

ARTS3097

7 i ffYbh'8 YVUhYgʻ]bʻA YX]UʻUbXʻ7 i `hi fY

HYfa 'C bY''##'&\$%-

7ci fgY[.]Cj Yfj]Yk

7ci fgY 8YHJ] g

7 fYX]hDc]bhg * *

Gi a a UfmcZh\Y 7ci fgY

Gi VYWn5fYU. ? WW[Sł 5g/fgdWS` V FWUZ` a 'aYk

#5 'fk]g'Wci fgY'mci 'k]```YI Ua]bY'W ffYbfz`']j Y' hcd]Wg']b'a YX]Uz'i g]b['fY`Yj UbhWcbWrdhg'UbX'ZfUa Yk cf_g'hc]bj Ygh][UhY'UbX'i bXYfghUbX'dfYgYbhXUma YX]U!fY`UhYX']ggi Yg"'8i f]b['fk,Y'Wci fgYz'mci 'k]``'VY'YbWci fU[YX hc 'Wdh fY'UbX'ghi Xma YX]U!fY`UhYX'Yj Ybhg'UbX'XYj Y`cda Ybhg'Ug'h\Ymi bZc`X"'5hh\Y'VY[]bb]b['cZh\Y gYa YghYf'h\fYY'hc 'Zci f'g][b]ZJWUbhz'Yl Ya d`Ufmz'cb![c]b['hcd]Wgz'bYk g'ghcf]Ygz'cf'XYj Y`cda Ybhg']bj c`j]b[h\Y'a YX]U'k]``'VY'WkcgYb"'H\Y'fYghcZh\Y'gYa YghYfz']bWi X]b['`YWh fYg'UbX'h hcf]U``fYUX]b[g'UbX X]gWi gg]cbgz'k]``'VY'cf[Ub]gYX'Ufci bX'hk c'hc'h\fYY'VfcUX'h\Ya Yg'h\UhUfY'VUgYX'cb'h\Y'WkcgYb'hcd]Wg"

7ci fgY @YUfb]b[Ci hWca Yg

%" Ufr]W `UhY h\Y]f'i bXYfqhUbX]b['cZh\Y'fc`Y'cZh\Y'a YX]U']b'WcbhYa dcfUfm`]ZY"

- &" XYd`cmUddfcdf]UhY`WcbWYdhg`UbX`a Yh\cXc`c[]Yg`Zcf`UbU`mg]b[`X]j YfgY`a YX]U`UbX`h\Y]f`]a dUWicb X]ZYfYbh'gcWiU`ž'W`hi fU``UbX#cf'dc`]h\WU``WcbhYl hg"
- '"Yad`cm]bXYdYbXYbh`YUfb]b[`'UbX`gY`Z|X]fYWYX`fYgYUfVX`UVVIji]hm'UbX`X]gd`Um`]bhY``YWiU``f][cif`]bh\X\Y]f`UbU`mg]g`UbX`fYdcfn]b[`cZaYX]U'`mX `cm `c Wi hi hi

h

5ggYgga Ybh

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
G\cfh9ggUm	')।	%*#\$' #&\$% `%%\$\$`DA	% <u>ž&</u> Ž
7`UggʻdfYgYbhUhjcbʻ	%) ၊	Bch5dd`]WUV`Y	%ž&Ž Ž(
7 UgY gh Xm) \$1	&+#\$(#&\$% `%%\$\$`DA	%ž&Ž

5ggYgga Ybh8YHJ]`g

5ggYgga Ybh% G\cfh9ggUm

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g. `Gh XYbhg`k]```gi Va]hU`%\$\$\$`k cfX`fYZYWhj Y`d]YWY`cb`U`hcd]WdYfrU]b]b[`hc`a YX]U`hYW\bc`c[]Yg`]b h\Y`df]j UhY`Xca U]b"`5`a Uf_]b[`g\YYhk]h\`k f]hhYb`Wca a Ybhg`UbX`a Uf_`k]``VY`dfcj]XYX`j]U`Hi fb]h]b"

Hi fb]h]b gYh]b[. `H\]g Ugg][ba Ybh]g gi Va]hhYX h\fci [\`Hi fb]h]b UbX gh XYbhg Xc bchgYY Hi fb]h]b g]a]`Uf]hmfYdcfhg"

5ggYgga Ybh& 7`Ugg dfYgYbhUh]cb

GhUfhiXUhY. K YY_'- 'h hcf]U`q

8 YHJ]`g. `Ghi XYbhg`k]```a U_Y`]bX]j]Xi U``dfYgYbhUh]cbg`cZUVci h)!+`a]bi hYg`YUW\`cb`h\Y]f`hcd]WZcf`h\Y 7 UgY`Ghi Xm''H\]g`k]```cWW f`]b`U`h hcf]U``WUgg``UhYf`]b`h\Y`gYa YghYf''H\]g`HUg_`]g`WcgY`mgWUZc`XYX`hc`h\Y 7 UgY`Ghi XmUbX`YbWci fU[Yg`ghi XYbhg`hc`di h'Zcfk UfX`Ub`Uf[i a Ybh]b`j YfVU``Zcfa "5 `a Uf_]b[`g\YYhik]h\ k f]hhYb`Wca a Ybhg`UbX`a Uf_`k]``VY`dfcj]XYX"

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh' . 7UgY ghi Xm

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g. `Gh XYbhg`k]```gi Va]hU`&\$\$\$`k cfX`YggUmcb`U`hcd]WdYfhU]b]b[`hc`h\Y`a YX]U`hYW\bc`c[]Yg`]b`h\Y di V`]WXca U]b"`H\]g`]g`h\Y`Z]bU``UggYgga YbhHJg_"`9UW\`Ugg][ba Ybhk]```VY`[]j Yb`U`bi a Yf]WU``a Uf_`UbX`U [fUXY"

