

ARTS2121

H\YUhfY'UbX'7i ffYbh'9j Ybhg

HYfa 'C bY''#'&\$%-

GWCC``CZhAY5fhg'UbX'AYXJU`##'IBGK '5fhg'UbX'GcWJU`GWJYbWYg

7ci fgY[.]Cj Yfj]Yk

7ci fgY[·]8YHJ]`g

7 fYX]hDc]bhg[·]*[·]

Gi a a UfmcZh\Y'7ci fgY

Gi VYW15fYU. FZWSfdWS` V BW0Xad_ S` UWE fgV[We

5 fh]ghg 'U'k Umg 'fYgdcbX'hc 'h Y ']ggi Yg 'cZh Y Jf 'h]a Y" 5 bX'h YUHY 'a U_Yfg 'UfY 'cZhYb 'h Y 'Z]fghhc 'fYgdcbX'k \Yb Ub ']ggi Y 'Ya Yf[Yg" '=b 'h]g 'Vci fgYžmci 'k]``Yl d`cfY \ck 'Ufh]ghg \Uj Y 'WYUHYX'k cf_g 'h UhYb[U[Y 'Ui X]YbWYg]b 'W ffYbhYj Ybhg 'UbX 'VcbhYa dcfUfmXYVUHYg" 'K]h 'YUW ']ggi Yžmci 'k]``Yl d`cfY ']hg 'gcW]U`cf][]bg 'UbX'_Ym h]b_Yfgž'UbX'h Y 'a Yh cXg'h Uh'h YUHY!a U_Yfg \Uj Y 'i gYX'hc 'fYgdcbX" 'K]h 'h Y 'g_]``g'mci 'XYj Y`cdž'mci k]``YY 'UV'Y 'hc 'UbU'mgY'\ck 'h YUHY!a U_Yfg ']bhYfj YbY']b 'ci f'W 'h fY 'UbX'XYj]gY 'dYfZcfa UbWYg'h Uh Yb[U[Y 'Ui X]YbWYg ']b ']ggi Yg 'h Uh'k bW fb 'mci 'hc XUm''H\Y'U]a ']g 'hc 'YUfb \ck 'h Y 'k cfX'k cf_g 'k]h]b h YUHY 'UbX'\ck 'h YUHY'k cf_g ']b 'h Y'k cfX" 'H\]g ']g 'U'WcfY '@Yj Y``& Wci fgY ']b 'h\Y'H\YUHY'UbX DYfZcfa UbWY Ghi X]Yg 'gffYUa '''

7cifgY`@YUfb]b[`CihWcaYg

- % X]gW ggih YigdYW]ZYXih YcfYh]WU``ZUa Yk cf_giUbXi\ck ih Ym\Uj Yi]bZi YbW/XidYfZcfa UbW/a U_]b[UbXifYW/dh]cb/
- &" YI d`U]b`\ck`f\Y`Uff]gh]WWUgY`ghi X]Yg`Ya VcXmW\Ub[Yg`]b`f\Y`Zcfa žWcbhYbfz`UbX`dc`]h]Wg`cZ K YghYfb`dYfZcfa UbWY`g]bWY`f\Y`% *\$g"
- ' "UbU`mnY`UbX`YI dYf]a Ybhk]h. WebhYa defUfmidYfZefa UbWY`UddfeUWXYg`hc`hYI hzgdUWYž'h]a Yž dYfZefa]b[`'UbX`gdYWUh]b["
- ("UggYgg'h\Y`k Ummci f`ck b`Yl dcgi fY`hc`WcbhYa dcfUfmih\YcfmUbX`dYfZcfa UbVW`dfUWf}VW`\Ug]a dUWfYX`cb`mci f`UddfcUVX`hc`h\Y`bUhi fY`UbX`Z bWfjcbg`cZ`]j Y`dYfZcfa UbVW

