hma UmYl W gY 'U'ghi XYbhiZica 'WUggYg'cf'UWjj]hjYg'Zcf'i d'hc'cbY'a cbh\"'<ck Yj Yfž'h\Ym a UmUgg][b'UXX]hjcbU``UbX#cf'U'hYfbUhj Y'HUg_g'hc'Ybgi fY'Wca d`]UbW"'5 '7 ci fgY'5 i h\cf]hmWcbg]XYf]b['h\Y [fUbh]b['cZUVgYbW'a i gh'VY'gUhjgZ]YX'U'ghi XYbhk]``'gh]``'VY'UV'Y'hc'a YYhh\Y'Wci fgY'g'`YUfb]b['ci hWca Yg UbX#cf'j c'i a Y'cZ'YUfb]b["'5 'ghi XYbhigYY_]b['Uddfcj U`'hc'VY'UVgYbhiZcf'a cfY'h\Ub'cbY'a cbh\'a i gh'Udd'm]b'k f]hjb['hc'h\Y'8YUb'UbX'dfcj]XY'U``cf][]bU`'cf'WYfljZ]YX'gi ddcfh]b['XcW a YbhUhjcb"

: cf'a cfY']bZcfa Uh]cb'UVci hih\Y'UhhYbXUbWY'dfchcWc`g']b'h\Y': UW\`hmcZ5fhg'UbX'GcW]U\ GWYbWYg.\\hdg.#k k k "Ufhg"i bak "YXi "Ui #W ffYbhqh XYbhg#UWUXYa]W]bZcfa Uh]cb#dfchcWc`a![i]XY`]bYg#

5 WUXYa]W = DZcfa Uhjcb

: cf`YggYbhJU``ghi XYbh`]bZcfa UhJcb`fY`UhJb['hc.`fYei Yghg`Zcf`YI hYbg]cb/fYj]Yk 'cZa Uf_g/cWW dUhJcbU``\YU`h\UbX`gUZYhm'YI Ua]bUhJcb'dfcWXi fYg/gdYWJU``Wcbg]XYfUhJcb`]b'h\Y'Yj YbhcZ]``bYgg`cf`a]gUXj Ybhi fY/ghi XYbhYei]hmUbX`X]gUV]`]hm/UbX`ch\Yf`YggYbhJU``UWUXYa]W]bZcfa UhJcbžgYY`

7ci fgY 8YW] g

7 fYX]hDc]bhg * *

Gi a a UfmcZh\Y 7ci fgY

Gi VYWh5fYU. ? W/S

H\]g`Wci fgY`dfcj]XYg`ghi XYbhg`k]h\`\][\!`Yj Y``UbU`mh]WU``g_]``g`]b`k f]hhYbžj]gi U``UbX`a i `h]!a cXU` Wca a i b]WUh]cb"K Y`k]``XYj Y`cd`U`gYhcZUbU`mh]WU``hcc`g`k \]W\`Yl d`cfY`]ggi Yg`cZa YUb]b[`]b`fY`Yj Ubh dfcZYgg]cbU``hYl hg"`A YUb]b[g`k]```VY`Yl d`cfYX`]b`hYfa g`cZ]ggi Yg`cZWfYUh]b[`WcbhYbhž`Wcbghfi Wh]b[`]XYbh]hm UbX`fY`Uh]cbgž'UbX`WfYUh]b[`Wc\Yg]j Y`UbX`Wc\YfYbhihYl hg"`GdYW]Z]WUbU`mh]WU``hcc`g`k]```VY`XYj Y`cdYX`k\]W\ ZcW g`cb`]ggi Yg`cZ[fUa a UfžX]gWci fgYžfY[]ghYfž'UbX`[YbfY"'H\YgY`hcc`g`k]```VY`i gYX`hc`]bhYffc[UhY`UbX Wf]h]ei Y`hYl hg`]b`fY`Uh]cb`hc`h\Y]f`gcW]U``UbX`dfcZYgg]cbU``WcbhYl hgž'UbX`hc`]bZcfa `ghi XYbhgfidfcXi Wh]cb`cZ h\Y]f`ck b`hYl hg`]b`dfcZYgg]cbU``Xca U]bg"

5 hh Y WcbWi g]cb cZh]g Wci fgY h Y gh XYbhk]``VY UV Y hc

- %" 5dd`mU'j Uf]YhmcZhcc`g`hc`Yb\UbWY'h\Y]f`dfcXi Wh]cbž'UbU`mg]g'UbX'Yj U`i Uh]cb`cZ_Ym Wca a i b]WUh]cb'UbX'`ci fbU`]ga 'hYl hg
- &" D`UWY`Wca a i b]\WUnjcb`UbX`ci fbU`]ga `dfUWnj\WYg`k]h\]b`fY`Yj Ubhgc\WjU`ž`]bghjhi hjcbU``UbX`Yh\]\WU` ZfUa Yk cf_q
- "8YacbghfuhY Uddfcdf]uhY XYj Y`cda YbhcZh\Y`g_]``g`cZgV\\c`Uf`mYbei]fm

HYUWX]b[GhfUhY[]Yg

FUNcbUY.

