

MDIA2013

AU_]b['J]fhiU`FYU`]mi8cWaYbhUf]Yg

Gi a a Yf"#"&\$&\$

7ci fgY[·]Cj Yfj]Yk

GHJZZ:7cbHJWh:8YHJ]`g`

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
? Yff]Y 8Uj]Yg				

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

HcXUmig`XcW a YbhUfmightfinhY``]b['Zcfa Uhg`bck ']bWi XY'' *\$'j]XYcgž5i [a YbhYX'FYU`]hmfbF Ł'UbX'J]fhi U`FYU`]hmflJ F Ł"'BYk g`cf[Ub]gUh]cbg`UfY'fY[i `Uf`mdfcXi W]b['g\cfh'J F 'ghcf]Yg`UWfcgg`X]j YfgY'hcd]Wg`UbX'Bch : cf!DfcZ|hcf[Ub]gUh]cbg`UfY`\UfbYgg]b['h\Y'dck Yf`cZj]fhi U``fYU`]hmUg`'Ya dUh\ma UW\]bYg''hc'dfca chY Uk UfYbYgg`cZ]ggi Ygž'XYYdYb`_bck `YX[Y`UbX'YbWci fU[Y`gi ddcfh''H\Y'J F 'Zcfa 'g`dcdi `Uf]hm]g i bXYfd]bbYX'VmUWWgg]V`Y'j]Yk]b['cb'Mci Hi VY'd`UhZcfa g`UbX'ga Ufhd\cbYg`Ug`k Y```Ug'J F 'XYX]WUhYX XYj]WYg"

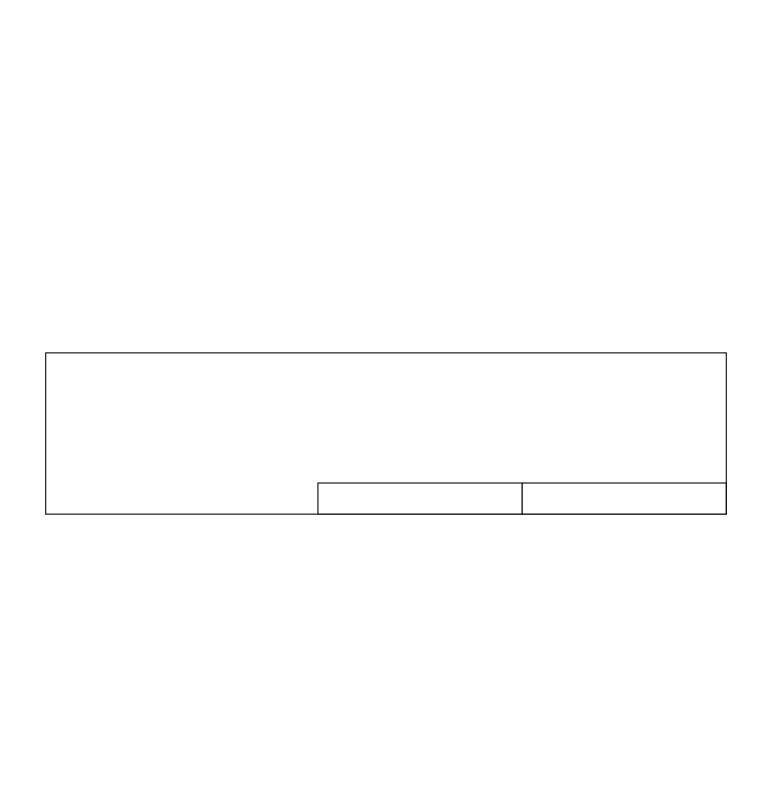
H\]g`Wci fgY`dfcj]XYg`mci `k]h\`h\Y`dfUWf)WU``g_]``g`]b`J]fhi U``F YU`]mmfJ F ŁXcW a YbHJfmdfcXi Wf]cb`h\Uh'UfY]bWfYUg]b[`mdUfhcZh\Y`a i `h]d`Uhzcfa 'a YX]U`Ybj]fcba Ybh'`Mci `k]```XYj Y`cd`\UbXg!cb`WfYUfj Y`UddfcUW\Yg hc`J F `]a a Yfg]j Y`XcW a YbhJfmghcfmhY``]b[`Zfca `]XYUZʻgWf]dhzj]XYc]b[`UbX`bUffUfjcb`UbX`YX]h]b[`hc`Z]bU` j]XYc"'H\Y`Wci fgY'g`dfUWf)WJ``Y`Ya Ybhg`k]```VY`gi ddcfhYX`Vmh\YcfYh]WJ``fYUX]b[gzïX]gW gg]cb`UbX`UbU`mg]g cZ]a a Yfg]j Y``cb[!Zcfa `XcW a YbhJf]Yg`UbX`h\Y]f`Wca a i b]WJh]cb`UbX`ghcfmhY``]b[`ghfUhY[]Yg"