HYUWX]b[GhfUhY[]Yg

DYfZcfa UbWr WUb bYj Yf VY X]j cfWr X Zrca `]ng gcW]U žW 'h fU ž\]ghcf]WU žUbX a UhYf]U`W]fW a gHUbWrg" <YbWr hc 'gh XmU dYfZcfa UbWr hYI h]g 'hc 'gh Xm]'ng WobhYI hhc c" H\YgY 'k]b 'HUg_gž'hc `YUfb \ck 'hc 'fYUX'hYI hg UbX WobhYI hgžfYei]fY a i `h]d Y a Yh\cXgžgca Y 'hYUW\Yf!X]fYWhYX UbX gca Y 'gh XYbh 'YX" H\]g Woi fgY i gYg U Woa V]bUh]cb cZ YWi fYg k]h 'gWfYb]b[gž'h hcf]U 'gžUbX U ZUW] [hUhYX WfYUh]j Y 'dfc Wrgg UbX'dfYgYbHUh]cb" =b 'UXX]h]cb žgh XYbhg UfY 'YI dYWhYX hc 'k cf_']bXYdYbXYbhmž ci hg]XY cZWUgg 'h]a YžfYUX]b[ž'hU_]b['bchYgž k UhW] [b['j] XYc 'W]dgžd `Ubb]b['dfUWf]WU`UWh]j]h]YgžUbX 'UfhYbX]b[``]j Y'dYfZcfa UbWrg"'H\Y 'YWi fYg 'gYhci h [YbYfU' 'h\YcfYh]WU` ZfUa Yk cf_g 'UbX ']bhfcXi Wr dUfh]W `Uf 'h\YUhf]WU` 'k cf_g" H\Y 'h hcf]U`g`UfY' cddcfhi b]h]Yg Zcf'h\Y '[fci d 'hc 'X]gW gg UbX fYZYWicb 'h\YgY WobWrdhg UbX 'WlgY ghi X]Yg 'Ug 'k Y``Ug 'hc 'Xc 'gca Y 'dfUWf]WU` YI YfW]gYg 'UbX 'YI dYf]a Ybhg": cf'hk c'k YY_g ']b 'h\Y `UhYf'dUfhicZh\Y 'Wci fgYž'h\Y `YWi fY UbX'h hcf]U`g`UfY fYd`UWrX 'Vmghi XYbh 'YXž'WfUh]j Y 'dfcWrgg UbX dfYgYbHJh]cbg"'H\]g 'XYj Y`cdg'h\Y]f`Uk UfYbYgg'cZh\Y]f`ck b UV]`]h]Yg 'UbX'fYgdcbg]V]`]h]Yg 'Ug 'W 'h fU`dfcXi Wrfg"

FZWSfdWS`V5gdWif7hWife'YIHYbXg'_bck'YX[Y'cZVcH\'h\YUHY'ghiX]YgžUg'WcjYfYX']b'h\Y'[UHYkUm WcifgY'3DFE##\$" '7jbWd[WU[`YFZWSfdWzUbX'dYfZcfaUbW'ghiX]YgžUg'WcjYfYX']b'h\Y'[UHYkUmWcifgY' 3DFE##\$#FZW>[XWaXBWdXad_S`UW'=hYIHYbXg'h\Y'Wiff]Wi`U'cZh\YgY'WcifgYg'VmZcWig]b['cb'U'jUf]YhmcZ dYfZcfaUbWY'[YbfYgž]bWiX]b['XfUaUh]Wd`Umgž'dcghXfUaUh]Wh\YUHYž'dYfZcfaUbWY'Ufhž'UbX'ghUbX!id WcaYXm''=h]bhY[fUhYg'gY'ZfYZYI]jY'UbX'dYfZcfaUh]jY''YUfb]b['UbX'hYUW}]b['ghfUhY[]Yg']b'Vch\'dYXU[c[m UbX'UggYggaYbh'

5ggYgga Ybh

 $\label{eq:linear} D`YUgY`igY`hY`7\]WU[c`5ihcf!8UhY`fYZYfYbW]b[`gmghYa`ff%+h`YX]h]cbL''$