H\]g`Wci fgY`Wcj Yfg`U`fUb[Y`cZa YX]U`HYI himdYg`]b`cfXYf`hc`dfcj]XY`ghi XYbhg`k]h\`U`gc`]X`Zci bXUh]cb`Zcf Zi fh\Yf`ghi XmUbX`dfcZygg]cbU``dfUWf]WV"'H\Y`Wci fgY`ZcW gYg`cb`Vch\`dfUWf]WV``UbX`UbU`mh]WU``g_]``g`hc YbUV`Y`ghi XYbhg`hc`dfcXi WV`UbX`Wf]h]WU``mYj U`i UhY`a YX]U`dfcXi Wf]cb"'H\Y`Wci fgY`]bWi XYg`h\YcfYh]WU` WcbWYdhg`hc`YbUV`Y`ghi XYbhg`hc`Yb[U[Y`dfc!UWfj Y`mUbX`WfYUhjj Y`m]b`U`fUb[Y`cZdfcZYgg]cbU``WcbHYI hgž Yei |dd|b[`h\Ya 'hc`VY`dchYbh]U```YUXYfq`]b`h\Y]f`Z]Y`X

HYUW]b[GhfUhY[]Yq.

 $H\ [g`WcifgY`igYg`ZJWY!hc!ZJWY`WUggfcca`hYUWX]b[žk]h\ `%I`-\$`a]bihY``YWhifY`UbX`%I`-\$`a]bihY`hihcf]U`YUWX`kYY_ž`]bWiX]b[`Wc``UVcfUh]jY`UbX`]bX]j]XiU``kcf_"`GhiXYbhg`dfYdUfY`fYUX]b[`gžX]gW gg]cb`dc]bhg UbX`YIYfW]gYg`]b`UXjUbWY`cZYUWX`hihcf]U``": cf`h\Y`acghdUfhžaUhYf]U`g`fYei]fYX`Zcf`hihcf]U``dfYdUfUh]cb UbX`Zcf`UggYggaYbhHUg_g`k]```VY`aUXY`UjU]`UV`Y`j]U`h\Y`WcifgY³&3g`AccX`Y`kYVg]hY"$

5ggYgga Ybh

5ggYgga Ybh& J]Yk g! ci fbU`]ga 'Wca dUf]gcb'd]YW

fH\]g`UggYgga YbhihUg_`]g`U`fYei]fYX`Y`Ya YbhicZh\Y`Wci fgY"`\Zmci `ZJ]``hc`gi Va]hih\]g`k cf_'!mci `k]```ZJ]``h\Y Wci fgY"Ł

DUfh'%

DUfh'&'!'Ufh]WY'f% \$\$'!'&\$\$\$'k cfXgŁ

⊋mci 'UfY'i bUV`Y'hc'Wca d`YhY'h\Y'dfYdUfUh]cb'k cf_'Zcf'U'[]j Yb'k YY_'cb'UWWci bhcZU'XcW a YbhYX']`bYgg fbf'ch\Yf'UWWdhUV`Y'fYUgcbŁ'h\Yb'mci '\Uj Y'U'Z fh\Yf'gYj Yb'XUmg'hc'Wca d`YhY'h\Y'hUg_'UbX'hc'gi Va]hhc A ccX`Y"'C Vj]ci g`mmci 'a i gh']bZcfa 'mci f'h hcf'cZmci f']`bYgg'UbX'gi dd`mh\Y'fYei]fYX'XcW a YbhUh]cb'Vm Ya U]`'hc'mci f'Wci fgY'Wcbj YbYf"'fa X]U&\$\$&4 i bgk Va YX]U"cf[Ł

D`YUgY`bchY`U`[Ybi]bY`YZZcfh]g`fYei]fYX`Zcfkcf_hc`WcibhUg`U`giVa]gg]cb"`G]a]`UfmžU```eiYgh]cbg#Ug_g Zcf'U`[]jYb`kYY_aighVY`Wcad`YhYX"`Mci`aighWcad`YhY`h\Y`kcf_cb`Ub`]bX]j]XiU``VUg]g`!`]"Y" Wc``UVcfUhj;Y# fcid`giVa]qq]cbg`UfY`bchdYfa]hhYX"

A Uf_]b[Wf]hYf]U

 $H\Y''HJg_fgL'gY'YWMYX'Zcf'a Uf_]b['k]``'fYWM']j Y'U'[fUXY'UWWcfX]b['hc'\ck'k Y``'h\Ym'a YYh'h\Y'Zc``ck]b[Wf]hYf]U''$

%"XYa cbglfUhYg'U'gci bX'i bXYfgHJbX]b['cZfY`Yj Ubh'fYUX]b[gžgYa]bUf'bchYg'UbX#cf'a UhYf]U`'dfYgYbhYX']b 'YWfi fYg"

 $\& "`Ubgk Yfg\#X]gW gg]cb`dc]bhg`UfY`k Y```Wcbg]XYfYX`UbX`]bg][\https://doi.org/10.1016/10.016/10.016$

