

MUSC4706

Aig]W9bgYaV`Y

HYfa "Hk c"#"&\$%

7ci fgY'Cj Yfj]Yk

Ghuzz 7 cbhuWh8 Yhu]`g

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`]m	@cWJhjcb	D\cbY
9a YfmGWNi VYfh	Y"gVXiVYfh4 ibgk"YXi"Ui		%\$%zi@YjY``% FcVYfh KYVgHYf	Ž*%&'-',) *,\$,

GWkcc~7cbhJWh~abZcfa Uh]cb

Fcca " &ž`Yj Y`" FcVYfhK YVghYf`6i]`X]b[

D\cbY.'f\$&E'-',)'(,)*

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. \hdg.#gUa "Ufhg"i bgk "YXi "Ui

 $\label{thm:constraint} $$HY'GWKcc``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'hc'FYgdYWZI``m5W_bck`YX[Y'h\Y'HfUX]h]cbU``7ighcX]Ubgžh\Y'6YXY[U``fPYbg]b[hcb'WJadigłž; UX][U``fP]hmUbX'5fhV'8Yg][b'7UadigYgŁ'UbX'h\Y'B[ibbUkU`dYcd'Y'fbighfU`]Ub'8YZYbWY': cfWY'5WUXYam]b'7UbVYffUŁ'cZh\Y'`UbXg'k\YfY'YUWX'WJadig'cZIBGK']g`cWJhYX''$

7ci fgY[.]8YHJ]`g

5ggYgga Ybh

9bgYa V`Yg`a Um\Uj Y`g`][\hmX]ZZYfYbhdfchcWc`g`Zcf`fY\YUfgU`g`UbX`WcbWYfhg`Zca 'YUWX`ch\Yf''``Ghi XYbhg g\ci `X`dUmWUfYZ ``UhhYbh]cb`hc`]bZcfa Uh]cb`dfcj]XYX`Vmh\Y`YbgYa V`Y`X]fYWhcf#JXa]b]ghUhcf`cf`dcghYX`cb A ccX`Y`UVci hh\Y`YbgYa V`Yg`mci `]bhYbX`hc`^c]b"

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'tUg_	K Y][\h	8iY'8UhY	Gh XYbh@YUfb]b[CihWcaYgʻ5ggYggYX
9bgYa V`Y`5`7cblf]Vi ljcb`UbX DUftl7\YW_]b[' \$I	Bch5dd`]WUV`Y	%
9bgYa V`Y`6`7cblf]Vi ljcb`UbX DUftl7\YW_]b[' \$I	Bch5dd`]WUV`Y	%
Di V`]WDYfZcfa UbWF	&\$ı	8YdYbXgʻcbʻh\YʻYbgYa VʻYg ʻc]bYX	%ž&
7 ca dUfUhji YʻYj Uʻi Uhjcbʻ	&\$ı	K YY_'%\$	ı

5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% 9bgYa V`Y`5 7cblf]Vi h]cb UbX DUfH7\YW_]b[

GhUfhXUhY. Bch5dd`]WUV`Y

8 YHJ]`g. `Gh XYbhgfidUfh]WdUh]cb`UbX`Wcbhf]Vi h]cb`hc`YbgYa V`Y`fY\YUfgU`g`k]```VY`UggYggYX`Zcfa Uh]j Y`mžVm h\Y`YbgYa V`Y`X]fYWcf`fY[i`Ufma cb]hcf]b['gh XYbhg`]b`fY\YUfgU`g`hc`Ybgi fY`h\Uh`YUfb]b[']g`HJ_]b[d`UWY"`Gi a a Uh]j Y`UggYgga Ybh`]g`UXa]b]ghYfYX`h\fci [\'U'dUfh\W\Y_]b['hYghUhh\Y`YbX`cZh\Y gYgg]cb": YYXVU_`8]fYWhUbX`cb[c]b['ZYYXVU_Xi f]b['fY\YUfgU`g`UbX`UZhYf`U'di V`]WdYfZcfa UbW\"

5XX]h]cbU`XYhU]`g.

