

MUSC4702

DYfZcfa UbWY @UVcfUhcfmi,

GYa YghYf'Hk c''#'&\$%

5WUXYa]W±bZcfaUh]cb

: cf`YggYbh]U``ghi XYbh`]bZcfa Uh]cb`fY`Uh]b[`hc.`fYei Yghg`Zcf`YI hYbg]cb/fYj]Yk `cZa Uf_g/cWW dUh]cbU``\YU`h\UbX`gUZ/hm/YI Ua]bUh]cb`dfcWXi fYg/gdYW]U``Wcbg]XYfUh]cb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybhi fY/ghi XYbhYei]hmUbX`X]gUV]`]hm/UbX`ch\Yf`YggYbh]U``UWUXYa]W]bZcfa Uh]cbž gYY`\hdg.#k k k "Ufhq"i bqk "YXi "Ui #W ffYbhqhi XYbhq#UWUXYa]W]bZcfa Uh]cb#dfchcWc`q![i]XY`]bYq#

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

Gi VYWh5fYU. ? ge[U

#John Jgr ZJbu ``Wci fgY c Zh Y 'DYfzcfa UbWY @UVcfUhcfmgYei YbWY zmci k]```ghfYb[h\Yb'UbX`XYa cbghfUhY 'mci f dYfzcfa UhJj Y 'g_]``g`]b`a i g]Wa U_]b['VmXYj Y`cd]b['UbX'dfYgYbhJb['U'dYfzcfa UbWY'dfc YWfY`UhJj Y 'hc'mci f WkcgYb'ghfYUa z'bUa Y`mA i g]WDYXU[c[mz'Gcb]W5fhg'cf'A i g]Wc`c[m''K cf_g\cdg'UbX'gYa]bUfg'k]`` gi ddcfhih\Y'dfYdUfUhJcb'cZdfc YWfg'UbX'zcW' g'cb'WcbhYl hi U``]ggi Yg`]b'dYfzcfa UbWY'dfYgYbhUhJcb'''Mci k]`` U'gc'dUffJWJdUhY']b'Ubdfcj YX'l BGK 'YbgYa V'Y'UbX'hU_Y'U'`YUXYfg\]d'fc`Y'Ug'Uddfcdf]UhY'''

5 hh Y WcbWi g]cb cZh]g Wci fgY h Y gh XYbhk]``VY UV Y hc

%" 5dd`mUXj UbWYX'g_]``g`]b`a i g]WU``dYfZcfa UbWY'Yl YW h]cb'UbX'dfYgYbhUh]cb"

5ggYgga Ybh

D`YUgY`gYY`A ccX`Y`Zcf`a cfY`XYhU]`g"`5`gcž`UggYgga YbhhUg_gž`XYhU]`g`cZa U'cf`dfc'YWgžXUhYg`UbX`ch\YffY`Yj Ybh]bZcfa Uh]cb`k]``VY`X]gW ggYX`]b`h\Y`Z]fghWUgg`cZh\Y`gYa YghYf"

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'tUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
H\fYY`fYdcffgfl \$`	' \$1	Bch5dd`]WUV`Y	B#5
DYfZcfa UbWY dfc YWff) \$") \$1	Bch5dd`]WUV`Y	B#5
9bgYa V`Y`dUff]WJdUf]cbf&\$`	&\$ı	Bch5dd`]WUV`Y	B#5

5ggYgga Ybh8YHJ]`g

5ggYgga Ybh% H\fYY fYdcfhgfl \$

GhUfhXUhY. Bch5dd`]WUV`Y

8 YHJ]`g. 'H\fYY`fYdcffg`cZ%\$\$\$'k cfXg`YUW.'k]``XcW a YbhiUgdYWfg`cZghi Xm'UbX`dfYdUfUh]cb`fck UfXg`h\Y ZbU``dfYgYbHJh]cbžYb[U[Ya Ybhk]h\`k cf_`cZdYYfg`UbX`a UhYf]U``dfYgYbHYX`]b`gYa]bUfg"'K f]HhYb`ZYYXVUW_Zca ``YWhifYf"

5XX]h]cbU`XYhU]`g.

 $\label{thm:condition} $$H\g''= W\d' g Y Y Who W Ub[Y'cf'a cX] Who who was $$ L^2Who Y Y Ub X'X]g W g Y X']b W Ug Y' Ub X'X]g W g Y X']b W Ug Y' Ub X'X]g W g Y X']b W Ug Y' Ub X'X]g W g Y X'Y Ub X'X]g W g Y X'Y Ub X'X [g Y W Ub X'X]g W g Y X'Y Ub X'X [g Y Y W Ub X'X]g W g Y X'Y Ub X'X [g Y Y W Ub X'X [g Y Y W Ub X'X]g W g Y X'Y Ub X'X [g Y Y W Ub X'X$

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh& DYfZcfa UbWY dfc YWYf) \$