' "'UbU`m\\WJ``ZfUa Yk cf_g`UfY`UWW fUhY`mUbX`Uddfcdf\UhY`mUdd`\YX

("'Ubgk Yfg'dfcj]XYX']b'[fUa a Uh]WJ'ž'k Y``'YI dfYggYX'`Ub[i U[Y

) "Zfyy czgdy"]b[ždi bWi Uhjcb Ubx ch/yf dfygybłUhjcb yffcfg

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

±Vci 'g\ci `X`k f]hY`a cfY`]b`Ub`ĉi fbU`]gh]Wh\Ub`Ub`UWUXYa]Wghm`Y`fgYY`Yl Ua d`Y`Ufh]WYg`]b`h\]g`A ccX`Y dUbY``Zcf`[i]XUbW']b`h\]g"Ł

+b 'lh]g'd]YWY mci ''Wca dUfY'UbX'WcblfUghik c'j]Yk g! 'ci fbU']ga ž'cd]b]cb'd]YWYg'cb'h\Y'gUa Y hcd]W#]ggi Y#XYVUhY'''flC Vj]ci g`mh\Yma i gh'VY'Zfca 'lh\Y'[YbYfU`'gi VYWh'UfYU'mci 'gY`YWh'X'Ug'mci f "fci bX"'''Ł'+b 'lh\]g'd]YWY'mci 'a i ghdfcj]XY'WcbWi g]cbgžVUW_YX'i d'VmYj]XYbWY'Zfca 'lh\Y'hYl hg'h\Ya gY`j Ygž Ug'hc'k\Yh\Yf'h\Y'd]YWYg'UfY'g]a]'Uf'cf'X]ZZYfYbhk]h\ fYgdYWh'nc'gi W\']ggi Yg'Ug'h\Y]f'ghm`Yž'h\Y'hmdYg'cZ Uf[i a YbhUh]j Y'gi ddcfh'Ya d'cm'Xž'h\Y'Uggi a dh]cbg'a UXY'UVci hh\Y'Ui X]YbWYfg'j U'i Yg'UbX'VY`]YZgž`]_Y`m YZYWMj YbYggž'UbX'gc'cb''fl\ cfY'XYhU]`'cb'h\]g'dfcj]XYX'VY`ck "Ł

H\Y`hk c`d]YWYg`mci `WkccgY`hc`Wca dUfY`UbX`WcbhfUghiWUb`Y]h\Yf`hU_Y`cddcg]b[`dcg]h]cbg`cb`h\Y`]ggi Y i bXYf`Wcbg]XYfUh]cb`fZcf`Yl Ua d`Y`Zcf`j Yfgi g`U[U]bgh`WkUb[Yg`hc`h\Y`a Uff]U[Y``Uk g/gi ddcfh]j Y`cf`Wf]h]WU`cZh\Y``cW_!ci h`Uk gžYhW`YhW'Ecfž'U`hYfbUh]j Y`miYggYbh]U``mh\Y`gUa Y`dcg]h]cb`fY"["'h\Yma][\h\Vch\`VY`]b ZUj ci f`cZVk\Ub[Yg`hc`h\Y`a Uff]U[Y``Uk gž'cf`Vch\`Wf]h]WU``cZh\Y`W ffYbh``cW_!ci h`Uk gžYhW`YhW'E

: cfZ ```XYHJ]`g`cZk \Uh]g`fYei]fYXžgYY`h\Y`A ccX`Y`a UhYf]U``Zcf`h\]g`Wci fgY"

A Uf_]b['Wf]hYf]U

 $\label{eq:local_problem} \begin{picture}(20,0) \put(0,0){\line(1,0){100}} \put(0,0){\line(1,0){100$

%"'H\Y'k c'h/I kg'WkcgYb'Zcf'Wca duf]gcb'UfY'Uddfcdf]UhY'Zcf'h\Y''kUg_ž'UbX'fY`UhY'kc'h\Y'WkcgYb'"fci bX""

&"'H\Y'd]YWY'XYa cbghfUhYg'U'gci bX'_bck `YX[Y'cZh\Y'a Yh\cXg'cZ'UbU`mg]g'ci h]bYX']b'h\Y'`YWh fYgž fYUX]b[g'UbX'h hcf]U`gž'UbX'Ub'UV]`]hmhc'YZZYWh]j Y`mUdd`mh\YgY'a Yh\cXg"

' "'H\Y'd]YWY'WYUf`mgHJHYg'k\Uh'UfY'H\Y'g]a]`Uf]H]Yg'UbX#cf`X]ZZYfYbWYg'VYlk YYb'H\Y'lk c'HYl hg']b'HYfa g'cZ h\Y'hmdYg'cZei YgH]cbg'ci h]bYX']b'H\Y'HJg_`XYgWf]dH]cb'fgYY'UVcj YŁ"