7 ci fgY @YUfb]b[Ci hWca Yg

- %" 8Ya cbglfUhY`dfUVVjVW)`ž'hYVX.b]\W\``UbX`Wca a i b]\WUhjcb`g_]``g`]b`h\Y`dfcXi Whjcb`cZJ F`XcW`a YbhUfm' &" 5dd`mh\YcfYhj\W\``i bXYfghUbX]b[g`cZ]a a Yfg]j Y`ghcfmhY```]b[`]b`XcW`a YbhUfm'UbX`Wca a i b]\WUhjcbg hc`dfcXi Whjcb`dfUVVj\W\"
- " DfcXi WY'U'J F 'j]XYc'Zcf'U'a YX]U'dfcXi Wfjcb'dcfhZc`]c'UbX'#cf'di V`]WW\UbbY`:gi W\'Ug'Mci Hi VY" (" 7 f]h]ei Y'UbX'g]hi UhY'h\Y'\]ghcfmUbX'h\YcfmcZ]a a Yfg]j Y'ghcfmHY``]b['k]h\]b']bX]j]Xi U``a YX]U dfUWfJWY"

HYUW]b['GhfUhY[]Yg'

H\]g`Wci fgY`dfcj]XYg`mci `k]h\`dfUWf]WU``g_]``g`WfYUh]b[`J F `ghcfmhY``]b[`Zcf`h\Y'`ci fbU`]ga `UbX Wca a i b]WUh]cb`]bXi ghf]Yg"`8i f]b[`h\Y`Wci fgYž`mci `k]```dfcXi WY`U`J F 'j]XYc`h\fci [\`\UbXg!cb``UV gYgg]cbg`UbX`Z]Y`X`k cf_"'Mci `k]```\Uj Y`Yei]hUV`Y`UWWgg`hc`Yei]da YbhUbX`dcghldfcXi Wh]cb`YX]h]b[ZUWf]h]Yg"`H\Y`dfUWh]WU``k cf_`k]```VY`gi dd`Ya YbhYX`k]h\`h\YcfYh]WU``X]gW gg]cb`UbX`UbU`mg]g`cZ]a a Yfg]j Y ghcfmhY``]b[`hc`Yb\UbW'`mci f`dfcXi Wh]cb`g_]``g"



8 YHU]`g. '9 X]h]b['UbX'gci bX'dcghdfcXi Wh]cb'cZh\fYY'hc'Z]j Y'a]bi hY'XcW a YbhUfm'J F'j]XYc'k]h\'bUffUh]cb" Gi Va]hhYX'k]h\'bUffUh]cb'gWf]dh'fUbgWf]dh'fUbddfcI "'%\$\$\$'k cfXgŁ": YYXVUW_'dfcj]XYX'j]U'@A G"

5XX]h]cbU`XYhU]`g.

⊋mci 'fYei]fY 'UXj]WY 'UVci hYX]h]b['cf'ch\Yf'hYW\b]WU``Ugg]ghUbWY 'fY`Uh]b['hc'mci f'dfc'YWhci hg]XY 'h\Y hi hcf]U'žd`YUgY 'gYY'h\Y 'HYW\b]WU``F Ygci fWY '7 YbhYY fHF 7 Ł"'

'*\$"WUa YfU'_]hg'gi dd`]YX'j]U'h\Y'HF7"cUbg'ci hg]XY'hi hcf]U'g"'GYY'A ccX`Y'Zcf'Z fh\Yf`]bZcfa Uh]cb'cb Vcffck]b["

FcVYfhK YVghYf`UVg'UfY'Uj U]`UV`Y ZcfYX]hjb[ci hg]XY hi hcf]U`g"

GdYVJU`Wcbg]XYfUh]cb'j]U'a mibgk']g'fYei]fYX'Zcf'Yl hYbg]cbg"

.