 $H\Y'dUfftWXYW_]b[\ 'hYghi \ gi \ U``m'tU_Yg'd`UW']b \ 'hXYk YY__'UZhYf'th\Y'Z]bU``fY\YUfgU`*di \ V`]WdYfZcfa \ UbW'fK \ YY__'%$`cf'f%ft''\``8YtU]`g`cZh\Y'dUfftWXYW_]b[\ 'hYghk]```VY'Ubbci \ bW'X''\`K \YfY'dcgg]V`Yž_YYd'U'WcdmcZU```a i g]WdUfftg#g\WcfYg'fY\YUfgYX'Xi f]b['h\Y'hYfa 'UbX'Vf]b['h\Ya 'hc'h\Y'dUfft\W\Y\W_]b['hYgh'$

Gi Va |qq|cb bchYq.GYY 5XX|hcbU XYHJ|q

5ggYgga Ybh& 9bgYa V`Y 6 7cbff]Vi h]cb UbX DUfH7\YW]b[

GhUfhXUhY. Bch5dd`]WUV`Y

8 YHU]`g. `Gh XYbhgfidUfh]WdUh]cb`UbX`Wcbhf]Vi h]cb`hc`YbgYa V`Y`fY\YUfgU`g`k]```VY`UggYggYX`Zcfa Uh]j Y`mž`Vm h\Y`YbgYa V`Y`X]fYWcf`fY[i`Ufma cb]hcf]b['gh XYbhg`]b`fY\YUfgU`g`hc`Ybgi fY`h\Uh`YUfb]b[']g'hU_]b[d`UW\"`Gi a a Uh]j Y`UggYgga Ybh`]g`UXa]b]ghYfYX`h\fci [\'U'dUfh\W\Y_]b['h\Ygh'Uh'h\Y`YbX`cZh\Y gYgg]cb": YYXVU_`8]fYWhUbX`cb[c]b['ZYYXVU_Xi f]b['fY\YUfgU`g`UbX`UZhYf`U'di V`]WdYfZcfa UbW\"

5XX]h]cbU`XYhU]`g.

 $H\Y'dUfftWXYW_]b[\ 'hYghi gi U``m'tU_Yg'd`UW']b'h\Y'k YY_'UZhYf'h\Y'Z]bU'`fY\YUfgU'*di V`]WdYfZcfa UbW' fK YY_'%$`cf'f%ft"''`8YtU]`g'cZh\Y'dUfftWXYW_]b[\ 'hYghk]```VY'Ubbci bW'X"''K \YfY'dcgg]V`Yž_YYd'U'WcdmcZU'``a i g]WdUfftg#g\WcfYg'fY\YUfgYX'Xi f]b['h\Y'hYfa 'UbX'Vf]b['h\Ya 'hc'h\Y'dUfft\W\Y\W_]b['h\Ygh'$

Gi Va]gg]cb bchYg.GYY 5XX]h]cbU``XYhU]`g

5ggYgga Ybh' Di V`]WDYfZcfa UbWY

GHJfhXUhY. Bch5dd`]WUV`Y

 $8 YHJ]^g. "Ghi XYbhg" UfY" fYei]fYX'hc" dUfh] W] dUhY"]b'di V'] WdYfZcfa UbWYg" cZh\Y]f'W\cgYb'YbgYa V'Ygž'k \]W\k]\"'VY'UggYggYX'Vmh\Y'YbgYa V'Y'X]fYWcf"": YYXVUW_."[]j Yb"]b'k f]h]b[''UZhYf'h\Y'di V']WdYfZcfa UbWY"'H\]g]g'h\Y'Z]bU`'UggYgga Ybh'hUg_"$

5XX]h]cbU`XYhU]`g.

9UWX 'YbgYa V'Y'Di V`]WDYfZcfa UbWY']g'U'[fci d'UWfj]]mUbX'**Wcbhf]Vi hYg'%\$i 'hc'h\Y'Wci fgY'a Uf_**'f&\$i]b'hchJ'Ł'''9UWX'Di V`]WDYfZcfa UbWY'k]``'VY'UggYggYX'Vmh\Y'YbgYa V'Y'X]fYWhcf'UbX#cf'Ubch\Yf'Yl Ua]bYf'' Ghi XYbhg'g\ci 'X'VY'Uk UfY'h\Uh'h\Y'Di V`]WDYfZcfa UbWY'UbX'gca Y'fY\YUfgU`'h]a Y'Zcf'YUWX'YbgYa V'Y'a Um hU_Y'd'UWY'ci hg]XY'h\Y'fY[i 'Uf'gWXYXi 'YX'k YY_`mfY\YUfgU`'h]a Y''

5ggYgga Ybh (. 7ca dUfUh]j Y Yj U`i Uh]cb

GHJfh XUhY.

@Yb[h\.'%\\$\$\$!\%\\$\$\$'kcfXq

 $8 YHJ]^g. H\g^fVei\g^fYei\g^fYei\g^fYei\g^fYei\g^fYei\g^fYeb'\g^fYeb$

5XX]h]cbU`XYHJ]`q.