("'7cbWig]cbg'Ug'hc'X]ZZYfYbWYg'UbX#cf'g]a]'Uf]h]Yg'UfY'Wcbj]bW]b[`mUbX'd`Uig]V`mVUW_YX'id'VmUddfcdf]UhY'Yj]XYbWY'Zfca'h\Y'hYl hg'h\YagYj Yg'ffYZYfYbWYg'hc'fY`YjUbhk cfXgž'd\fUgYg'cf'ch\Yf'ZYUhifYg cZh\YjhYl hc'UbX'VmYld`UbUh]cbg'Ug'hc'h\Y'dchYbh]U`'Wcaaib]WUh]j Y'YZZYWIg'cZh\YgY'fYZYfYbWY'hYlhiU`YYaYbhg"

) "'H\Y'd]YWY']g'k Y```k f]HYbž'k Y```ghfi Wi fYXžYUgmhc'fYUXž'k]h\'h\Y'X]gW gg]cb'cf[Ub]gYX'Ug'U'`c[]WU``UbX Wc\YfYbhgi WWgg]cb'cZWU]a g'UbX'gi ddcfh]b['Yj]XYbW!"

*"'H\Y`k f]h]b[`]g`ZYY`cZYI dfYgg]cb`UbX`dfYgYbhUh]cb`Yffcfg`UbX`]bZY`]W]h]Yg"`fB chY`h\Uh]h]g`j]hU``hc Y`]a]bUhY`Ubmgi W\`Yffcfg`cZYI dfYgg]cb"'H\Y`k cf_`k]```bYYX`hc`VY`j Yfm`Uf[Y`mZYY`cZgi W\`dfcV`Ya g`]Z]h]g`hc`UW\]Yj Y`U`dUgg`[fUXY`!`fF YdYUhYX`YI dfYgg]cb`UbX#cf`dfYgYbhUh]cb`Yffcfg`k]```fYgi `h`]b`U`ZU]`"Ł

+"'H\Y'k cf_']g'dfYgYbhYX'UW\cfX]b['hc'gdY\\]Z\Wh]cb"'@]b_g'UbX'\cd]Yg'fUg'Uddfcdf]UhYE'nc'h\Y'd]Y\Wg i bXYf'\cbg]XYfUh]cb'k YfY'gi dd`]YX'UbX'k YfY'U\W\gg]V`Y"'f\Zmci 'Xcbf\\dfc]]XY'U\W\gg'Zcf'h\Y'a Uf_Yf'nc h\Y'd]Y\Wg'i bXYf'\cbz'mci 'k]``fY\W]j Y'U'Z\J]'[fUXY"E

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh' . A YX]U 5bU mg]g DfYgYbhUh]cb

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g. '%!%) 'a]bi hY`dfYgYbHJh]cb`d`i g`) '--', 'a]bi hYg`cZX]gWigg]cb`]"Y"`&\$`a]bi hYg`]b`hcHJ`"; fUXYX U[U]bghU`gYhcZUggYgga YbhWf]hYf]U/k f]hhYb`Wca a YbhUfmcb`dfYgYbHJh]cb"

5XX]h]cbU`XYhJ]`g.

DYfgi Ug]j Y'f]]Yk g! ci fbU`]ga Łd]YWY!c!WJa YfU'f%) ı Ł

fl/ci 'a i gh'Wca d'YhY'h\]g'UWfjj]hm'': U]'i fY'hc'gi Va]hik]``fYgi `h']b'mci 'fYWY]j YX'U'ZU]``Zcf'h\Y'Wci fgY"Ł

: cf'h\Y'hi hcf]U`Xi f]b['k\]W\mci 'UfY'g\W\YXi \YX'hc'dfcXi \W'mci f'd]Y\W\z'mci 'a i gh\\can Y'dfYdUfYX'k]h\'Ub U'fYUXmk f]h\Yb'g\Wf]dh! Zcf'U'd]Y\W'cZUfci bX'Zj Y'a]bi h\Yg'f\Y\k Y\b'('a]bi h\Yg'' \$'g\Y\\cbXg'UbX')
a]bi h\Yg'' \$'g\Y\\cbXg\z'''\h\]g'k]``\YY'Zcf'Ub'cd]b]cb'd]Y\W'\Vmmci 'cb'Ub']ggi Y\\cd]\W\Uggc\W]Uh\X'k]h\ mci f
\W\cg\Yb'\ci fbU']gh]\W'\fci bX''''fi cf']b\Zcfa Uh]cb'cb'h\]g'''fci bX''g\YY'h\Y'\\ci fg\Y'cj \Yfj]\Yk 'U\Ycj \Y'!'UbX'hc
\YY'\I d'U]\bYX'\UbX'\X]g\W gg\X'\Xi f]b['h\Y'\Y\Ufm'\Y\W\f f\Yg'\UbX'hi h\Yg''\z