Gi Va]gg]cb bchYg. D`YUgY fYZYf hc A ccX`Y

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh' . FYZYWncb DUdYf fllox]j]Xi U'Ł

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. '%) \$\$ k cfXg

8 YHJ]`g. '%) \$\$!k cfX`fYZYWIJcb"': YYXVUW_'dfcj]XYX`j]U'@A G"

5XX]h]cbU`XYHJ]`g.

< Ufj UfX F YZYfYbWb['fYei]fYX"

GYY'A ccX`Y'Zcf'Z fh\Yf'UggYgga Ybh]bZcfa Uh]cb"

#i]g fYWca a YbXYX mci k f]hY mci f fYZYWf]cb df]cf hc h\Y Xi Y XUhY"

Gi Va]gg]cb bchYg.gYY A ccX`Y

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb & fb & gYh &$

5 http://bxubww.F.Yei]fYa Ybhg

 $\label{eq:continuity} D`YUgY`bchY`h\Uh`YWh fY`fYWcfX]b[g`UfY`bchUj U]`UV`Y`Zcf`h\]g`Wci fgY"`Ghi XYbhg`UfY`ghfcb[`miYbWci fU[YX`hc UhhYbX`U```WUggYg`UbX`WcbhUWhh\Y`7ci fgY`5i h\cf]mnhc`a U_Y`U`hYfbUh]j Y`UffUb[Ya Ybhg`Zcf`WUggYg a]ggYX"$

7ci fgY'GWXYXi `Y

J]Yk Wugg'ha Yhuv'y

H]a YHJV`Y

8 UHY	HmdY	7 cbhYbh	
*`>Ubi Ufm!`%\$`>Ubi Ufm	@YWi fY	BHFC8I7H€B' @AF7, FZWdWSdWfZdWWWUfgdWeWhWdkiWW]ł #"Ž##S_ žDWWdfa? aaVWXadS**WUfgdWdW\$V[Yež @97HIF9'ACB85M'>UbiUfm'*.'±blfcXiWf]cb'hc'h\Y WtifgY @97HIF9'HI9G85M'>UbiUfm'+'Z['a]b['kcf_g\cd @97HIF9'K 98B9G85M''>UbiUfm', 'gdUh]U' ghcfmtY``]b[
	Hi H@UV	BHFC8I7H€BK99? @AF7, FZWdWSdWfZdWWfgfad(S'e WhWdki WW) ł ##Ž#b_ ž HIH!@56'ACB85M'>UbiUfm*. +bHfcXiWf)cbhch\Y HF7'UbX''*\$'WUaYfUg HIH!@56'HI9G85M'>UbiUfm+. 'Z]'a]b['kcf_g\cd.' HIH!@56'K98B9G85M'>UbiUfm,+bHfcXiWf)cbhc JF'YX]h]b['DfYa]YfY'Dfc'	
% `>Ubi Ufm!'%+`>Ubi Ufm		JF'GHCFMH9@B; 'GHM@9G DWWdfa? aaV'WXaddWSV['Ye XadS\\WdfgdWez' @97HIF9'ACB85M\\''>Ubi Ufm'JF''\\''*\\$ bUffUhj Yg'UbX'bYk g'a YX]U' @97HIF9'HI9G85M'\('>Ubi Ufm''JFz''bch'ZcfdfcZhUbX'UWjj]ga @97HIF9'K98B9G85M'\\')'>Ubi Ufm''JF'Ui X]c]a a Yfg]cb'	
	Hi H@UV	HI H!@56'A CB85M'% '>Ubi Ufm'J F 'ghcfm k cf_g\cd'fYgYUfW\'UbX'Uddfcj U'g HI H!@56'HI 9G85M'%('>Ubi Ufm'bUffUh]j Y'UbX]bhYfj]Yk g']b'h\Y'' *\$'Ybj]fcba Ybh HI H!@56'K 98B9G85M'%) '>Ubi Ufm'Ui X]c	