H\Y`E`i`]W_`;``i`]XY`]g`Uj`U]`UV`Y`\YfY.`

<u>\htd.#k k k "VX]WU[ca Ubi U`cZghm`Y"cf[#cc`gSWJhUh]cb[i]XY#/VhUh]cb![i]XY!&"\ha``</u>

H\Y': i ```; i]XY']g'Uj U]`UV`Y'j]U'h\Y'I BGK `@]VfUfm

<u>\htd.#k k k "VX]WU[ca Ubi U`cZghmY"cf["k k k dfcl m%"`]VfUfm'i bgk "YXi "Ui #/cc_#/X%+#Zfcbha UhhYf#ncW\ha``</u>

5ggYgga YbhHUg_g

5ggYgga YbhHJg_	K Y][\h	8iY'8UHY	GhiXYbhi@YUfb]b[CihWcaYgʻ5ggYggYX
5bU`mhjWU``9b[U[Ya Ybh)\$I	\$, #\$' #&\$% ⁻ 'UbX`&- #\$' #&\$%-	%ž&ž ž(
; fci d'dYfZcfa UbWY	&) I	%+#\$(#&\$%-`\$\$\$`5A	%ž
FYZYWAjiY`9ggUmi	&) I	&*#\$(#&\$%-`%%)-`DA	%ž(

5ggYgga Ybh8YHJ]`g

5ggYgga Ybh'% 5bU`mh]WU`9b[U[Ya Ybh

GHUfh'XUHY.

8YHU]`g. '&'l '%\$\$\$!%&) \$'k cfXg; fUXYžFi Vf]WUbX'7 ca a Ybh

5XX]h]cbU``XYhJ]`g.

D`YUgY`bchY`h\Uhih\]g`UggYgga YbhWcbg]ghg`cZhk c`YggUmg`cZ%z\$\$\$!%z&) \$`k cfXg`YUW\"`H\Y`Z]fgh`]g`Xi Y`Uh h\Y`YbX`cZK YY_`' ž`cb`: f]XUmž', `A UfW\`&\$% `Vm`%%') - da "`H\Y`gYWcbX`]g`Xi Y`Uhih\Y`YbX`cZK YY_`* ž`cb : f]XUmž`&- `A UfW\`&\$% "

5ggYgga Ybh'%"%" K f]hY 'U'%ž\$\$\$!k cfX'UbU`mg]g'cZcbY fYWYbhZYa]b]ghdYfZcfa UbWY ffZca 'h\Y``]ghcZWUgY ghi X]Yg'dfcj]XYX']b'WUggŁ" F i Vf]W]bX]WUh]b['`Yj Y``cZdYfZcfa UbWY 'U[U]bghYUVX 'UggYgga Ybh'Wf]hYf]Už k f]hhYb'Zcfa Uh]j Y'ZYYXVUW_ž'UbX'dYfWYbhU[Y`[fUXY"'8 i Y'Uhh\Y'YbX'cZK YY_'' žcb': f]XUmž, 'A UfW\'&\$%-Vm'%%') - da "

5ggYgga Ybh%"&"`K f]hY`U`%ž\$\$\$!k cfX`UbU`mg]g`cZcbY`fYWYbhdcghWc`cb]U``dYfZcfa UbWY`fZfca `h\Y``]ghcZ WUgY`ghi X]Yg`dfcj]XYX`]b`WUggŁ"`F i Vf]W]bX]WUh]b[``Yj Y``cZdYfZcfa UbWY`U[U]bghYUVX`UggYgga YbhWf]hYf]Už k f]hhYb`Zcfa Uh]j Y`ZYYXVUW_ž`UbX`dYfWYbhU[Y`[fUXY"`8 i Y`Uhih\Y`YbX`cZK YY_`*žcb`: f]XUmž&- `A UfW\`&\$%-Vm'%%") - da "

Hi fb]h]b'gYh]b[.`H\]g'Ugg][ba Ybh]g'gi Va]hYX'h\fci [\`Hi fb]h]b'UbX'ghi XYbhg'Xc'bchgYY'Hi fb]h]b g]a]`Uf]hmifYdcfhg"

5ggYgga Ybh&; fciddYfZcfa UbVW

GHUfh'XUhY.