GhUfhXUhY. Bch5dd`]WUV`Y

8 YHU]`g. Mci 'k]``VY'fYei]fYX'hc'dfYgYbh'U'gc`c'cf'[fci d'dYfZcfa UbWY'cZ&) !' \$`a]bi hYg`]b`Xi fUhjcb'h\Uh g\ck WUgYg'UV]`]hjYg`]b`a i g]Wa U_]b['UbX#cf'dYfZcfa UbWY!fY`UhYX'g_]``g"'H\Y'dYfZcfa UbWY'a UmHU_Y'h\Y Zcfa 'cZU'gc`c'dYfZcfa UbWYž'U'[fci d'cf'Wc``UVcfUhj Y'dYfZcfa UbWYž'YWh fY'fYWjhU`cf'Ub'Uddfcdf]UhY dYfZcfa Uhj Y'Yj Ybh'Uddfcj YX'Vmh\Y'Wci fgY'Wcbj Ybcf"''H\Y'dYfZcfa UbWY'k]``VY'dfYgYbhYX'Ug'Ub YI Ua]bUhjcb"'K f]hhYb'fYdcfh'Zca 'YI Ua]bYf

5XX]h]cbU`XYhU]`g.

Hi fb]hb gYhb[. H\]g]g bchU Hi fb]hb Ugg][ba Ybh

5ggYgga Ybh' . 9bgYa V`Y'dUfhJWJdUhJcbf&\$

GHJfhXUhY. Bch5dd`]WUV`Y

 $8YHU]`g. `DUffHVW.YW_]b[`fHYghcb'a i g]WU``a UfHYf]U``gfh X]YX`UbX`dYfZcfa YX`]b`Ub`Uddfcj YX`I BGK `YbgYa V`YXi f]b[`fh.Y`gYgg]cb''``H\]g`]g`fh.Y`Z]bU``UggYgga YbhZcf'fh.Y`Wci fgY''`8]fYWfZYYXVUW_`Zfca `X]fYWfcf'fh.fci [\ci hh.Y`gYgg]cb`UbX`X]fYWfZYYXVUW_`cb'fh.Y`dUffHVW.YW_]b[`fhYgfi'$

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gh XYbhg UfY Yl dYWYX hc di hh\Y]f bUa Yg UbX gh XYbhbi a VYfg cb Yj YfmdU[Y cZh\Y]f Ugg][ba Ybhg"

Hi fb]h]b Gi Va]gg]cb

#mci YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dr]b['rc'gi Va]hmci f'Ugg][ba Ybht\fci [\'Hi fb]r]bž'd'YUgY
hY'Yd\cbY'9I hYfbU'Gi ddcfhcb'-',) '''' %cf'Ya U]''h\Ya 'cb'<u>Yl hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui"</u>"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft\"'=\maintention
UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]h]b'mci 'a Um'Udd'm\Zcf'Ub'YI hYbg]cb\ZVi hmci
a i ghfYhJ]b'mci f'h]\W_Yhbi a VYf\Zfca '9I hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW'a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' hc'gi ddcfhmci f'Yl hYbg]cb'Udd']\WJh]cb'''\=\maintention
Uh\WJ'`mfYWY']j Y'U'h]\W_Yhbi a VYf\ZVi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]\Z\WJ'`mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'gfUh' g'cb'Hk]hYf"

; YbYfU`mž'UggYgga YbhitUg_g'a i ghVY'gi Va]hhYX'Y`YWfcb]WU``mj]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bghUbWfg'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'ghUhYX'cb'mci f'Wci fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYhU]`g"

@UhY 5ggYgga Ybh'DYbU`h]Yg

Ghi XYbhgʻUfYʻfYgdcbg]V`YʻZcfʻh\Yʻgi Va]gg]cbʻcZUggYgga Ybhilug_gʻVmh\YʻfYei]fYXʻXUhYgʻUbXʻh]a Ygʻ' 8YdYbX]b[ˈcZh\YʻYl hYbhcZXY`Um]bʻh\Yʻgi Va]gg]cbʻcZUbʻUggYgga Ybhilug_ʻdUghih\YʻXi YʻXUhYʻUbXʻh]a Yž cbYʻcZh\YʻZc``ck]b[ʻ`UhYʻdYbUʻh]Ygʻk]```Udd`mi b`YggʻgdYVJU`'Wcbg]XYfUh]cbʻcfʻUʻV`Ub_YhYl hYbg]cbʻXi YʻhcʻU hYV\\b]WJ``ci hU[Yʻ]gʻ[fUbhYX'': cfʻh\Yʻdi fdcgYʻcZ`UhYʻdYbUʻhmWJʻW``Uh]cbžʻUʻʻXUmiʻ]gʻXYYa YXʻhcʻVYʻYUV\\&(!\ci fʻdYf]cXʻfcfʻdUfhih\YfYcZcdUghh\Yʻgh]di `UhYX`XYUX`]bYʻZcfʻgi Va]gg]cb''