]a a Yfg]cb'Yl dYf]a YblUh]cb'
		9 dagb3eeWee_WifBSdf#VgW8d[VSk#) <s`gsdk /#', fi</s`gsdk
&\$`>Ubi Ufmi!`&(`>Ubi Ufm	@YWhifY	JF DfcXiWh]cb UbX 9h\]WU`]ggiYg
		@97HIF9"ACB85M'&\$">Ubi Ufm"9aVcX]aYbh'UbX DfYgYbW\" @97HIF9"HI9G85M"&%">Ubi Ufm"IL"UbX"JF @97HIF9"K98B9G85M'&&">Ubi Ufm"H\Y]aaYfg]jY'k]hbYgg
	Hi M@UV	
		HI H!@56"A CB85M'&\$">Ubi Ufm"GdUh]U" ghcfmhY"]b["k cf_g\cd HI H!@56"HI 9G85M'&%">Ubi Ufm"I L"YX]h]b[k cf_g\cd HI H'@56"K 98B9G85M'&&">Ubi Ufm"YX]h]b[k cf_g\cd
		9 DA GB 3EE7EE? 7@FBSdf\$/#' _fiVgW8D;63K \$& <s`gsdk< td=""></s`gsdk<>
&+`>Ubi Ufm!'' %`>Ubi Ufm	@YWn fY	51; A 9BH98 F 95@±HM5B8: I HI F 9 8 F 97H € BG
		? A @63K \$) <s`gsdx bg4="">;5 : A >;63K `a 'WUfgd\\ S`V U'SeeWe</s`gsdx>
		@97HLF9"HL9G85M1&, ">Ubi Ufm"5F "UbX"A]LYX FYU"]hm
		@97HIF9"K98B9G85M"&-'>UbiUfm":ihifY X]fYW¶cbg'
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		? A@63K\$) <s`gsdkž@a 5="">3EE HI H'@56'HI 9G85M'&, '>Ubi Ufm'YX]h]b['k cf_g\cd Hi H@UV'K 98B9G85M'&- '>Ubi Ufm'YX]h]b[k cf_g\cd</s`gsdkž@a>
		-88∋ -815@5GG9GGA9BH'819''‰")-

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

#mci YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dr]b['rc'gi Va]hmci f'Ugg][ba Ybht\fci [\'Hi fb]r]bž'd'YUgY
hY'Yd\cbY'9I hYfbU''Gi ddcfhcb'-',) '''' %cf'Ya U]''h\Ya 'cb'YI hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft\"'=\maintention
UfY'i bUV'Y 'rc'gi Va]hmci f'Ugg][ba YbhXi Y'rc'U'ZJi 'hk]h\'Hi fb]r]b'mci 'a Um'Udd'm\Zcf'Ub'YI hYbg]cb\ZVi hmci
a i ghfYhJ]b'mci f'r]\W_Yhbi a VYf'\Zfca '9I hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW a Ybfg\L'rc
]bWi XY'Ug'Yj]XYbW' 'rc'gi ddcfhmci f'YI hYbg]cb'Udd']\WJh]cb'''=\maintention
UfY'' bUYY''mfYWY']j Y'U'r]\W_Yhbi a VYf\Zfvi h]Zmci 'hY'Yd\cbY'mci 'k]'''bYYX'rc'gdYW]\ZfwJ''mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'gfUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mj]U'Y]h\Yf'Hi fb]hjb'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]``'VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\ U`hYfbUhj Y'gi Va]gg]cb'XYHJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va]hUgq][ba Ybha ccXiY

5 WUXYa | W < cbYglmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i\\identity a Ubm Zcfa qžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiU\wbck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b[WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