#5 'h\]g'd]YWY 'mci 'g\ci 'X'gYY_'hc'Uf[i Y'Wcbj]bW]b['Zcf'U'dUfh]W\ `Uf'j]Yk dc]bh''Mci 'g\ci `X'UfhYa dh'hc'Wca Y i d'k]h\ 'gca Yh\]b['cf][]bU``UbX'ZfYg\ 'Vmk UmcZU'dYfgi Ug]j Y'hU_Y'cb'h\Y'hcd]W! ']"Y"'ffmhc'Uj c]X'fYdYUh]b[h\Y'gUa Y'Uf[i a Ybhg'Ug'k]``'UfYUXm\Uj Y'U'fYUXmVYYb'UXj UbW'X']b'di V']g\YX'cf'VfcUXWUgha UhYf]U'"5h 'YUghWca Y'i d'k]h\ 'U'bcj Y``UbX'Yb[U[]b['k UmcZUf[i]b['h\]g'WUgY"'C ZWci fgYž']hk]``'VY'[fYUh]Zmci 'Wbb]XYbh]ZmU'hcd]Whc'VY'dYfgi Ug]j Y'UVci hk \]W\ `\UgbffiVYYb'Wcj YfYX'Uh'U``'dfYj]ci g`mžcf'\Ug'fYWY]j YX'cb`m a]b]a U``a YX]U'UhYbh]cb"'fH\]g']g'i d'hc'mci ž'cZWci fgY"'>i ghYbgi fY'h\Uhmci 'Wca Y'i d'k]h\ 'gca Yh\]b[cf][]bU``]Zmci 'Xc'WccgY'hc'XYU'k]h\ 'U'hcd]Wk \]W\ \\UgvYYb'XYU'hk]h\ YI hYbg]j Y`mVYZcfY"Ł

$: cf'Z) ```XYhU]`g'cZ'k \ \ Uh']g'fYei \]fYX \"zgYY'h \ Y'Wci \ fgY'A \ ccX`Y'g]hY$

; fUX]b['k]``VY VmfYZYfYbWY 'hc 'h\Y 'Zc``ck]b['Wf]hYf]U

 $\begin{tabular}{l} % \begin{tabular}{l} % \begin$

&"`H\Y`d]YWY`]g`cf][]bU``UbX`ZfYg\`Y]h\Yf`]b`]hg`gi VYWha UhhYfž``Uf[i a YbhUh]j Y`UddfcUW\`cf`]b`h\Y`a UbbYf cZdfYgYbhUh]cb

' "'H\Y'Uf[i a Ybh]g'Wc\YfYbhž`c[]WU``mghfi Whi fYX'UbX'k Y``!Zci bXYXž'Uj c]X]b['VY]b[']bUddfcdf]UhY`m "Ya ch]j Y"'UbX'Uj c]X]b['gi ddcfh]b['cf'fY`m]b['cb'dfY^X]WYXžX]gWf]a]bUhcfmcf'ghYfYchmd]b['VY`]YZg'UbX j U`i Yg"

("'H\Y'd]YWY'Ujc]Xg'h\Y'igY'cZY[fY[]cig`mibkUffUbhYX'cf'cVj]cig`mibgcibX'''c[]WU``ZU``UW]Yg''fhc'VYX]gW ggYX'Xif]b['th\Y'WcifgY"Ł

) "'H\Y'Uf[i a Ybh]g'dfYgYbhYX`]b'U``]j Y`mUbX'Yb[U[]b['a UbbYf

*"'H\Y'd]YWY']g'k Y```XY`]j YfYX'UbX'YUgmhc'Zc``ck #Wca dfY\YbX'!']"Y"'h\Y'dfYgYbhYf'Ybi bWJUhYg'WYUf'mž\Ug Ub'Uddfcdf]UhY'[UnY'f]"Y"'X]fYWhmhc'h\Y'WJa YfUłž'gdYU_g'Uh'Ub'Uddfcdf]UhY'gdYYXž'k]h\'Uddfcdf]UhY`m Yb[U[]b[']bhcbUh]cbž'UbX'k]h\ci hhcc'a Ubm\Yg]hUh]cbg'cf'ZU'gY'ghUfhg"

 $+ \tilde{z} \, H \ Y \, fY \, Wc \, fX \,]b[\]g \, fY \, Wk \, b] \, WU \, \tilde{m} \, C? \, ?! \,]"Y" \, Uh \, Ub \, Udd \, fc \, df] \, UhY \, \tilde{z} \, Wc \, bg] \, ghY \, bh'j \, c \, i \, \, a \, \, Y \, / \, WU \, a \, \, Y \, fU \, 'ghY \, UXm' \, 'k \,]h \, 'h \, Y \, dfY \, gY \, bh'f \, f' \, U'k \, Umg \, Y \, Ugmhc \, `\ Y \, Uf/k \,]h \, c \, i \, hY \, WY \, gg] \, y \, Y \, V \, UW \, [\, fc \, i \, b \, X' \, bc \,]gY \, \tilde{z} \, Ub \, X' \, gc \, c \, b""$

Hi fb]h]b gYh]b[. H\]g]g bchU Hi fb]h]b Ugg][ba Ybh

5ggYgga Ybh(. A YX]U 5bU mg]g D]YVW &

GHJfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g. '>ci fbU`]gh]WWca a YbHJfm!ghmY`Ufh]WY`cf`dcX#, cXWJgh-Yei]j U`Ybh'hc`&\$\$!&) \$\$`k f]hhYb`k cfXg"

