

ARTS2095

8][]hu``HYW\bc`c[]Yg`UbX'h\Y`GY`Z

HYfa 'C bY''##'&\$%

7ci fgY[·]Cj Yfj]Yk

GHJZZ:7cbHJWh:8YHJ]`g`

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`] m	@cWJh]cb	D\cbY

7ci fgY 8YHJ] g

7 fYX]hDc]bhg *

Gi a a UfmcZh\Y 7ci fgY

Gi VYWn5fYU. ? WW[Sł 5g/fgdWS` V FWUZ` a 'aYk

H\]g`Wci fgY`Yl d`cfYg`h\Y`k Umg`h\UhX][]hU``a YX]U`UbX`hYW\bc`c[]Yg`g\UdY`ci f`Wc``YWfj Y`UbX`]bX]j]Xi U` i bXYfghUbX]b[g`cZ'h\Y`gY`Z"`#iXfUk g`Vch\`h\YcfYhJWU``UbX`dcdi `Uf`UW\ci bhg`cZh\Y`fY`UhJcbg\]d`VYhk YYb hYW\bc`c[]Ygž]bX]j]Xi U`g`UbX`W``h fYg"`Nci `k]```Yl Ua]bY`h\Y`WcbhJbi]hJYg`UbX`X]gWcbhJbi]hJYg`VYhk YYb`'c`X' UbX`'bYk '`gY`Zla U_]b['dfUWfJWYg`gi W\`Ug`ci fbU`]b[#V`c[[]b[/UbX`gY`ZlacfhfU]hi fY#gY`ZJYgž'hc`Wcbg]XYf`h\Y k Umg`h\Uha YX]U`UbX`hYV\bc`c[mVch\`g\UdYž'UbX`UfY`g\UdYX`VmgcWJc!Wi`hi fU``i bXYfghUbX]b[g`cZ]XYbhJm Zcfa UhJcb"`Nci `k]```Yl d`cfY`h\Y`k Umg`h\UhiHY\bc`c[]Yg`UbX`a YX]U`d`UhZcfa g`WcbgHU]b`gca Y`gY`j Yg``UbX YbUV`Y`ch\Yfg"`K Y`k]```U`gc`Wcbg]XYf`h\Y`k Umg`h\UhigY`Zla U_]b[`dfUWfJWYg`]bhYfgYWhk]h\`XUhU![Uh\Yf]b[dfUWfJWYgžXYg][b`dfcWYggYgžVi g]bYgg`a cXY`gžUbX`di V`]Wdc`]Wh'`Hcd]Wg`]bWi XY.``]ZY!\UW_]b[`UbX`h\Y ei UbhJZYX`gY`Za cj Ya Ybhža]WfcWYYfJhmUbX`VfUbX]b[ža YX]UhYX`]bhJa UWJYg`]b`XUhJb[`UbX`\cc_!i d`Udd W``hi fYgžUbX`V]ca Yhf]Wg`UbX;gi fj Y]``UbW\"

7ci fgY @YUfb]b[Ci hWca Yg

- %" 9j U`i UHY`U`fUb[Y`cZX][]HU``HYV\\bc`c[]Ygž]bWi X]b[`gcV]U``a YX]U`d`UHZcfa g`UbX`dfUWf]W\g" &" 5dd`mH\YcfYH]WU``ZfUa Yk cf_g`]b`cfXYf`hc`Yl d`U]b`H\Y`]a dUWicZa YX]U`UbX`HYV\\bc`c[mcb`]bX]j]Xi U``UbX`Wc``YV\fj Y`Yl dfYgg]cbg`cZ]XYbH]m"
- ' " 7 ca a i b]\WUHY`\WYUf`m\UbX`d\Yfgi Ug]j Y`m]b`\Vch\`j Yf\VU``UbX`k f]\HYb`\\CbHYl \mg"
- ("8Yg][b'UbX'WcbXi WhVUg]WfYgYUfWXž'Vch\`]bXYdYbXYbhmUbX`]b'HYUa gž'XfUk]b['cb'UWUXYa]WUbX bcb!UWUXYa]WfYgci fWYg"