• K cf_'gi Va]hhYX``Ygg'h\ Ub'%\$`XUmg'UZhYf'h\ Y`gh]di `UhYX`XYUX`]bY']g'gi VYVWhc'U`XYXi Wi]cb'cZ) i 'cZh\Y'hchU``Uk UfXUV`Y'a Uf_'Zfca 'h\Y'a Uf_'h\ Uhk ci `X'\Uj Y'VYYb'UW\]Yj YX']Zbch'Zcf'h\Y dYbU`hm'Zcf'Yj YfmXUmdUghh\Y'gh]di `UhYX`XYUX`]bY'Zcf'gi Va]gg]cb"'H\Uh]gz'U'ghi XYbhk \c'gi Va]hg Ub'Ugg][ba Ybh'k]h\ 'U'gh]di `UhYX`XYUX`]bY'cZ(.\$\$da 'cb'% 'A Um&\$%* 'Uh(.%\$da 'cb'%('A Um&\$%* k]``]bW f'U'XYXi Wi]cb'cZ%\$i "

HUq_k h U bcb dYfWbhJ[Ya Uf_

⊋h\Y'hUg_`]g'a Uf_YX'ci hcZ&) žh\Yb``UhY'gi Va]gg]cb'k]```UhtfUWhU'dYbU`hmcZU'XYXi Wf]cb'cZ%"&) 'Zfca 'h\Y a Uf 'Uk UfXYX'hc'h\Y'gh XYbh'Zcf'Yj Yfm&(!\ci f'dYf]cX'fbf'dUfhh\YfYcZd'dUghh\Y'ghdi `UhYX'XYUX`]bY"

- K cf_'gi Va]hhyX'%\$'hc'% 'XUmg'UZhYf'h\Y'gh]di `UhYX'XYUX`]bY'k]``'VY'UggYggYX'UbX'ZYYXVUW_dfcj]XYX'Vi h'U'a Uf_'cZnYfc'k]``'VY'fYWcfXYX'''=Zh\Y'k cf_'k ci `X'\Uj Y'fYWf]j YX'U'dUgg'a Uf_'Vi h'Zcf h\Y'`UhYbYgg'UbX'h\Y'k cf_']g'U'Wca di `gcfm'Wci fgY'Wca dcbYbh'f\i fX`Y'fYei]fYa Ybh'z'U'gh XYbh'k]`` VY'XYYA YX'hc'\Uj Y'a Yhh\UhfYei]fYa Ybh'
- K cf_'gi Va]hhYX`&\$`cf`a cfY`XUmg`UZhYf`h\Y`gh]di `UhYX`XYUX`]bY`k]``bchVY`UWWdhYX`Zcf UggYgga YbhUbX`k]``fYWY]j Y`bc`ZYYXVUW_ža Uf_'cf`[fUXY"'=Zh\Y`UggYgga YbhHUg__']g`U`Wca di `gcfm Wca dcbYbhcZh\Y`Wci fgY`U'ghi XYbhk]``fYWY]j Y`Ub`l bgUh]gZJJWcfm: U]`fl: Ł'[fUXY`Ug`U'fYgi `hcZ i bgUh]gZJJWcfmdYfZcfa UbWY`]b`Ub`YggYbh]U``Wca dcbYbhcZh\Y`Wci fgY"

H\]g']bZcfa Uh]cb']g'U`gc'Uj U]`UV`Y'Uh

\hdg.#k k k "Uffg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWUXYa]W]bZcfa Uhcb#dfchcWc`g![i]XY`]bYg#

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'\lefi\Wb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UVW]XYbhU`mWcdm]b['Zfca `U`gci f\W' k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidgdu[]Uf]ga]bhch\YZc``ck]b[WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

 $7c^{i}$ g]cb. 'k cf_]b['k]th. 'cth. Yfg'Vi h'dUgg]b['c\Zth. Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U''k cf_"'7c''i g]cb'U'gc

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWfg'cZ9@G9'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "f\hd.#gi VYWf[i]XYg"]VfUfm"i bgk "YXi "Ui #\`]gY#JVci hY`]gY

FYgci fWfg

DfYgWf]VYX F Ygci fWYg

H\YgY`k]``VY`dcghYX`cb`AccX`Y"

FYWca a YbXYX FYgci fWYg

H\YgY`k]``'VY`dcghYX`cb`AccX`Y"

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

 $H\]g'k\]```VY'UXXfYggYX']b'h\ Y'Z]fghgYa\]bUf'cZh\ Y'Wc'i\ fgY''$

a U[Y'7fYX]h

6cX]Yg:/ **b\YfZJ\W\g*&\$\%*

7F = 7CG

7 F = 7 C G 'Dfcj]XYf'7 cXY. '\$\$\$-,;