Y`gYk\YfY`]b`h\Y`a YX]U"cf`gcW]U``a YX]U"'=hh\YfYVm\Ug`gca Yh\]b[`k cfh\k\]`Y`UbX`bYk `hc`Wcblf]Vi hY`Vm k UmcZUb`Uf[i a Ybh'`fD]YWYg`k\]W\``UW_`h\]g`fYei]fYX`cf][]bU`]lmž`ZYg\bYgg`UbX`W ffYbh]bhYfYghj U`i Y`k]``fYWY]j Y``ck Yf`[fUXYg`! `cb`UWWci bhcZh\YfYVmbchVY]b[`di V`]g\UV`YŁ"

("'H\Y'gi ddcffjb['Uf[i a Ybhg'UbX'Yj]XYbWY'UfY'k Y``!Zci bXYXž'd`Ui g]V`Y'UbX'k ci `X'\Uj Y'h\Y'dchYbhJU``hc]bZi YbWY'h\Y']bhYbXYX'Ui X]YbWY''''? YmWU]a g#WcbhYbhJci g'UggYffJcbg'UfY'UXYei UhY`mVUW_YX'i d'k]h\Uddfcdf]UhY'^ ghJZ]WUhJcbg#[fci bXg'''5bmYj U`i UhJj Y'dfYgi a dhJcb']g'bch`]_Y`mhc'VY'hcc'U`]YbUhJb[#Ubbcm]b[hc'Ub'i b`]_Y!a]bXYX'cf'i bWca a]hYX'fYUXYf'''5g'Uddfcdf]UhYz'_YmdchYbhJU`'Wci bhYf'Uf[i a Ybhg'\Uj Y'VYYbfYWc[b]gYX'UbX'YZYWf] Y`mfYZ hYX''

-) "'H\Y'cd]b]cb'd]YW']g'k f]HYb']b'U'ghm]gh]WU``mUddfcdf]UhY'a UbbYf'f[]j Yb'h\Y']bhYbXYX'Ui X]YbW'#ci hYhZ' gc'Ug'hc'VY'YUgmhc'fYUXZ'k]h\'h\Y'Uf[i a Ybhg'Zc``ck]b[']b'U'Wc\YfYbhUbX'Wc\Yg]j Y'a UbbYf'!']"Y"``]_Y`mhc\cc_'UbX'a U]bHJ]b'h\Y']bHYfYghcZU'WJgi U``fYUXYf"
- *"'H\Y'd]YWY']g'ZYY'cZ[fUa a Uh]WU'žYI dfYgg]cbžgdY``]b['UbX'ch\Yf'dfYgYbhUh]cb'Yffcfg"'\\Ug'VYYbdfcXi WYX'UbX'qi Va]hhYX'UWWcfX]b['hc'qdYWZ]WUh|cb"

GY`Z5bU`mg]g D]YWY

%"H\Y`gY`Z\UbU`mg]g`d]Y\\Y`XYa cbglfUhYg`U`gci bX`_bck `YX[Y`cZz`UbX`YZZY\Mj Y`m'Udd`]Ygz`h\Y`UddfcU\\Yg hc`UbU`mgYg`cZ^ci fbU`]gh]WdYfgi Ug]j YbYgg`XYj Y`cdYX`Xi f]b[`'h\Y`\\circ\fantarian fay"

&"'H\Y'gY`ZUbU`mg]g`YZZYWfj Y`mUbX`gmghYa UhjWU``mUXXfYggYg`h\Y']ggi Yg`ci h`]bYX`]b'h\Y']bghfi Wfjcbg`Ug`hc h\Y'WcbXi WhcZh\Y'UbU`mg]g``f]"Y"'ei Yghjcbg`Ufci bX'h\Y'cd]b]cb'd]YWYfg`WYbhfU``WU]a g/Ufci bX'h\Y Uggi a dhjcbg`UggcWjUhYX'k]h\`_Ym'\ ghjZjWUhjcbg`cZZYfYXž'Ufci bX`Uggi a dhjcbg`UVci hi\Y'Ui h\cffg`UbX]bhYbXYX`fYUXYffgik cfX'j]Yk ž'UbX`gc`cb'! gYY`"\deltaghfi Wfjcbg`DUfh&"``UVcj Y"Ł

""H\Y"WcbWi g]cbg'fYUWXYX'cb'h\YgY'UbX'Ubmich\Yf']ggi Yg'UfY'WYUf'midfYgYbhYX'UbX'Wcbj]bWjb[`miUbX d'Ui g]V`mVUW_YX'i d'VmUddfcdf]UhY'Yj]XYbWY'Zfca 'h\Y'cd]b]cb'd]YWY'hYI h]lrgY`ZfWmfYZYfYbWYg'rc'fY`Yj Ubh k cfXgž'd\fUgYg'cf'ch\Yf'ZYUhi fYg'cZh\Y'hYI hE'UbX'VmYI d`UbUh]cbg'Ug'hc'h\Y'dchYbh]U``Wca a i b]\WJh]j Y YZYWng'cZh\YgY'fYZYfYbWXX'hYI hi U``Y`Ya Ybhg"

("'H\Y'UbU'mg]g'd]YWY']g'Yb[U[]b[žkY```kf]thYbžkY```gtfi WtifYXžYUgmthc'fYUXžk]th\th\Y'X]gW gg]cbcf[Ub]qYX'Uq'U'`c[]WU'`UbX'Wc\YfYbhqi WWYqq]cbcZcVqYfj Ut|cbžWcbWi q]cbg'UbX'qi ddcft|b['Yj]XYbW''

) "'H\Y'ghm`Y'cZk f]h]b[']g'Uddfcdf]UhY'UbX'h\Y'`Ub[i U[Y']g'ZfYY'cZYl dfYgg]cb'UbX'dfYgYbhUh]cb'Yffcfg'UbX]bZY`]W]h]Yg"

*"H\Y k cf_]g dfYgYbhYX UWVcfX]b[hc gdYV]ZWUh]cb"

Hi fb]hb gYhb[. H\]g]g bchU Hi fb]hb Ugg][ba Ybh

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

 $Gh\ XYbhg\ 'UfY\ 'YI\ dYWhYX\ 'hc\ 'di\ hh\ Y]f\ 'bUa\ Yg\ 'UbX\ 'gh\ XYbhbi\ a\ VYfg\ 'cb\ 'Yj\ YfmdU[\ Y\ 'cZh\ Y]f\ 'Ugg][\ ba\ Ybhg\ ''$

Hi fb]h]b Gi Va]gg]cb

- K cf_'gi Va]hhyx'%5'hc'% 'XUmg'UZhYf'h\Y'gh]di `Uhyx'XYUX`]bY'k]``'VY'UggYggYX'UbX'ZYYXVUW_dfcj]XYX'Vi h'U'a Uf_'cZnYfc'k]``'VY'fYWcfXYX'''=Zh\Y'k cf_'k ci `X'\Uj Y'fYWf]j YX'U'dUgg'a Uf_'Vi h'Zcf h\Y'`'UhYbYgg'UbX'h\Y'k cf_']g'U'Wca di `gcfm'Wci fgY'Wca dcbYbh'f\i fX`Y'fYei]fYa Ybhz'U'gh XYbh'k]`` VY'XYYA YX'hc'\Uj Y'a Yhh\Uh'fYei]fYa Ybh/
- K cf_'gi Va]hhYX`&\$`cf`a cfY`XUmg`UZhYf`h\Y`gh]di `UhYX`XYUX`]bY`k]```bchVY`UWWdhYX`Zcf UggYgga YbhUbX`k]```fYWY]j Y`bc`ZYYXVUW_ža Uf_'cf`[fUXY"'=Zh\Y`UggYgga YbhHUg_']g`U`Wca di `gcfm Wca dcbYbhcZh\Y`Wci fgY`U`gh XYbhk]```fYWY]j Y`Ub`l bgUh]gZJWcfm: U]``fl : Ł'[fUXY`Ug`U`fYgi `hcZ i bgUh]gZJWcfmdYfZcfa UbWY`]b`Ub`YggYbh]U``Wca dcbYbhcZh\Y`Wci fgY"

H\]g']bZcfa Uh]cb']g'U`gc'Uj U]`UV`Y'Uh

\hdg.#k k k "Uffg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa]W]bZcfa Uhcb#JfchcWc`g![i]XY`]bYg#

GdYWJU`7cbg]XYfUhJcb5dd`JWUhJcbg

 $\label{thm:condition} $$Mci `WUb`Udd`m'Zcf'gdYW]U``Wcbg]XYfUh]cb`k \Yb`]``bYgg`cf`ch\Yf'W]fW a ghUbWfg`]bhYfZYfY`k]h\`mci f UggYgga Ybh'dYfZcfa UbWf''$

G]W_bYggža]gUXj Ybhi fY'cf'ch\Yf'W]fW a ghUbWrg'VYmcbX'mci f'Wcbhfc``a Um

ł DfYj Ybhmci Zfca Wca d`Yhjb['U'Wci fgY'fYei]fYa Ybhž

ł? YYd mci Zfca 'UttYbX]b['Ub'UggYggUV'Y'UWt]]tmž

ł Ghod moi igi Va]hh]b[iUggYggUV Y k cf_ Zcf U Wci fgYž

ł G][b]ZJWUbhmiUZZYWimci fidYfZcfa UbWijbiUggYggUV`Yik cf_žVYi]hUiZcfa U``YbX!cZgYa YghYfiYl Ua]bUhjcbž'U WUggihYghž'U``UVcfUhcfmhYghž'U`gYa]bUfidfYgYbhUhjcbicf'Ubmch\YfiZcfa icZUggYgga Ybh'

cfila fh\YfiXYfU]`gi]bifY`Uf]cbihciGdYW]U`i7 cbg]XYfUf]cbi]bWi X]b[iiK \Ybihci5dd`m´izii<ckihci5dd`m´iUbXiiGiddcfi]b[ii8cWia YbfUf]cbiid`YUgY`fYZYfihcih\YiGdYW]U`i7 cbg]XYfUf]cbik YVgf]Y.ii\\hdg.#gfi XYbfii bgk "YXii"Ui #gdYW]U`!Wcbg]XYfUf]cb