=\uddfcdf]UhY 'dUfUd\fUg]b[.'7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU`
ghfi Whi fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU`ž'UbX']bZcfa Uh]cb'k]h\ci h\UW_bck `YX[Ya Ybh''H\]g'U`gc
Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY 'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h\UhYX]hUbX'hc
d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yž'k]h\ci h\Uddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb.'k cf_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhiUbmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_ž'ghYU']b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d'Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[`gcifWrg`k\]W\`\UjY`bchVYYb`fYUXžk]h\cihUW_bck`YX[]b[`h\Y`´gYWcbXUfm´gcifWrZfca`k\]W\`_bck`YX[Y`cZh\Ya`\Ug`VYYb`cVhU]bYX"

8 i d`]\W\n]cb`ffgY`Z\d`U[]\Uf]ga "Ł`gi Va]\n]b[`mci f`ck b`k cf_z`]b`k \c`Y`cf`]b`dUffz`k \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\nYX`Zcf`Ubch\Yf`UggYgga Ybhcf`\\cf\\text{i fgY`UhI_BGK_`cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWnfYZYfYbWlb['dfUWnWyg.

- DUfUd\fUg]b[žgi a a Uf]g]b[žYggUmk f]h]b['UbX h]a Y a UbU[Ya Ybh
- 5ddfcdf]UhY'i gY'cZUbX'Uhtf]Vi h]cb'Zcf'U'fUb[Y'cZa UhYf]U`g']bWi X]b['hYl hž']a U[Ygž'Zcfa i `UY'UbX WbWYdhg"

±0X]j]Xi U``Ugg]ghUbWr`]g`Uj U]`UV`Y`cb`fYei YghZfca `H\Y`@YUfb]b[``7 YbhY`f\hd.#k k k "`Wi bgk "YXi "Ui #£" Gh XYbhg`UfY`U`gc`fYa]bXYX`h\UhWUfYZ ``ha Ybu[Ya Ybh]g`Ub`]a dcfhUbhdUfhcZgh XmUbX`cbY`cZh\Y]XYbh]Z]YX`WUi gYg`cZd`U[]Uf]ga `]g`dccf`h]a Y`a UbU[Ya Ybh'`Gh XYbhg`g\ci `X`U``ck `gi Z]W]Ybh'h]a Y`Zcf fYgYUfW\žXfUZn]b[`UbX`dfcdYf`fYZYfYbW]b[`cZgci fW'g`]b`dfYdUf]b[`U```UggYgga Ybh']hYa g"

IBGK @]VfUfmU`gc`\Ugʻh\Y`9@G9`hcc``UjU]`UV`Y`hc`Ugg]ghmci`k]h\`mcif`ghiXmUhIBGK "'9@G9`]gXYg][bYX`hc`]bhfcXiVW`bYk`ghiXYbhg`hc`ghiXm]b[`UhIBGK`Vih]hWUb`U`gc`VY`U`[fYUhfYZfYg\Yf`Xif]b[`mcifqhiXm"

7 ca d'Yhlb['h\Y'9@&9'hi hcf]U`'UbX'ei]n'k]`'YbUV'Y'mci 'hc.

 UbU`mgY`hcd]\Mgž'd`Ub`fYgdcbgYg`UbX`cf[Ub]gY`fYgYUf\W\`Zcf'U\WXYa]\Wk f]hjb[`UbX`ch\Yf UggYgga YbhiHUg_g

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi V'YWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUXYa]W = b Zcfa Uh cb ·

: cf`YggYbhJU``gh XYbh`]bZcfa UhJcb`fY`UhJb['hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY`gi Va]gg]cbg`[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK '< YU`h\ 'UbX'GUZYhm'dc`]W[Yg/
- YI Ua]bUhcb dfcWXi fYg/
- gdYWJU``Wcbg]XYfUHjcb`]b`H\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybhi fY/
- gh XYbhYei]lmUbX X]gUV]`]lm/
- UbX'ch\Yf'YggYbhJU`'UWUXYa]W]bZcfa UhJcbž'gYY

\hdg.#k k k "Uffg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa]W]bZcfa Uf]cb#dfchcWc`g![i]XY`]bYg#

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a U[Y'7fYX]h

D\chc VmA Ulth\Yk ? k cb['cb' | bgd'Ug\

7F = 7CG

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