HYUW]b['GhfUhY[]Yg'

5ggYgga Ybh

 $H\ [g.7ci\ fgY.Cj\ Yfj\]Yk\ "dfcj\]XYg\ "mci\ "k\]h\ "h\ Y\ VUg]W]b\ Zcfa\ Uh]cb\ "UVci\ h'h\ Y\ Wci\ fgY\ "UbX\ YI\ d\ "U]bg\]a\ dcfhUbh\ I\ B\ GK\ "dc\]Wh" 8\ Yh\ [Jg\ Zcf\ YUWX\ "UggYgga\ Ybh'UfY\ Uj\ U]\ "UV\ Y\ "cb\ "A\ cc\ X\ Y"$

Hc dUgg h\]g Wci fgYžmci a i gha U_Y U gYf]ci g UhhYa dhUh5 ggYgga Ybhg %UbX " "

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hJg_	K Y][\h	8iY'8UhY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
K f]h]b['HUg_'	(\$1	%\$#\$' #&\$% '%%) - 'DA	%ž&Ž
±oX]j]Xi U`F YgYUfW\Dfc YWh 8Yj Y`cda Ybh	%) ၊	Bch5dd`]WUV`Y	% <u>%</u> &ž(
±bX]j]Xi U`F YgYUfW\`Dfc YWh	()।	%(#\$(#&\$% `\$, .\$(`DA	%ž&Ž Ž(

5ggYgga Ybh8YhJ]`g

5ggYgga Ybh% K f]h]b[HUg_

GhUfhiXUhY. Bchi5ddi]WUVY

@Yb[h\. &**ž**\$\$\$ k cfXg

8YHJ]jg. '&ž\$\$\$!k cfX'k f]hYb 'Ugg][ba Ybhž'ZYYXVUW_'dfcj]XYX'j]U'Hi fb]h]b"

 $\label{thm:continuous} \begin{tabular}{ll} Hi fb]h]b gYh]b[. Hi]g Ugg][ba Ybh]g gi Va]hh X h fci [in the fill b]h]b Ub X gh XYbhg Wub gYY Hi fb]h]b g]a] Uf]hm fYdcffhg"$

5ggYgga Ybh' . '₽X]j]Xi U`FYgYUfW\ Dfc YWh

GHJfhXUhY. Bch5dd`]WUV`Y

@Yb[h..&z)\$\$ k cfXgz cf Yei] U Ybh

8YHJ]`g. '&ž) \$\$!k cfX`cf`Yei]j U`YbhfYgYUfW\\`dfc\Y\\\\\\ZYYXVU_`dfcj]XYX`j]U`Hi fb]h]b"

 $\begin{tabular}{ll} \textbf{Gi Va } \begin{tabular}{ll} \textbf{Gi Va } \be$

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' \end{tab$

5 hmy b X Ub WY F Yei] f Ya Yb hg

5 thY b X U b WY c Z @Y W i f Y g ' U b X ' H i | tc f] U `g `] g `a U b X U hc fm] b `th] g ` Wc i f g Y " I b Y I W g Y X ` U V g Y b W ` Z f ca `a c f Y `th U b &\$ i `k] `` f Y g i `th] b `th Y ` U k U f X ` c Z U ` Z U] `` [f U X Y "

± h hcf]U`g`mci k]```UWhj Y`mYb[U[Y`k]h\`WcfY`Wci fgY`WcbhYbhh\Uhk]```YbUV`Y`mci hc`XYa cbghfUhY UHtU]ba YbhcZfcf`\Y`d`mci hc`UHtU]bŁ7ci fgY`@YUfb]b[`Ci hWca Yg`%ï&ž' `UbX`("

