

# 7ci fgY'Cj Yfj ]Yk

## Ghuzz 7 cbhuWh8 Yhu]`g

#### '7cbj Ybcfg'

BUa Y	9a U]`	5j U]`UV]`]m	@cWJhjcb	D\cbY
8f <sup>-</sup> >cX] <sup>-</sup> 6fcc_g		: f]XUmgʻ‰!‱Ua UbXʻch\Yfʻh]a YgʻVm Uddc]bha Ybh	&' % 'K YVghYf 6i]'X]b[	-',))*')

#### 'Hi hcfg'

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUhjcb	D\cbY
>cX] <sup>-</sup> 6fcc_g		: f]XUmg:'%\$!:%WJa UbX:'ch\Yf'h]a Yg:'Vm Uddc]bha Ybh	&' % 'K YVghYf 6i ]'X]`b[	-',))*')

# GWXcc`7cbHJWh =bZcfa Uh]cb

Fcca " %ž~Yj Y`" FcVYfhK YVghYf~6i ]`X]b[

D\cbY. f\$&£'-',) '(,)\*

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. \hdg.##qUa "Ufhq"i bgk "YXi "Ui

 $\label{eq:hamiltonian} $$HY'GW.cc``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5\Lock\`YX[Y'h\Y'HfUX]tjcbU``7i\gtcX]Ubg\cong h\Y'6YXY[U'`fP\Ybg]b[tcb'WJa\di\gt\cong \cong m5\Lock\`YX][U'`fP\Ybg]b[tcb'WJa\di\gt\cong \cong m5\Lock\`Y'B[i\bbUk\U'\dYcd'Y'fbi\gth\'Y'B[i\bbUk\U'\dYcd'Y'fbi\gth\'Y'\UW\\`WJa\di\g'\cong \cong \cong m5\Lock\Y'\UbXg'k\YfY'YUW\\`WJa\di\g'\cong \cong \cong\cong \cong \cong \cong \cong \cong \cong \cong \cong \cong \cong$ 

 $\verb|h|UhXYj|Y`cd'h|Y]f'g_]``g']b'Y|dfYgg]b['h|Y]f']XYUg'UbX']b|YfYg|g/$ 

# XUbWFYei]fYaYbhg

¢i5ftghYb]g{UXbVV}KYcf@xX/hWfcfa[Y\```YVWiffYgžhifbcf]U`gžgWfYYb]b[gžUbX`cb`]bY`UWh]j]h]Yg"``

5 hh/bXUbWrcZhi hcf]U`g`]g`a UbXUhcfm]b`h\]g`Wci fgY''5 hh/bXUbWr]g`a UbXUhcfmVYWUi gY`]b`mci f h hcf]U`g`mci k]``UWhj Y`mYb[U[Y`k]h\`WcfY`Wci fgY`WcbhYbhh\Uhk]``Vi]`X`mci f`g\_]``g`]b`!!`UbX YbUV`Y`mci hc`XYa cbghfUhY`UhtU]ba YbhcZ!!`Ub`]a dcfhUbhWci fgY``YUfb]b[`ci hWca Y`f7@CŁh\Y`UV]`]mhc`"HYghUbX`Udd`mgca Y`cZh\Y`\_YmWcbWrdhg`UbX`XYVUhYg`]b`h\Y`X]gWJd`]bY`hc`j Uf]ci g`Z]`a UbX`a YX]U`hYI hg`UbX`WcbhYI hg'''I bYI W gYX`UVgYbWrZfca 'a cfY`h\Ub`&\$i `cZh`hcf]U`g`k]``fYgi`h]b b\b\g\V\X\UffYbZ\Uj\Zh\X\Uj\YfXch\Uj\Y`j]hk`i gYXk]g`Z]ba hh\UUj\Y`z`g\Yfi gYh]` h\g`\Xci fgYK\]jYbX

	_	

Hi hcf]U`	A cbhY]fcz'GhYd\Yb'f&\$%( Ł"'": ]h'hc'ZfUa Y.']a U[Y'UbX YX[Y']b'WcbhYa dcfUfm]bhYfZUWYgz"GWfYYbz'J c'i a Y )) z'=ggi Y'' z'%GYdhYa VYf'&\$%( z'DU[Yg'' *\$-' + 8 YNU]'g'UVcl higl dd'Ya YbhUfmfYUX]b['UbX'fYgcl fWfo cb'A ccX'Y" D`Um]b['k]h\'h\Y'ZfUa Y'!!']b!WUgg'Yl‰fV]gYg'fd'YUgY

