

# **MDIA3009**

6fUbX.7caaib]WUh]cbg.UbX.7i hifYg

HYfa 'C bY''##'&\$%-

### 7ci fgY'Cj Yfj ]Yk

#### Ghuzz 7 cbhuWh8 Yhu]`g

#### '7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D\cbY
>]b[	ſb["n\Uc4 i bgk "YXi "Ui	K YXbYgXUmg	FcVYfh	Ž*%'&'-',)
		f% .' \$!%( .' \$Ł	KYVghYf&' %8	, \$**

#### 'Hi hcfg'

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWJhjcb	D\cbY
B]X]U^A UffjbYn		5j U]`UV`Y`cb fYei Ygh"`D`YUgY Ya U]``]b`UXj UbVW"		

#### GWXcc~7cbHJWh~abZcfa Uh]cb

Fcca " %&ž~Yj Y~" FcVYfhK YVghYf~6i ]`X]b[

D\cbY.'f\$&Ł'-',)'(,)\*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. \hdg.##gUa "Ufhg"i bgk "YXi "Ui

 $\label{eq:hamiltonian} $$HY'GW.cc``cZh,Y'5ftg'UbX'AYX]U'k ci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck``YX[Y'th,Y'HfUX]tjcbU``7i gtcX]Ubgž h,Y'6YXY[U'`fPYbg]b[tcb'WJa di gŁZ'; UX][U'`fP]tmUbX'5fti/'8Yg][b'7Ua di gYgŁ'UbX'th,Y'B[i bbUk U` dYcd`Y'fbi gtfU`]Ub'8YZYbW': cfW'5WJXYa m]b'7UbVYffUŁ'cZth,Y'`UbXg'k \YfY'YUW\cong WJa di g'cZI BGK ']g`cWJtYX''$ 

### 7ci fgY 8YW] g

7 fYX]hDc]bhg \*

Gi a a UfmcZh\Y'7ci fgY

Gi VYWh5fYU. A YX]U

H\]g`Wci fgY`k]```YI d`cfY`fY`Yj Ubhih\Ycf]Yg`UbX`dfUWf]WYg`cZVfUbX`Wca a i b]WUf]cbg`UbX`W`hi fYg`Zfca `Ub ]bhYfX]gW]d`]bUfmdYfgdYWf]j Y`]bZcfa YX`Vma YX]U`UbX`Wca a i b]WUf]cbgž`W`hi fU``ghi X]Ygž'UbX`Vi g]bYgg ghi X]Yg"`Mci `k]``XYj Y`cd`U`dfUWf]WU``UbX`Wf]h]WU``i bXYfghUbX]b[ `cZh\Y`]bWYUg]b[ `]a dcffUbWYcZUbX UddfcUWXYg`hc`VfUbX`Vi ]`X]b[ `]b`gcW]U`ž`W`hi fU`ždc`]h]WU``UbX`YWcbca ]WWcbhYl hg`]b`WcbhYa dcfUfm gcW]Yh]Yg"`H\Y`Wci fgY`k]```U`gc`Wcj Yf`\_Ymh\Ya Yg`UbX`XYVUhYg`UVci hih\Y`fY`Uh]cbg\]d`VYhk YYb`VfUbX]b[ UbX`Wcbgi a Yf`W`hi fYž]bhY``YWi U``dfcdYfmž'h\Y`W\Ub[]b[ `a YX]U`Ybj ]fcba Ybhž'UbX`h\Y`[ `cVU``YWcbca m'

#### 7 ci fqY @YUfb]b[ Ci hMca Yq

%" 9 I d`U]b`\_YmWcbWfdfgž]ggi Yg`UbX`ffYbXg`fY`Yj Ubhhc`VfUbX`Wcaaib]WUf]cbg`UbX`W`hifYg" &" 7 cbWfdhiU`]gY`h\Y`g][b]Z\WbWfcZVfUbX]b[i]b`j Uf]cig`gYfh]b[g"