K filmyb'Yj U'i Unichg'Ufy'gi Va ilmyx'j ilu'hi fbilmib'Ubx'ilbWi xy'h\y'Zc``ck ib[.

%" H]hY du[Y. fUŁgh XYbhfg Z ``bua Y UbX =8 fVŁbua Y cZYUW. YbgYa V Yž UbX fWŁk cfX Wci bh &" Ub]bhfcXi WcfmdUfU[fUd\ža U]bhYl hz UbX WcbWi g]cb'

: cf'h]dg'cb'k f]h]b['UVci ha i g]WWcbgi `h'h\Y' < Ufj UfX'k f]h]b['[i]XY Uh '\hdg.#k f]h]b[dfc Υ Wh'ZUg"\Ufj UfX"YXi #] Υ g#\k d#] Υ g#\k d#] Υ g#\k d#]

5XX]hjcbU``]bghfi Whjcbg'cb'k f]hjb['U'Wca dUfUhj Y'Yj U'i Uhjcb'k]``'VY'dfcj]XYX"

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b"gYhh]b" . "H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\ 'Hi fb]h]b"UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a]"Uf]hmfYdcfhg" \\ \end{tabular}$

5 hmy b X Ub WY F Yei] f Ya Yb hg

9bgYa V`Y`dUff]WdUbhg`a i ghUffYbX`U```fY\YUfgU`g`UbX'di V`]WdYfZcfa UbWYg"

?YYd']b'a]bX'h\Uh

fleGca Y YbgYa V Yg fYei]fY 'Ui X]hcbg df]cf hc k YY_'%"

f&E'GYj YfU`fYei]fY'mci 'hc'\Uj Y'Ub']bglfi a Ybhih\Uhmci 'WUb'd`Um'

fl Ł'Gca Y'\Uj Y'`]a]hUhjcbg']b'bi a VYf'cZdYcd`Y'k \c'WUb'VY'UWWYdhYX"

f(EDiV)WdYfZcfa UbWrg'UbX'UXX]hcbU`fY\YUfgU`g'UfY'fYei]fYX'cihg]XY'h\Y'fY[i`Uf'fY\YUfgU`h]a Yg"

Ghi XYbhgʻa i ghiYbgi fYʻh\Uhih\YgYʻUXX]hJcbU``fY\YUfgU`gʻXcʻbchWUg\`k]h\`ch\Yf`Wci fgYg#/bgYa V`Yg"

8 YHJ] YX`]bZcfa Uf]cb`UVci hYUW\`YbgYa V`Y`]g``]ghYX`\<u>YfY</u>"`C b`mUddfcj YX`YbgYa V`Yg`&]bYX`]b`h\Y`W ffYbh hYfa `WUb`VY`i gYX`Zcf`Wci fgY`UggYgga Ybh'

FYYj Ubh]bZcfa Uh]cb'UVci hYUW. YbgYa VY']g'i gi U`mi dXUhYX'VmC!K YY_'UbX'ghi XYbhg'g\ci `X'W\YW_'Zcf Ubmi dXUhYg'Uhh\]g'h]a Yž'UbX']ZZ fh\Yf']bZcfa Uh]cb']g'fYei]fYXž'dY`UgY'WcbhUWh\Y'GW\cc`'cZ5fhg UbX'A YX]U'C ZZWY. '\hdg.#gUa "Uffg"i bgk "YXi "Ui #WcbhUWhi g

Ghi XYbhgʻgY`YWNb['h\Y]fʻhk cʻYbgYa V`YgʻZcfʻh\]gʻWci fgY`Xcʻgcʻcbʻh\Yʻi bXYfghUbX]b['h\Uhih\Ymi\Uj YfYUX`UbX`i bXYfghccX'h\]gʻ]bZcfa Uhjcb"

7ci fgY'GWXYXi `Y

J JYk 'WUqq'Ha YHUV'Y

H]a YHJV`Y

8UHY	Hmd Y	7 cbhYbh
K YY_'%'' '>i bY'!'+'>i bY		7 ci fgY`gWkYXi `Y`]bZcfa Uh]cb`j Uf]Yg`XYdYbX]b[`cb h\Y`YbgYa V`Yg`WkcgYb"

Fygci fWyg

DfYgWf]VYX F Ygci fWYg

Bch'Udd`]WUV`Y"

FYWca a YbXYX FYgci fWYg

Bch'Udd`]WUV`Y"

