

ARTS2122

DYfZcfa UbWY DfcXi Wgcb

HYfa 'C bY''##'&\$%

7ci fgY[·]Cj Yfj]Yk

`GHJZZ'7cbHJWh'8YHJ]`g`

7cbj Ybcfg

BUa Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D∖cbY
7 Ufc`]bY'K U_Y				
		1		
	+			

7ci fgY 8YW] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

Gi VYWh5fYU. FZWSfdWS` V BWdAd_S` UWEfgV[We

H\]gʻdfUWfJWJ``ghi X]c!VUgYX`Wci fgY`[]j Ygʻmci ʻh\Yʻcddcfhi b]hmhcʻVY`]bj c`j YX`]bʻh\Y`WfYUh]cbʻcZUʻdi V`]W h\YUhfYʻdfcXi Wfjcbʻ''Mci ʻk]``hU_YʻdUfh`]bʻUbʻYI Wfljb[`UbX`]bhYbg]j YʻdfcWrggʻcZghU[]b[`UʻdYfZcfa UbWYʻhYI h k]h\`UʻdfcZYgg]cbU``X]fYWhcfžXYg][bYfža UbU[YfʻUbX`dfcXi Wfjcbʻa UbU[Yfʻ'H\]gʻWci fgYʻcZZYfgʻUbʻUi h\Ybh]W YI dYf]YbWYʻcZk cf_]b[`UgʻUʻh\YUhfY!a U_Yfž'Ugʻk Y```Ugʻh\Yʻcddcfhi b]hmhcʻXYj Y`cdʻUbU`mfjWJ``a Yh\cXgʻZcf

Webhybh'r (\]'Y'h\Y'g\cfnfy\YUfgU''dYf]cX'cZhyb'dfyWi XYg'h\Y'dcgg]V]']hmZcf'Yl hybg]j Y']bdi hhc'h\Y g\Udy'cZh\Y'Z]bU''dYfZcfa YX'dfcXi Whžmci f'gi [[Ygh]cbg'UbX'dYfgdYWh]j Yg'cb'h\Y'Ybei]fmUfY fY`]YX'i dcb'Uh'U``'h]a Yg'''H\Uh']gžh\Y'Wci fgY'\Y'dg'mci 'hc "YUfb'Vch\ 'hc'hU_Y'X]fYWh]cb'Zfca 'U dfcZYgg]cbU''dfUWh]h]cbYfž'UbX'hc''YUX'UbX']b]h]UhY'XYW]g]cbg'UbX'UWh]cbg''''DUfhcZmci f'hUg_']g'hc bUj][UhY'h\]g'Wca d'Yl''dfcWygk]h\'[YbYfcg]hmUbX']a U[]bUh]cb''

H\]g Wci fgY c ZZYfg U fUfY cddcfhi b]mk]h\]b h\Y 65]b H\YUhfY UbX DYfZcfa UbW Ghi X]Yg hc WcgY mYl Ua]bY UbX a Ub]ZYghU Wca d Yl fYgdcbgY hc Ub Yl hUbhdYfZcfa UbW hYl h'

5ggYgga Ybh

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
7 cbhf]Vi h]cb hc DfcXi Wh]cb i	*\$1	: fca 'h\Y'Z fghfY\YUfgU``hc h\Y'Z bU``dYfZcfa UbW	%ž&Ž ž(ž)
FYZYW¶jY'>cifbU`	' \$ı	%(#\$' #&\$% `\$) .\$\$`DA	%ž&Ž Ž*
DfcXi Wjcb'6`c[%\$I	Cbimci fiUgg][bYXiXUmg	&ž) ž*

5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh'% '7cbhf]Vi h]cb hc DfcXi Wh]cb

GhUfh'XUhY.

8YHJ]`g. '8Yj Y`cdYX'h\fci [\ci hfY\YUfgU``dfcWrgg'UbX'di V`]WdYfZcfa UbWrg": cfa Uh]j Y`UbX'gi a a Uh]j Y ZYXVUW_'fYWr]j YX'k]h\]b'' 'k YY_g'cZZ]bU``dYfZcfa UbWr"

5XX]h]cbU``XYhU]`g.