- " 7 f]h]WU``mYj U`i UhY`\ck 'VfUbXg`fY`UhY`hc`h\Y]f`gcV]U`ž'W``hi fU``UbX`dc`]h]WU``WcbhYl hg"
- ("5dd`mih\YcfYh]WU``\_bck `YX[Y'Ug'k Y```Ug'dfUWf]WU``g\_]``g'hc'VfUbX'Wcaaib]WUh]cbg'UbX'Yj U`i UhYYZYWf]j YbYgg'cZVfUbX'WJadU][bg"

#### HYUWX]b[ GhfUhY[]Yg

# 5ggYgga Ybh

## 5ggYgga Ybh'HUg\_g'

**8YHJ]`g**. '&\$\$\$ k cfXgž'ZYYXVUW\_'j ]U'Hi fb]h]b"

 $5\,XX]h]c\,b\,U^{\,\cdot}\,XYh\,J]\,\hat{}g.$ 

 $\hbox{D`YUgY`gYY`XYHJ]`YX`UggYgga~Ybh[i~]XY`]bYg`cb`A~ccX`Y"}$ 

 $\label{thm:continuous} \begin{tabular}{ll} Hi fb]h]b "gYhh]b" . "H\]g"Ugg][ ba Ybh]g"gi Va ]hhYX"h\fci [ \ 'Hi fb]h]b "UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a ]`Uf]hmfYdcfhg" \\ \end{tabular}$ 

## Fygci fWyg

DfYgWf]VYX F Ygci fWYg

5j U]`UV`Y`cb`A ccX`Y"

FYWca a YbXYX FYgci fWYg

5j U]`UV`Y`cb`A ccX`Y"

7 ci fgY '9j U'i Uh]cb 'UbX '8 Yj Y`cda Ybh

H\]g`Wci fgY`k]``i bXYf[ c`Wcbh]bi U``XYj Y`cda Ybhž'j ]U`ZYYXVUW\_'dfcWYggYg`gi W\`Ug`cb`]bY`gi fj Ymg`UbX

#### Gi Va |gg|cb cZ5ggYgga YbhHUg\_g

### Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Hi fb]h]b Gi Va ]gg]cb

#mci 'YbWci bhYf'U'dfcV'Ya 'k \Yb'UhYa dhJb[ 'rc'gi Va ]hmci f'Ugg][ ba YbhNhfci [ \'Hi fb]hJbž'd'YUgY
hY'Yd\cbY'9I hYfbU'Gi ddcfhcb'-', ) '' ' ' %cfYa U] 'h\Ya 'cb'YI hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY\_YbXg'f1 \*) 'XUmg'U'mYUft'' #c'ii
UfY'i bUV'Y 'rc'gi Va ]hmci f'Ugg][ ba YbhXi Y 'rc'U'ZJi 'hk ]h\ 'Hi fb]hJb 'mci 'a Um'Udd'mZcf'Ub'YI hYbg]cbž'Vi hmci
a i ghfYhJ]b 'mci f'hJW\_Yhbi a VYf'Zfca '9I hYfbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fY'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbWY 'rc'gi ddcfhmci f'YI hYbg]cb'Udd']WUh]cb'''#mci 'Ya U]''9I hYfbU''Gi ddcfhmci 'k ]'`
Ui hca UhJWJ'`mfYWY]j Y'U'hJW\_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]'``bYYX'hc'gdYWJZJWJ'`mUg\_'Zcf'cbY''
Hi fb]hJb'U'gc'dfcj ]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hYf''

; YbYfU``mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\W)``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]``'VY'gHJhYX'cb'mci f'\\ci fgY'g'A ccX`Y'g]hY'k ]h\ U`hYfbUh]j Y'gi Va ]gg]cb'XYhJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va ]hUgq][ ba Ybha ccXiY

#### 5 WUXYa ]W<cbYghmUbX D`U[ ]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"'=hWJb'hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UWW]XYbhU`mWcdm]b[ 'Zfca `U`gci fWY'k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidgdu[]uf]ga ]bhch\YZc``ck]b[ WuhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfX]h'