	Hi lcf]U`	DUfh%: ]fghUggYgga YbhHU_Yg'd`UWY']b'h hcf]U`g'h\]gkYY_"'GYY'UggYgga YbhdU[Y'cb'A ccX`Y'Zcf'XYHJ]`g"
		DUfn&. gWfYYbžghcfmUbX d`chXi fUhjcbg
K YY_`).`% `A UfVX`!`&( A UfVX	@YWhifY	F Ya U_Ygž±bhYfa YX]U`]hmUbX h\Y 5 ZhYf`]j Yg c Z7 ]bYa U
	GWfYYb]b[	DgmWXcffligJUbGUbhž%-,Ł
		d`i g`YI WYfdhg`Zfca `DgmWXc`fk]hWXWc\W_ž%*\$Ł
	F YUX]b[	9ggYbhJU`fYUX]b[ .
		7 Uff][ mžA Y[ Ubˈf&\$%&Ł" "F Y!ghU[ ]b[ 'h\Y W]bYa U. BekUZał: ]`a 'GdYWfUhcfg\]džUbX'h\Y F YXi bXUbhBYk 'F Ya U_Yž" E UdWW [ Y fZW BSeffGdYW]U`]ggi Y "I bh]a Y`m7 ]bYa U"ŁŻj c`", ž UWWggYX'% '8 YWWa VYf'&\$% žO\hnd.#k k k "gWfYYb]b [ h\YdUgh"Wca #&\$%&#\$, #fY!ghU[ ]b[!h\Y!W]bYa U! dgmW\c!Z]`a!
		8YM]`gʻUVci hgi dd`Ya YbMfmfYUX]b[ 'UbX'fYgci fWfgcb'AccX`Y"
	Hi lcf]U`	9  Ua ]b]b[ 'Uf[ i a Ybhg'UVci hfYa YX]Uh]b[ 'W]bYa U h\fci [ \ 'gYei YbWY'UbU`mgYg'UbX';
		7 cbW/dhUbX:XYVUhY:a Udd]b[:'UbX:'dfYdUfUhcfmk cf_ Zcf'[fci d'dfcYWhUW]j]h]Yg:fZcf'k YY_'*Ł
K YY_'*.'&) 'A UfWX'!'' % A UfWX	Cb`]bY`5W]j ]hm	FYgYUfW\ KYY GYYAccX`YZcfcb`]bY fYgYUfW\ HJg_g hcVYi bXYfHJ_Yb h\]g kYY_"Cb`]bYUWh]j]h]Yg fYd`UWY``YWhi fYg UbX hi hcf]U`g"
K YY_'+.'%'5df]`'!'+'5df]`	@YWhi fY	5 Wci gh]Wg Ub X gd UWY. 7 ]b Ya U ]b Ub 91 dUb XYX : ]Y `X `
	GWfYYb]b[	; ]f`\ccX`f7Y`]bY`GW]Ua a Už&\$%(ŁfH67Ł
	F YUX]b[	9ggYbiJU`fYUX]b[.
		7\]cbž'A]WXY``f% - %L"`"E i]YhF Yj c`i h]cb"""'5bX F][]X'GHJ[bUh]cb""'HfUbg"'6Yb 6fYk gHYf"'

		dd"‰-!%(, " 8YHJ]`g'UVci hgi dd`Ya YbHJfmfYUX]b[g'UbX fYgci fWfg'cb'A ccX`Y"
K YY_', .', '5df]`'!'%( '5df]`	Hi hcf]U` @	"Hi hcf]U``dfYdUfUh]cb. "Wcbgi `hA ccX`Y`Zcf'h hcf]U` dfYdUfUh]cb"HJg_g"Zcf'h\]g"k YY_"

# FYgci fWyg

#### DfYgWf]VYX F Ygci fWYg

#### 9ggYbh]U`fYUX]b[.

 $H\Y'gYhk\ YY\_mfYUX]b[\ g'Zcf'h\]g'Wci\ fgY'UfY'Uj\ U]'UV'Y'h\ fci\ [\ \ @Y[\ Ubhc'j\ ]U'A\ ccX'Y'!!'d'YUgY'a\ U\_Y'gi\ fY'mci\ 'Vf]b[\ 'Y'YWfcb]Wcf'\UfX'Wcd]Yg'cZh\Y'fY'Yj\ UbhfYUX]b[\ g'hc'WUgg'YUW\'k\ YY\_'gc'h\Uh'h\Ym'WUb'VY' dfcdYf'mX]gW gqYX'UbX'XYVUhYX'''$ 

Cb`]bY`WcifgY`fYgcifWY.'5FHG&\$\*%]g`cb`AccX`Y—d`YUgY`Wcbgi`h']hfY[i`UfmiZcf`]adcftUbh']bZcfaUh]cbUVcih'h\Y`WcifgY"

9UW\`k\YY\_'g`hcd]W\Ug`]hg`ck\b`gYWf]cb`]b`A\ccX`Y"\#b`h\YgY`gYWf]cbg`mci\`k\]``\ZbX.