 $\label{thm:limit} \begin{array}{ll} \mbox{Hi fb]h]b"gYhh]b" . "H\]g"Ugg][ba Ybh"]g"gi Va]hhYX"h\fci [\ \ 'Hi fb]h]b"UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a]"Uf]hmfYdcfhg" \\ \end{array}$

5 h Yb X Ub WY F Yei] f Ya Yb hg

Gh XYbhgʻUfYʻYl dYWYXʻhcʻUhhYbXʻU```YWi fYgž'UbXʻh hcf]U`gʻUfYʻWta di `gcfm'ʻGh XYbhgʻUfYʻfYei]fYXʻhc UhhYbXʻUʻa]b]a i a 'cZ, \$ı 'cZU``'h\Y`h hcf]U`g"'H\]gʻfYei]fYa Ybh]gʻfY`UhYXʻhcʻU```'h\Yʻ@YUfb]b['Ci hWta Ygž YgdYW]U``mCi hWta Ygʻ%'UbXʻ("

7ci fgY'GWXYXi `Y

J]Yk Wugg'ha Yhuv'y

H]a YHJV`Y

8 UhY	HmdY	7 cbh/bh	
K YY_'%'%': YVfi Ufm!'&(: YVfi Ufm	@YWhifY	7 i ffYbhXYVUhYg. Hi h\ UbX XYa cWfUWhža YX]U di V`]WgžHUbgbUh]cbU`a cV]`]h]YgžX][]hU`UWh]i]ga `	
K YY_^&.^&) ': YVfi Ufm!'' A UfVX	@YWi fY	H\Y`U`hlfi h\`g\U```gYhi g`ZfYY3`Dcghlfi h\ždi V`]W _bck `YX[Yž`W]h]nYbg\]d`	
	Hi hcf]U`	7 cbhYa dcfUfm`]ZY`Ug`U`a YX]U`YI dYf]YbW\`	
K YY_" ."("A UfWX"!"%\$	@YWhi fY	8][]hU`]bh]a UV()Yg'!']g'h\Y'df]j UhY'bck'di V`]W8'	
A UfVX	Hi hcf]U`	8][]hU``Ui X]YbWYg`UbX`Z[`hYf`Vi VV`Yg`	
K_YY_^(_^%%^A_UfVX^'!^%+	@YWhi fY	GcWJU``a YX]U`UbX`j]c`YbWY`	
A UfVX	Hi hcf]U`	BYhkcf_YX`U``]UbWYgžcb`]bY`fY`Uh]cbg\]dg`	
K YY_').'% 'A UfVX'!'&(@YWhi fY	H\Y'; `cVU``Gci h\'!'X]ghUbhUbX'dfcl]a UhY'gi ZYf]b['	
A UfVX	Hi hcf]U`	Hfc``]b['UbX'HYffcf'	
K YY_'*.'&) 'A UfW\'!'' %	@YWhifY	DfYWJf]ci g``]j Yg'!`\i a Ub]hJf]Ub'Wca a i b]WJh]cb'	
A UfW	Hi hcf]U`	6YUf]b['k]lbYgg'	
K YY_'+.'%'5df]`'!'+'5df]`	@YWhi fY	9a chlobgžiUWhji]ga iUbXigcWjc!dc`]hJWJ`iWkUb[Yi	
	Hi lcf]U`	HfUbgbUhjcbU`di V`jWg`UbX`h\Y`Wcaaib]rUfjUb]aU[]bUhjcb`	
K YY_', .', '5df]`!'%('5df]`	@YWhifY	K \c'a UXY'a m']D\cbY3'HfUbgbUhjcbU'X][jhU' 'UVci f'	
	Hi hcf]U`	5 X][]HJ`a YX]U'di V`]Wgd\YfY3'	
K_YY_''%)	@YWhifY	H\Y`a YX]U`f`i g`—`fYj]g]h]b[`h\Y`XYVUhYg`	
5df]`	Hi hcf]U`	5gg][ba Ybh-'WUgg'dfYgYbrUr]cbg	

Fygci fWyg

DfYgWf]VYX F Ygci fWYg

 $\label{eq:K-YY_miffux} $$K YY_mfYUX]b[g^*Zcf^*h^*hcf]U^*g^*k]^*^*VY^*id^*cUXYX^*cb^*AccX^*Y''$

FYWca a YbXYX FYgci fWYg

7 ci fgY '9j U'i Uh]cb 'UbX '8 Yj Y`cda Ybh

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-',) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'' \(\frac{1}{2}\)mci
UfY'i bUV`Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU`'Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd']\(\frac{1}{2}\)mci 'Ya U]`'9! hYfbU`'Gi ddcfhmci 'k]``

5 WUXYa | W < cbYghmUbX D`U[| Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hUW_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUXYa]W = b Zcfa Uh cb.

: cf`YggYbhJU``gh XYbh`]bZcfa UhJcb`fY`UhJb['hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY`gi Va]gg]cbg`[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK '< YU`h\ 'UbX'GUZYhm'dc`]W[Yg/
- YI Ua]bUhcb dfcWXi fYg/
- gdYWJU``Wcbg]XYfUHjcb`]b`H\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybhi fY/
- gh XYbhYei]lmUbX X]gUV]`]lm/
- UbX'ch\Yf'YggYbhJU`'UWUXYa]W]bZcfa UhJcbž'gYY

\hdg.#k k k "Uffg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa]W]bZcfa Uf]cb#dfchcWc`g![i]XY`]bYg#

.

a U[Y'7fYX]h

5 i gh|b *&\$% 'G\]`]b['K i

7F = 7CG

7F = 7CG Dfcj]XYf 7cXY. \$\$\$-,;