H\fci[\cihh\Y`fY\YUfgU`g`UbX`dYfZcfa UbWYg`Zcf`\\f@'7 ca d`]\WhYX`mci `a i gh`Wcbhf]\Vi hY`hc`h\Y\\WbWY]j]b[ža U_]b[`UbX`dfYgYbh]b[`cZmci f`[fci d'g`hYl h\VUgYX`h\YUhfY`dfcXi Wh]cb"`Hc`h\]g`YbX`mci `a i gh

5XX]h]cbU`XYhJ]`g.

H\Y'>ci fbU`

H\fci [\ci hih\Y`fY\YUfgU``UbX`dYfZcfa UbWY`dYf]cX`mci `UfY`YI dYWYX`hc`_YYd`U`XU]`mi>ci fbU`fYWcfX]b[`mci f`dYfgcbU``cVgYfj Uh]cbg"`H\Y`>ci fbU``Wcbg]ghg`cZ

- %" mci f'ck b'fYWcfX'cZYj Ybhg'UbX'X]gW gg]cbgž'UbX'hUg_g'mci 'a Um'VY'[]j Yb'Xi f]b['fY\YUfgU'/ &" mci f'ck b'fYZYWhj Yž'dYfgcbU'ž'UbX'g\]Zhjb['cVgYfj Uhjcbg'cb'h\Y'dfcXi Whjcb'dfcWrgg'UbX']hg'fYgi `hg/ UbX
- ' "gca Y`fYZYfYbWYg`hc#bchYg`cb`UXX]h]cbU``fYUX]b[`UbX`fYgYUfWX`h\Uhmci ii bXYfhU_Y`ci hg]XY`cZ h\Y`fY\YUfgU``fcca `fbYY`A ccX`YŁ"'

H\Y'>cifbU``g\ci`X`VY`_Ydh]b`U`bchYVcc_'cZmcif`W\c]\W`UbX`mci`a Um`Z|bX`]h\Y`dZ``hc`Vf]b[`]hhc YUW\fY\YUfgU`": cf`YI Ua d`Yž`]h\\ci`X`VY`5(`cf`5'`]b`g]nYž\Uj Y`fi`YX`UbX\cf`V`Ub_`dU[Yg"`\\ci f`fY\\cfXg UbX`fYZY\M]cbgž'dUff]\W``Uf`mth\cgY`mci`k]g\`hc`\Uj Y`UggYggYXž'g\ci`X`VY`YI dfYggYX`]b`k f]\hYb`Zcfa" < ck Yj Yfž'mci`\\Ub\U`gc`]b\Wi XY`ch\Yf`j]gi U``a UhYf]U`g`gi \W`Ug`]a U[Ygž'X]U[fUa gž'a Udg`]Zmci`Z]bX`gi \W a UhYf]U`j U`i UV`Y`Zcf`mci f`cVgYfj Uh]cb`UbX`fYZY\M]cb`dfc\W`gg"

C b 'gca Y 'XUmgž'h\Y '>ci fbU``k]``ZYY``h]a Y!Wcbgi a]b['UbX'Vcf]b["'<ck Yj Yfž']h']h']g'k cfh\ 'dYfgYj Yf]b[ž'Ug h\]g'dfYj]ci g'h\]fX!mYUf'ghi XYbhYl d`U]bg.

"5 Xa]hhY X`mž_YYd]b['h\Y``c[Vcc_'O-ci fbU`O]b]h]U``mZY`hhY X]ci g`UbX`_]bX`cZha Y`Wbgi a]b[ž\ck Yj Yf' = gccb`VYWJa Y`Uk UfY`h\Uhh\Y`]XYUg`UbX`fYU`]gUh]cbg`=\UX`Ug`U`fYgi `hcZZcfW]b['a mgY`Zhc`k f]hY k ci `X`bch \Uj Y`VYYb`Uj U]`UV`Y`Ubmch\Yf`k Um'`< Uj]b[`U`W\UbW`hc`g]a d`mWcbgc`]XUhY`U```h\Y`Z]hh]b[g`@j]WDcZ]XYUgži bfYUgcbYX`Wca di `g]cbgžUbX`fYgcbUbhWcbWidhgžbchhc`a Ybh]cb`h\Y`f]W\`]bgd]fUh]cbg`Zfca 'h\Y a Ubmg\ck g`UbX`Yj Ybhg`UbX`k \Uhbchk Y`YI dYf]YbWi`h\fci [\ci hh\Y`k YY_žVYWJa Y`YggYbh]U`''H\Y`hfi Y a U[]WcZh\Y``c[Vcc_'dfcWigg`k Ug`bchfYU``mUddUfYbhi bh]``h\Y`YbX`h\ci [\ž'k \Yb``YUZ]b['VUW_'h\fci [\']h'=k Ug`UV`Y`hc`hfUWI`U`dfcWigg`h\Uh=\UX`bchfYU``mVYYb`WcbgW]ci g`cZUg`=k Ug`g` VWdfS] [`Y [f'O'a d\Ug]g UXXYXQ`UbX`Wci `X`cb`mfYWc[b]gY`]hh\fci [\`YbWci bhYf]b[']hUg`U`k \c`Y`VmfY!fYUX]b[`UbX`fYa Ya VYf]b[']h h\fci [\`h\Y``c[Vcc_'''=\UX`h\fYY`hf]Yg`Uhk f]h]b['a mZ]bU``F YZYWf] Y`5bU`mg]gž'h\Y`Z[fghhcc`fYa cj YX`Zfca a mUWi U`YI dYf]YbW'ža cfY`cZU'fYdcfhcb'dYfZcfa Uh]]hm/h\Y`gYWcbXž'U`Wf]h]WU``fUbhih\UhX]XbfhidUmhf]Vi hY hc'a mdfcWigg`k \UhgcYj Yf'''H\Y`h\]fX'!`Z]bU``mUb`\cbYgh]bj Ygh][Uh] Y`fYgdcbgY`hc`k \Uh=\UX``YUfbh!`k Ug cb`mdcqq]V`Y`VYWJi qY`=Wci `X`fYZYf`hc`h\Y``c[Vcc_'''ffYdfcXi WYX`k]h\`dYfa]qg]cbŁ"

¬Emci `WUb`\Ub[`]b`h\YfYžfYWcfX`h\Y`Wf]gYgž'hi fb]b[`dc]bhgž'gi fdf]gYgž'ei Ygh]cbgž'UbX`ZJ]`i fYgž'mci `k]`
\Uj Y`gca Yh\]b[`a i W\`a i W\`f]W\Yf`]b`A UfW\" ¬ZU``Y`gY`ZJ]`gž'hfmh\YgY`hk c`ghfUhY[]Yg.

%" WfYUhY h\fYY ei Ygh]cbg'Uhh\Y YbX'cZh\Y XUmih\UhfYZYWhk\Uh]g'i f[Ybhcf'Uf]g]b['Zcf'mci "'H\Ym a][\h'VY'gdYW]Z]Whc'gca Yh\]b['mci 'UfY'Xc]b[z'cf'a cfY'[YbYfU''ei Ygh]cbg'UVci ha U_]b['h\YUhfY/ &" fYh fb'hc'U'dfYj]ci g'XUm'UbX''cj Yf!k f]hY''mci f'Ybhfmk]h\'bYk 'cVgYfj Uh]cbg"'Mci 'g\ci 'X'U'gc'XUhY Ubm'bYk 'Ybhf]Yg"

$H \setminus Y \cdot F Y Z \cdot Y W \cap Cb$

F Uh\Yf'h\Ub`\UbX]b[']b'h\Y'Ybh]fY'&i fbU`hc'h\Y'7ci fgY'7cbj Ybcfž'h\]g'UggYgga YbhUg_g'mci 'hc'gY`YWi_YmgYWl]cbg"'H\YgY'a][\h'VY'h\Y'Wf]gYg'UbX'ei Ygh]cbg'a Ybh]cbYX'UVcj Yžcf'h\Yma][\h']bj c'j Y gca Yh\]b['Y'gY'U[U]b"' \dispression UXX]h]cbž'Uh'YUghcbY'Ybhfmg\ci 'X'fYgdcbX'hc'cbY'cZh\Y'gYhfYUX]b[g'fcb A ccX'YŁ"

 $>\!\!ci\,fbU``\check{Z}\,F\,YZYW\!h\!]\!cb`1\,F\,YZYW\!h\!]\!j\,\,Y\;\!>\!\!ci\,fbU`$

H\YfY`UfY`hk c`Wca dcbYbhg`Zcf`gi Va]gg]cb.

 $\% > ci fbU``GY`Z15ggYgga Ybh: cfa "`H\]g']g'BCH'; F5898'Vi h'fYei]fYX'hc'dUgg'h\Y'UgggYa Ybh'$

 $\label{eq:weaksub} \mbox{Wea d'YhYX'UZhYf'a]Xb][\hik] $$``UVWfi Y'ghUbXUfX'`UhYbYgg'dYbU'h]Yg'' $$$

Hi fb]h]b gYh]b[. H\]g]g bchU Hi fb]h]b Ugg][ba Ybh

5 https://bxubwww.Fyei]fya Ybhg

 $\label{thm:continuity:continuity:bound} H\Y'UhhYbXUbWY'cZU```fY\YUfgU`g'UbX'dYfZcfa UbWY']g'a UbXUhcfm']b'h\]g'Wci fgY'''I bYI W'gYX'UVgYbWYg'k]``fYgi `h']b'h\Y'Uk UfX'U': U]`'[fUXY''$

7ci fgY GWX YXi `Y

J]Yk 'WUgg'ha YhUV'Y

H]a YtUV`Y

8UhY	HmdY	7 cbh/bh
K YY_'%'%': YVfi Ufm!'&(: YVfi Ufm		H\]g`Wci fgY`fi bg`]b`: YVfi UfmUbX`A UfW\`UbX`a U]b`m df]cf`hc`h\Y`Wca a YbWYa YbhcZGYgg]cb`%''`GYY A ccX`Y`Zcf`h\Y`fY\YUfgU``UbX`dfcXi W¶cb`gW\YXi `Y"

FYgci fWyg

DfYgWf]VYX F Ygci fWYg

GYY'A ccX`Y'UbX'Cb`]bY'FYUX]b['@]gh''

FYWca a YbXYX FYgci fWfg

GYY'A ccX`Y'UbX'Cb`]bY'FYUX]b['@]gh'

7ci fgY 9j U'i Uhjcb UbX 8Yj Y'cda Ybh

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-',) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k]``

5 WUXYa | W < cbYghmUbX D`U[| Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUigci fWik]h\ci hiUW_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfglUbX`mci f`f][\hg`UbX`fYgdcbg]V]`]h]Yg`Ug`U`gh XYbh'Uhi BGK
- VY'Uk UfY'cZd`U[]Uf]ga ž'Wcdmf][\hzil BGK 'Gh XYbh7cXY'cZ7cbXi WhUbX'5WWdhUV`Y'I gY'cZ I BGK '= H'F Ygci fWfg'Dc`]Wh
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWhYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wcaaib]hm
- `cWUHY 'gYfj]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

5 WUXYa]W = b Zcfa Uh cb.

: cf`YggYbhJU``gh XYbh`]bZcfa UhJcb`fY`UhJb['hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY 'gi Va]gg]cbg '[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK '<YU`h\ 'UbX'GUZYhm'dc`]W[Yg/
- YI Ua]bUh]cb dfcW/Xi fYg/
- gdYWJU``Wcbg]XYfUhjcb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybhi fY/
- gh XYbhYei]hmUbX X]gUV]`]hm/
- UbX'ch\Yf'YggYbhJU`'UWUXYa]W]bZcfa UhJcbz'gYY

\hdg.#k k k "Uffig"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWUXYa]W]bZcfa Ufjcb#dfchcWc`g![i]XY`]bYg#

a U[Y'7fYX]h