- 5b']blfcXi Wjcb'hc'h\Y'hcd]WVY]b['Yl Ua ]bYX/
- Gi [ [ YghYX Z fh\Yf fYUX]b[
- @]b\_g hc fY Yj Ubhik YVg]hYg
- @YWi fY g`]XYg
- Hi hcf]U`dfYdUfUh]cb']bZcfa Uh]cb

#### FYWca a YbXYX FYgci fWYg

# FYWca a YbXYX fYUX]b[.

- DUa '7 cc\_'ffYXŁ'H\Y'7 ]bYa U'6cc\_'f@cbXcb.'6: ₹&\$\$+ž' fX'YX]r]cbŁ"'H\]g']g'Ub'YI WY``YbhfYZYfYbWYVcc\_'Zcf'UbmcbY'Xc]b['U': ]`a 'Ghi X]Yg'a U'cf"' ≠ndfcj ]XYg'i gYZ ``]bZcfa Ur]cb'cb'X]ZZYfYbhh\YcfYr]WU`UddfcUWXYg'hc'Z]`a 'UbX'W]bYa U'ghi X]YgžX]ZZYfYbhW]bYa Ug'UbX'[YbfYgž'UbX'X]ZZYfYbhdYf]cXg'cZZ]`a '\]ghcfm"'7 cd]Yg'Uj U]`UV'Y']b'hXY`]VfUfm"
- •; `mb'8Uj ]gž? Um8 ]W\_]bgcbž@]gU'DUh]žUbX'5a mJ ]``UfY'cž: ]`a 'Ghi X]Yg.'5'; `cVU``+bhfcXi Wh]cb fBYk 'Mcf\_'UbX'@cbXcb.'F ci hYX[Yž&\$%) Ł"'H\]g'bYk 'di V`]WUh]cb']g'U'i gYZ ``fYgci fWY'Vch\ 'Zcf h\cgY'bYk 'hc': ]`a 'Ghi X]Yg'UbX'Zcf'h\cgY'k \c'UfY']a a YfgYX']b'h\Y'Z]Y`X"'Ghfcb[`mfYWca a YbXYX'!! YVcc\_'Uj U]`UV'Y'h\fci [\h\Y'`]VfUfmž`]b\_'cb'A ccX`Y
- 8 "B "F cXck ]W\_'H\Y'J ]fhi U''@]ZY'cZ: ]`a ''f@cbXcb''UbX'7 Ua Vf]X[ Yž'A Ugg". '< Ufj UfX'l Dž'&\$\$+Ł"'5b ]a dcfhUbhUbX'\_YmhYl h`]b''WcbhYa dcfUfmZ]`a 'h\Ycfmž'UbX'U'\_YmhYl h`Zcf'h\]g'Wci fgY"'9! 'Vcc\_Uj U]`UV'Y'h\fci [ \ 'h\Y'`]VfUfmž'`]b\_'cb'A ccX'Yž

# 7 ci fgY '9j U`i Uh]cb 'UbX '8 Yj Y`cda Ybh

7 ci fgY'a UhYf]U`g`UbX`Wci fgY`ghfi Wi fY`f]bWi X]b[ 'h\Y`fYUX]b[ gž`gWfYYb]b[ gž`cfXYf]b[ 'cZhcd]Wg`UbX`hcd]Wgž UbX`h\Y`Zcfa g`cZUggYgga Ybhz'UfY`fY[ i `Uf`mfYj ]gYX`VUgYX`cb`ghi XYbh'ZYYXVUW\_'Ug`k Y```Ug`bYk `X]fYWf]cbg ]`a `\]ghY\Y`c`fYUYYb]b[ g!X`cb`gf]U`g`Uba ]bYX/

Wci fgYz'bch'cb`mh\fci [\'h\Y'Zcfa U`'ZYYXVUW_'dfcW/gg'Vi	hU`gc`]b`WUgg`X]gW gg]cbg`Xi f]b[ 'h\Y`gYa YghYf"

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Hi fb]h]b Gi Va ]gg]cb

\_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb[ 'hc'gi Va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZUi 'hk ]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW\_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb[ 'k ]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k ]``

### 5 WUXYa ]W<cbYghmUbX D`U[ ]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"'\lefi\WJb'hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UVW]XYbhU`m\Wcdm]b[ 'Zfca `U`gci f\W' k ]h\ci h\U\W\_bck `YX[ Ya Ybh'

IBGK [fcidgdu[]Uf]ga ]bhch\YZc``ck]b[ WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b[ 'U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b[ 'h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k ]h\ci hUW\_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci hWfYX]hUbX'hc d]YW]b[ 'hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b[ "

7 c``i g]cb. 'k cf\_]b[ 'k ]th 'cth Yfg'Vi hidUgg]b[ 'cZZth Y 'k cf\_'Ug'U'dYfgcb'g']bX]j ]Xi U``k cf\_"'7 c``i g]cb'U`gc ]bWi XYg'dfcj ]X]b[ 'mci f'k cf\_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'Uh'l BGK "'\\hd.##gi V'YWfi |XYg"`]VfUfm'i bgk "YXi "Ui #\`]qY#JVci hY`]qY