7ci fgY '9j U`i Uh]cb 'UbX '8Yj Y`cda Ybh

8] fYW h ub X cb [c] b [2YYXV uw Xi f] b [2YYX

Gi Va |gg|cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

#mci YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dr]b['rc'gi Va]hmci f'Ugg][ba Ybht\fci [\'Hi fb]r]bž'd'YUgY
hY'Yd\cbY'9I hYfbU''Gi ddcfhcb'-',) '''' %cf'Ya U]''h\Ya 'cb'YI hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft\"'=\maintention
UfY'i bUV'Y 'rc'gi Va]hmci f'Ugg][ba YbhXi Y'rc'U'ZJi 'hk]h\'Hi fb]r]b'mci 'a Um'Udd'm\Zcf'Ub'YI hYbg]cb\ZVi hmci
a i ghfYhJ]b'mci f'r]\W_Yhbi a VYf'\Zfca '9I hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW a Ybfg\L'rc
]bWi XY'Ug'Yj]XYbW' 'rc'gi ddcfhmci f'YI hYbg]cb'Udd']\WJh]cb'''=\maintention
UfY'' bUYY''mfYWY']j Y'U'r]\W_Yhbi a VYf\Zfvi h]Zmci 'hY'Yd\cbY'mci 'k]'''bYYX'rc'gdYW]\ZfwJ''mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'gfUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]``'VY'gHJhYX'cb'mci f'\\ci fgY'g'A ccX`Y'g]hY'k]h\ U`hYfbUh]j Y'gi Va]gg]cb'XYhJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va]hUgq][ba Ybha ccXiY

5 WUXYa | W < cbYglmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'=hWJb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhU`mWcdm]b['Zfca `U`gci fWY'k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidgd'U[]Uf]ga]bhch\YZc``ck]b[WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`'hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW''cf'i g]b['ei chUh]c<u>b'a Uf_g</u>"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

♣Uddfcdf]UhY 'dUfUd\fUg

\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \] Y 'a cghmfYhU]b]b['h\Y'cf][]bU`

ghfi Wh fY 'UbX#cf'dfc[fYg

Z]XYUg'cZh\Y'cf][]bU`žUbX ']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh''H\]g'U`gc

Udd`]Yg']b'dfYgYbhUh]cbg

d]Y\Mb['hc[Yh\Yf'ei ch'g

Uf

\[
\text{X} k \c`Yžk]h\ci h'Uddfcdf]UhY 'fYZYfYb\M

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\[
\text{X} k \c`Yžk]h\ci h'Uddfcdf]UhY 'fYZYfYb\M

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\[
\text{X} k \c`Yžk]h\ci h'Uddfcdf]UhY 'fYZYfYb\M

\]

\[
\text{Y} k \c'Yzk |h\ci h'Uddfcdfd]UhY 'fYZYfYb\M

\]

\[
\text{Y} k \c'Yzk |h\ci h'Uddfcdfd]UhY 'fYZYfYb\M

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\[
\text{Y} k \c'X k \c'Yzk |h\ci h'Uddfcdfd]UhY 'fYZYfYb\M

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\[
\text{Y} k \c'X k \c'Yzk |h\ci h'Uddfcdfd]UhY 'fYZYfYb\M

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\text{Y} k \c'X k \c'Yzk |h\ci h'Uddfcdfd]UhY 'fYZYfYb\M

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\text{Y} k \c'X k \c'Yzk |h\ci h'Uddfcdfd]UhY 'fYZYfYb\M

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\[
\text{Y} k \c'X k \c

g|cb'U`gc h\Ya 7 c``i g]cb. 'k cf_]b['k]h\ ' ridUgg]b['cZih]bWi XYg'dfcj]X]b['mci f' h\Y`XiY`XUhYž`cf`Zcf`h\Y`difd Ubch\Yfgh XYbh\ d`U[]Uf]q]b[UhUbmha Yž 'Ubch\Yf'dYfqcb'hc a Ub UWUXYa |WhUq_žghYU`i UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXY hc Wca d'YhY Ubch\Yf dYfgi dUma YbhZcfWca d`Yhb[/a]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7
gcifWYg'k\]WX`\UjY'
yb`fYUXžk]h\cih'UW_bck`YX

Zh\Ya`\Ug`VYYb'

YX"

7 cffYWhfYZYfYbWb[dfUWhWyg.

- i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]]hJYg'Ug'U'ghi XYbh'Uhi BGK VY'Uk UfY'cZd'U[]Uf]ga žWcdmf][\hz| BGK 'Ghi XYbh'7cXY'cZ7cbXi Wh'UbX'5VWWdhJV'Y'I gY'cZ I BGK "=7 H"F Ygci fWYg"Dc`]Wm