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]g`i g]b[ih\Yik cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ih\Ya iUg`mci f`ck b"'=hWJb'hU_Y`a Ubm Zcfa gžZfca iXY`]VYfUhY`W\YUh]b[ihc`UVW]XYbhU`mWcdm]b[iZfca iU`gci fWYik]h\ci hUW_bck iYX[Ya Ybh'

IBGK [fcidgdu[]Uf]ga]bhch\yzc``ck]b[WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b[...7\Ub[]b[...7\Ub[]b[...7\Ub[]b[...7\Ub]b]b[...7\Ub]b]b[...7\Ub]b]b[...7\Ub]b]b[...7\Ub]b]b[...7\Ub]cfa CfXg`UbX`d\fUgYg`k\]'Y 'a cghmfYhJ]b]b[...h\Y`cf][]bU`gfi Wh fY 'UbX#cf'dfc[fYgg]cb`cZ]XYUg`cZh\Y`cf][]bU`žUbX`]bZcfa Uh]cb`k]h\ci hU_bck`YX[Ya Ybh'H\]g`U`gc Udd`]Yg`]b`dfYgYbhUh]cbg`k\YfY`gca YcbY`dUfUd\fUgYg`Ubch\Yf'g`]XYUg`cf`k cfXg`k]h\ci h`WfYX]h`UbX`hc d]Y\\]b[...hc[Yh\Yf`ei chYg`UbX`dUfUd\fUgYg`]bhc`U`bYk`k\c`Yžk]h\ci h`Uddfcdf]UhY`fYZYfYb\\]b[..."

7 c``i g]cb.'k cf_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhiUbmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_ž'ghYU']b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d'Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[ˈgci fWYg'k\]WX`\Uj Y`bchVYYb`fYUXžk]h\ci hUW_bck`YX[]b[`h\Y`″gYWcbXUfm″gci fWY`Zfca 'k\]WX`_bck`YX[Y`cZh\Ya`\Ug'VYYb`cVhU]bYX"

8 i d`]\W\r\cb`ffgY`Z\d`U[]\Uf]ga "Ł`gi Va]\r\b[`mci f`ck b`k cf_ž]b`k \c`Y`cf`]b`dUffzk\YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\r\X`Zcf`Ubch\Yf`UggYgga Ybhcf\\cki fgY`UhI_BGK_`cf`Ubch\Yf`i b]j Yfg]\r\d

7 cffYWnfYZYfYbWlb[dfUWnWyg.

- DUfUd\fug]b[ž'gi a a Uf]q]b[ž'YggUmk f]hb['UbX'ha Y'a UbU[Ya Ybh
- 5ddfcdf]UhY'i gY'cZUbX'Uhtf]Vi h]cb'Zcf'U'fUb[Y'cZa UhYf]U`g']bWi X]b['hYl hž']a U[Ygž'Zcfa i `UY'UbX WbWYdhg"

±0X]j]Xi U``Ugg]ghUbWr`]g`Uj U]`UV`Y`cb`fYei YghZfca `H\Y`@YUfb]b[`7 YbhY`f<u>\hd.#k k k "`Wi bgk "YXi "Ui #</u>" Gh XYbhg`UfY`U`gc`fYa]bXYX`h\UhWUfYZ ``hja Y`a UbU[Ya Ybh]g`Ub`]a dcfhUbhdUfhcZgh XmUbX`cbY`cZh\Y]XYbh]Z]YX`WUi gYg`cZd`U[]Uf]ga `]g`dccf`hja Y`a UbU[Ya Ybh'`Gh XYbhg`g\ci `X`U``ck `gi Z]W]Ybhhja Y`Zcf fYgYUfW\žXfUZjb[`UbX`dfcdYf`fYZYfYbW]b[`cZgci fWrg`]b`dfYdUf]b[`U``UggYgga Ybh']hYa g"

IBGK @]VfUfmU`gc`\Ugʻh\Y`9@G9`hcc``UjU]`UV`Y`hc`Ugg]ghmci`k]h\`mcif`ghiXmUhIBGK "'9@G9`]gXYg][bYX`hc`]bhfcXiVW`bYk`ghiXYbhg`hc`ghiXm]b[`UhIBGK`Vih]hWUb`U`gc`VY`U`[fYUhfYZfYg\Yf`Xif]b[`mcifqhiXm"

7 ca d'Yhlb['h\Y'9@&9'hi hcf]U`'UbX'ei]n'k]`'YbUV'Y'mci 'hc.

٥

F Ygci fWYg

DfYgWf]VYX F Ygci fWYg