8YHU]`g. %\$`a]bi HYg; fUXYž'Fi Vf]WZUbX`7ca aYbh

5XX]h]cbU``XYhU]`g.

l

	Hi hcf]U`	C bY`\ci f`X]gW gg]b[`dYfZcfa UbW``WUgY`ghi X]Yg/ cbY`\ci f`cZdYfZcfa UbW``YI YfWjgYg"
K YY_`, .`, `5df]``!`%(`5df]`	@YWhi fY	FY\YUfgU``Zcf'; fci d`DYfZcfa Ub₩
	Hi hcf]U`	FY\YUfgU``Zcf'; fcid`DYfZcfaUb₩
K YY_'`%)	@YWhifY	; fci d`DYfZcfa UbWYg°
5df]`	Hi hcf]U`	: YYXVUW_`cb`; fci d`DYfZcfa UbWYg
	5ggYgga Ybh	5ggYgga Ybh&.; fci d`DYfZcfa UbWY`Xi Y`Xi f]b[`h\Y `YWMi fY`g`chž`]"Y"`K YXbYgXUmž`%+`5df]``&\$% `Uh-`Ua "
K_YY_`%\$.`&&`5df]``!`&, 5df]`	5ggYgga Ybh	5ggYgga Ybh' . F YZYWjj Y 9ggUmXi Y cb': f]XUmž&* 5df]`&\$% 'Vm'%") - 'da "

FYgci fWfg

DfYgWf]VYX F Ygci fWyg

H\Y`gYhfYUX]b[g`UfY`UjU]`UV`Y`]b`h\Y`WcifgY`AccX`Y`g]hY`UbX`j]U`h\Y`IBGK`aU]b``]VfUfm'`H\YfY`]g`bc GhiXm`?]h'Zcf`h\]g`WcifgY"

FYWca a YbXYX FYgci fWYg

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

=Zmci YbWci bhYf'U'dfcV'Ya 'k \Yb'UhYa dhjb['hc'gi Va]hmci f'Ugg][ba YbhN fci [\ 'Hi fb]hjbž'd'YUgY hY'Yd\cbY'9I hYfbU'`Gi ddcfhcb'-',) '' ' %cf'Ya U]`'h\Ya 'cb'YI hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '" Gi ddcfh \ci fg'UfY', .\$\$Ua '--'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '--) .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUfŁ'" =Zmci UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi `hk]h\ 'Hi fb]hjb'mci 'a Um'Udd`mZcf'Ub'YI hYbg]cbž'Vi hmci a i ghfYhU]b'mci f'hjW_Yhbi a VYf'Zfca '9I hYfbU``Gi ddcfhfU`cb['k]h`Ubm'ch\Yf'fY`Yj UbhXcW a YbhgŁ'hc]bWi XY'Ug'Yj]XYbWi'hc'gi ddcfhimci f'YI hYbg]cb'Udd`]WUh]cb''`=Zmci 'Ya U]``9I hYfbU``Gi ddcfhimci 'k]``

5WUXYa]W<cbYghmUbX`D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b[`h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[`h\Ya `Ug`mci f`ck b"`=hWUb`hU_Y`a Ubm Zcfa gžZca `XY`]VYfUhY`W\YUh]b[`hc`UWW]XYbhU``mWcdm]b[`Zca `U`gci fWY`k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd`U[]Uf]ga]bhc h\Y Zc``ck]b[WUhY[cf]Yg.