=\uddfcdf]UhY 'dUfUd\fUg]b[ .'7\Ub[ ]b[ 'U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b[ 'h\Y'cf][ ]bU`
ghfi Whi fY 'UbX#cf'dfc[ fYgg]cb'cZ]XYUg'cZh\Y'cf][ ]bU`ž'UbX']bZcfa Uh]cb'k ]h\ci h\UW\_bck `YX[ Ya Ybh''H\]g'U`gc
Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY 'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci h\UhYX]hUbX'hc
d]YW]b[ 'hc[ Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yž'k ]h\ci h\Uddfcdf]UhY'fYZYfYbW]b[ "

7 c``i g]cb.'k cf\_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf\_'Ug'U'dYfgcb'g']bX]j ]Xi U``k cf\_"'7 c``i g]cb'U`gc ]bWi XYg'dfcj ]X]b['mci f'k cf\_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhiUbmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa ]WhUg\_ž'ghYU']b['cf'UWei]f]b[ Ubch\Yf'dYfgcb'g'UWUXYa ]Wk cf\_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf\_'cf'gYY\_]b[ dUma YbhZcf'Wca d'Yh]b['UWUXYa ]Wk cf\_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[`gcifWrg`k\]W\`\UjY`bchVYYb`fYUXžk]h\cihUW\_bck`YX[]b[`h\Y`´gYWcbXUfm´gcifWrZfca`k\]W\`\_bck`YX[Y`cZh\Ya`\Ug`VYYb`cVhU]bYX"

8 i d`]\W\n]cb`ffgY`Z\d`U[]\Uf]ga "Ł`gi Va]\n]b[`mci f`ck b`k cf\_z`]b`k \c`Y`cf`]b`dUffz`k \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\nYX`Zcf`Ubch\Yf`UggYgga Ybhcf`\\cf\\text{i fgY`UhI\_BGK\_`cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWnfYZYfYbWlb[ 'dfUWnWyg.

- DUfUd\fUg]b[žgi a a Uf]g]b[žYggUmk f]h]b[ 'UbX h]a Y a UbU[Ya Ybh
- 5ddfcdf]UhY'i gY'cZUbX'Uhtf]Vi h]cb'Zcf'U'fUb[Y'cZa UhYf]U`g']bWi X]b['hYl hž']a U[Ygž'Zcfa i `UY'UbX WbWYdhg"

±0X]j ]Xi U``Ugg]ghUbWr`]g`Uj U]`UV`Y`cb`fYei YghZfca `H\Y`@YUfb]b[ `7 YbhY`f\hd.#k k k "`Wi bgk "YXi "Ui #£" Gh XYbhg`UfY`U`gc`fYa ]bXYX`h\UhWUfYZ ``hja Y`a UbU[ Ya Ybh]g`Ub`]a dcfhUbhdUfhcZgh XmUbX`cbY`cZh\Y ]XYbh]Z]YX`WUi gYg`cZd`U[ ]Uf]ga `]g`dccf`hja Y`a UbU[ Ya Ybh'`Gh XYbhg`g\ci `X`U``ck `gi Z]W]Ybh'hja Y`Zcf fYgYUfW\žXfUZh]b[ `UbX`dfcdYf`fYZYfYbW]b[ `cZgci fWfg`]b`dfYdUf]b[ `U``UggYgga Ybh']hYa g"

IBGK @]VfUfmU`gc`\Ugʻh\Y`9@G9`hcc``UjU]`UV`Y`hc`Ugg]ghmci`k]h\`mcif`ghiXmUhIBGK "'9@G9`]gXYg][bYX`hc`]bhfcXiVW`bYk`ghiXYbhg`hc`ghiXm]b[`UhIBGK`Vih]hWUb`U`gc`VY`U`[fYUhfYZfYg\Yf`Xif]b[`mcifqhiXm"

7 ca d'Yhlb[ 'h\Y'9@&9'hi hcf]U`'UbX'ei ]n'k ]`'YbUV'Y'mci 'hc.

 UbU`mgY`hcd]\Mgž'd`Ub`fYgdcbgYg`UbX`cf[Ub]gY`fYgYUf\W\`Zcf'U\WXYa]\Wk f]hjb[`UbX`ch\Yf UggYgga YbhiHUg\_g • i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK