

# **MDIA5029**

±blfcXi Whcb hc 5Xj Yfh]g]b[ H\YcfmUbX DfUWh]WY

GiaaYf"#"&\$&%

## 7ci fgY<sup>-</sup>Cj Yfj ]Yk

### GHJZ:7cbHJWh8YHJ]`g

#### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]m	@cWJhjcb	D\cbY
>Ua YgʻK ][[]bg	0 0	J ]XYcWJ```a YYh]b[ g Uj U]`UV`Y`Vm Uddc]bha Ybh'		

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GWXcc``cZhXY`5fhg`UbX`AYX]U

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D\cbY. f\$&Ł'-',) '(,) \*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'k ci`X``]_Y'tc'FYgdYWM\cong ``m5W_bck``YX[Y'thY'HfUX]tjcbU``7i gtcX]Ubgž thY'6YXY[U'`fPYbg]b[tcb'WJa di gŁż'; UX][U'`fP]tmUbX'5fth/'8Yg][b'7Ua di gYgŁ'UbX'thY'B[i bbUk U` dYcd'Y'fbi gtfU']Ub'8YZYbWY': cfWY'5WJXYa m']b'7UbVYffUŁ'cZthY'`UbXg'k \YfY'YUW\cong WJa di g'cZl BGK ']g `cWJtYX''$ 

## 7ci fgY 8YHJ] g

7 fYX]hDc]bhg\*\*

Gi a a UfmcZh\Y 7ci fgY

H\]g`Wci fgY`]bhfcXi W/g`mci 'hc`h\Y`XmbUa ]Wk cf`X`cZUXj Yff]g]b[ "'=iYa d\Ug]gYg`bchcb`mih\Y`d`UW/`cZ UXj Yff]g]b[ ']b`\]ghcfmUbX`W`'hi fYž'Vi hYei ]dg`mci 'k ]h\`h\Y`hcc`g`bYW/ggUfmhc`Ub`]b!XYdh\`fYUX]b[ 'cZ UXj Yff]g]b[ 'hYl hg'''Nci 'k ]```YbWci bhYf`UXj Yff]gYa Ybhg`Zfca 'X]ZZYfYbhYdcW\g`UbX`k ]```YUfb`hc`]bhYffc[ UhY

### 5ggYgga Ybh

### 5ggYgga YbhHUg\_g

5ggYgga Ybh'hJg_	K Y][ \h	8i Y'8UHY	Gh XYbh@YUfb]b[ Ci hWca Ygʻ5ggYggYX
7 f]r]WJ`'UbU`mg]g'	(\$1	&&#\$%#&\$&%`%%\$\$`DA</td><td>&</td></tr><tr><td>5 Xj Yfhjg]b[ 'h\fci [ \ 'h]a Y 'UbX d`UWY'</td><td>' \$ı</td><td>Bch5dd`]WUV`Y</td><td>%</td></tr><tr><td>≠ggi Ygʻ]bʻUXj Yfhg]b[ʻ</td><td>' \$ı</td><td>\$&#\$&#&\$&%`%%`\$\$`DA</td><td>%ž''</td></tr></tbody></table>	

### 5ggYgga Ybh'8YHJ]`g

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8 YW] g.

±bX]j ]Xi U`
7 f]h]WU``UbU`mg]g`cZU`df]bh'UXj Yfh]gYa Ybh
%) \$\$`k cfXg
: YYXVUW\_'j ]U`@A G

#### 5XX]h]cbU`'XYhU]`g.

D`YUgY`fYZYf`hc`AccX`Y`Zcf`UXX]h]cbU``XYhU]`g"

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb & fb & gYh &$ 

5ggYgga Ybh& 5Xj Yfh]g]b[ 'h\fci [\'h]a Y'UbX'd`UWY

8YHJ]`g.

; fci d'dfYgYbhJhjcb. '5Xj Yfhjg]b[ 'h\fci [ \ 'h]a Y'UbX'd`UW\H\fci [ \ ci hh\Y'h\fa gž/2fca 'K YY\_g'( h\fci [ \ '%\$žgh XYbhg'dfYgYbhcb'cbY'dUfhj\W'`Uf'YdcW\]b'cbY'dUfhj\W'`Uf'^\fg]gX]\Mjcb"'Gh XYbhg'h\Yb``YUX'WUgg'X]g\W'gg]cb'UbX'fYZY\Mjcb" : YYXVU\M\_j ]U'@A G

#### 5XX]h]cbU`XYhJ]`g.

D`YUgY`fYZYf`hc`AccX`Y`Zcf`UXX]h]cbU``XYhU]`g"

Hi fb]h]b gYh]b[. H\]g ]g bchU Hi fb]h]b Ugg][ ba Ybh

5ggYgga Ybh' =ggi Yg ]b UXj Yfhg]b[

8YHJ]`g.