7 cdm]b[..i g]b[.ih,Y`gUa Y`cf`j Yfmig]a]`Uf`k cfXg`hc`h,Y`cf][]bU``hYI h`cf`]XYU`k]h\ci hUW_bck `YX[]b[`h\Y gci fWF`cf`i g]b[`ei chUh]cb`a Uf_g"`H\]g`U`gc`Udd`]Yg`hc`]a U[YgžUfhUbX`XYg][b`dfc´YWfgž'Ug`k Y```Ug dfYgYbhUh]cbg`k \YfY`gca YcbY`dfYgYbhg`Ubch\Yf*g`]XYUg`cf`k cfXg`k]h\ci hWfYX]h"

=bUddfcdf]UhY`dUfUd\fUg]b[..7\Ub[]b[`U`ZYk`k cfXg`UbX`d\fUgYg`k \]`Y`a cghmfYhU]b]b[`h\Y`cf][]bU` ghfi Whi fY`UbX#cf`dfc[fYgg]cb`cZ]XYUg`cZh\Y`cf][]bU`žUbX`]bZcfa Uh]cb`k]h\ci hUW_bck`YX[Ya Ybh`H\]g`U`gc Udd`]Yg`]b`dfYgYbhUh]cbg`k \YfY`gca YcbY`dUfUd\fUgYg`Ubch\Yf'g`]XYUg`cf`k cfXg`k]h\ci hWfYX]hUbX`hc d]YW]b[`hc[Yh\Yf`ei chYg`UbX`dUfUd\fUgYg`]bhc`U`bYk`k \c`Yžk]h\ci hUddfcdf]UhY`fYZYfYbW]b["

7 c``i g]cb. k cf_]b[k]h ch Yfg Vi hdUgg]b[cZth Y k cf_'Ug U dYfgcb'g]bX]j]Xi U`k cf_'' 7 c``i g]cb U gc]bWi XYg dfcj]X]b['mci f k cf_'hc Ubch Yf gh XYbhVYZcfY h Y Xi Y XUhYžcf Zcf'h Y di fdcgY cZh Ya

- i bXYfgHJbX'mci f`f][\hg`UbX'fYgdcbg]V]`]h]Yg`Ug`U`ghi XYbh'Uh'I BGK
- VY Uk UfY cZdU[]Uf]ga žWcdmf][\lbzi BGK Ghi XYbh7 cXY cZ7 cbXi Wh0bX 5WW/dhUVY I gY cZ I BGK '=7 H'F Ygci fW/g'Dc']Wh
- VY Uk UfY cZh Y gHUbXUfXg cZVY Uj]ci f YI dYWYX cZYj YfmcbY]b h Y I BGK Wca a i b]m
- `cWUHY'gYfj]WYg'UbX']bZcfa UHjcb'UVci hI BGK 'UbX'I BGK '@VfUfm

Gca YicZh\YgYiUfYUgik]``VYiZJa]`]Ufihcimci žich\Yfgik]``VYibYk"'; U]b]b[`Uigc`]Xii bXYfgHUbX]b[`cZU```h\Y fY`UhYXiUgdYWfgicZ9@=G9ik]``\Y`dimci ia U_Yih\Yia cghicZmci fighi X]YgiUhil BGK"' <u>\hhd.#qi VYWfii]XYg"`]VfUfm"i bgk"YXi"Ui #Y`]qY#UVci hY`]qY</u>

5WUXYa]W=bZcfaUh]cb[·]

: cf'YggYbhJU``ghi XYbhi]bZcfa UhJcb`fY`UhJb['hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UHY'gi Va]gg]cbg'[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK '<YU`h\ UbX'GUZYhmidc`]W}Yg/
- YI Ua]bUh]cb'dfcWYXi fYg/
- gdYWJU`Wcbg]XYfUh]cb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybh fY/
- gh XYbhYei]hmUbX`X]gUV]`]hm/
- UbX'ch\Yf'YggYbh]U``UWUXYa]W]bZcfa Uh]cbž'gYY

<u>htdg.#k k k "Ufng"i bgk "YXi "Ui #W ffYbHgh XYbhg#UWUXYa]W]bZcfa Uh]cb#dfchcWc`g![i]XY`]bYg#</u>

∋a U[Y[·]7fYX]h