± ``YWi fYg`mci k]```UWij Y`mYb[U[Y`k]h\`WcfY`Wci fgY`WcbhYbh'h\Uhk]``YbUV`Y`mci hc`XYa cbghfUhY
UhtU]ba YbhcZfcf`\Y`d`mci hc`UhtU]bŁ'h\Y`UV]`]mhc`Yj U`i UhY`U`fUb[Y`cZX][]hU``hYW\bc`c[]Ygž]bWi X]b[`gcW]U`
a YX]U'd`Uhzcfa g`UbX`dfUWjWfg`fl7ci fgY`@YUfb]b[`Ci hWca Y`%Ł'UbX`Udd`mh\YcfYh]WU``ZfUa Yk cf_g`]b`cfXYf
hc`YId`U]b`h\Y`]a dUWicZa YX]U`UbX`hYW\bc`c[mcb``]bX]j]Xi U``UbX`Wc``YWij Y`YIdfYgg]cbg`cZ]XYbh]hm
fl7ci fgY`@YUfb]b[`Ci hWca Y`&Ł"

7ci fgY'GWXYXi `Y

J]Yk 'WUgg'hja YHJV'Y

FYgci fW/g

DfYgWf]VYX F Ygci fWYg

D`YUgY`gYY`A ccX`Y`Zcf`U```fYei]fYX`fYUX]b[g`UbX`fYgci fWYg"

FYWca a YbXYX FYgci fWYg

D`YUgY`gYY`AccX`Y`Zcf`U```fYei]fYX`fYUX]b[g`UbX`fYgcifWYg"

7ci fgY 9j U'i Uhjcb UbX 8Yj Y'cda Ybh

Gh XYbhZYXVUW_]gij U`i YX`]b h\]g Wci fgY`UbX`\Y`dgii gi]a dfcj Y`mci f``YUfb]b[`YI dYf]YbW\"' ⊕ h\Y'dUghž UbU`mg]g`cZA m9I dYf]YbW\"gi fj YmfYgi `hgžUg`k Y```Ug`X]fYWhZYXVUW_Zfca `gh XYbhgž`\Ug``YX`hc`fYXi W\X bi a VYfg`cZfYUX]b[gž'bYk `Wci fgY`hcd]Wg`UbX`h\Y`fYZ]bYa YbhcZUggYgga Ybh]bghfi Wf]cbg": cf'YI Ua d`Yž h\Y`YI dYWhJhcbg`Zcf'5ggYgga Ybh' . '⊕X]j]Xi U``F YgYUfW\`DfcYWh\Uj Y`VYYb`a cfY`WYUf`mgYhci hZc``ck]b[gh XYbhZYYXVUW_"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwb: bh/f'u'dfcv'ya k \yb'uhh/a dh/b['hc'gi va]hmci f'ugg][ba Ybhh\fci [\ 'Hi fb]h/bž'd'YugY

HY'Yd\cbY'9| h/fbu''Gi ddcfhcb'-',) '' ' ' %cf'Ya U] 'h\Ya 'cb'Yl h/fbu'h/hgi ddcfh4 i bgk "YXi "Ui "'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'\\$\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUfŁ'' Image
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h/b'mci 'a UmUdd'mZcf'Ub'Yl h/bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h/W_Yhbi a VYf'Zfca '9| h/fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'YY] UbhXcW a YbhgŁ'hc
]bWi XY'Ug'Yj]XYbW''hc'gi ddcfhmci f'Yl h/bg]cb'Udd']\WJh/jcb'' Image 'Ya U]''9| h/fbU''Gi ddcfhmci 'k]'`
Ui hca Uh/WJ'`mfYW']j Y'U'h/W_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'`'bYYX'hc'gdYW/ZWJ'`mUg_'Zcf'cbY''
Hi fb]h/b'U'gc'dfcj]XYg'i dXUh/g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hYf''

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mj]U'Y]h\Yf'Hi fb]hjb'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]``'VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\ U`hYfbUhj Y'gi Va]gg]cb'XYHJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va]hUgq][ba Ybha ccXiY

5 WUXYa | W < cbYghmUbX D`U[| Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUigci fWik]h\ci hiUW_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'Uh'l BGK "'\\hd.##gi V'YWfi |XYg"`]VfUfm'i bgk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUXYa]W±bZcfa Uhjcb

: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['hc. • fYei YghgʻZcfʻYl hYbg]cb/