⊕X]j ]Xi U` %\$\$!k cfXg Ghi XYbhg`cZYf`U`g\cfhfYZYWhjcb`cb`h\Y`]a dUWhcf`]a dUWhg`cZUXj Yfhjg]b[ `fcf`U`dUfhjW\`Uf UXj YfhjgYa Ybhcf`WJa dU][ bŁcb`Ub`Ub'UgdYWhcZgcWjYhmž'h\Y`YWcbca mž'dc`]hjWgž'YhW : YYXVUW\_j ]U`@A G

#### 5XX]h]cbU`XYHJ]`g.

D`YUgY`fYZYf`hc`AccX`Y`Zcf`UXX]h]cbU``XYhU]`g"

## 5 https://bxubwww.Fyei]fYaYbhg

Ghi XYbhgʻUfYʻghfcb[`miYbWci fU[YX'hcʻUhhYbX'U```WUggYgʻUbX'fYj ]Yk '`YWhi fY`fYWcfX]b[g"

## 7ci fgY'GWXYXi `Y

### J ]Yk 'WUgg'h]a YhUV'Y

### H]a YtUV`Y

8 UHY	HmdY	7 cbMbh
( `>Ubi Ufm!`, `>Ubi Ufm	@YWhifY	@YWifY`%fAcbXUmL`H\Y`gcWjc`c[]WU``cf][]bg`cZ UXjYfhjg]b[`UbX`h\Y`f]gY`cZWcbgiaYf`W`hifY"
		GYa ]bUf'% 'D`YUgY'fYZYf'hc'A ccX`Y"
		@YWifY`&`fH\ifgXUmL`H\Y`Yjc`ihjcb`cZYUfm UXjYfhjg]b["
		GYa]bUf'&.'D`YUgY'fYZYf'hc'AccX`Y"
%%`>Ubi Ufm'! '%) '>Ubi Ufm	@YWhi fY	@YWhifY" fAcbXUmŁ?YmUXjYfh]g]b[ h\Ycf]Yg"
		GYa ]bUf'' . 'D`YUgY'fYZYf'hc'A ccX`Y"
		@YWnfY'('fH\ifgXUmL'H\Y'gcifWrg'UbX'igYg'cZdckYf']b'UXjYfn]g]b["
		GYa ]bUf'(.'D`YUgY'fYZYf'hc'A ccX`Y"
% '>Ubi Ufm!'&&'>Ubi Ufm	@YWn fY	@YWi fY') 'fA cbXUmŁ'? YmgcWjc`c[]WJ`'UbX dgmWkc`c[]WJ`'UXj Yfnjg]b['hcc`g'UbX'h\Y'W`hi fU` WcbhYl hcZUXj Yfnjg]b[.'gYa]chjWgža mh\c`c[]Ygž a YhUd\cfž'UbX'bUffUnjj Y"
		GYa ]bUf') . 'D`YUgY'fYZYf'hc'A ccX`Y"
		@YWn fY`*fH\ifgXUmL'8]ZZYfYbhhmdYgcZUXjYfhjg]b[a YggU[Yg'UbX'h\Y'UXjYfhjg]b['WUadU][b"
		GYa ]bUf'*. 'D`YUgY`fYZYf`hc'A ccX`Y"
&) `>Ubi Ufm!`&- `>Ubi Ufm	@YWhifY	@YWi fY'+'fA cbXUmŁ'H\Y'UhiYbhjcb'YWcbca mUbX X][]hU``UXj Yfhjg]b["
		GYa ]bUf'+.'D`YUgY'fYZYf'hc'A ccX`Y"
		@YWi fY', 'fH\i fgXUmŁ'GcWjU`a YX]U'UbX'UXj Yfhjg]b[ h\Ycfm'
		GYa]bUf', .'D`YUgY'fYZYf'hc'A ccX`Y"

## FYgci fWfg

DfYgWf]VYX F Ygci fWYg

FcX[YfgžG\Y``m'UbX`H\cfgcbž'9gh\Yf`ff9X]hcfgt/5Xj Yfh]g]b[`H\Ycfm'&bX`9X]h]cbž'&\$% žFci h`YX[Y"Df]bhj Yfg]cb.

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8][]hU`'j Yfg]cb.

FYWca a YbXYX FYgci fWYg

DYUgY'gYY'AccXY'Zcf'XYHJ]g"

7ci fgY '9j U'i Uh]cb 'UbX '8Yj Y'cda Ybh

D`YUgY`gYY`AccX`Y`Zcf`XYhU]`g"

### 5 WUXYa ]W<cbYglmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"`±hWJb`hJ\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UVW]XYbhJ``mWcdm]b[ 'Zfca `U`gci fW' k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a ]`Uf'k cfXg'hc'h\Y'cf][]bU`h\I hcf']XYU'k ]h\ci hUW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U'gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfc YWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci h'WfYX]h'