

ARTS2021

7 fYUhjj Y'K f]hjb['DfcYWh

HYfa 'Hk c''# '&\$&\$

7ci fgY'Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g

7cbj Ybcfg

BUa Y	9a U]`	5j U]`UV]`] m	@cWUhjcb	D∖cbY
FcUbbU'; cbgU'j Yg	fcUbbU4 i bgk "YXi "Ui			

GWXcc~7cbhJWh~abZcfa Uhlcb

GWXcc`cZh\Y'5fhg'UbX'AYX]U

Fcca " %&ž@Yj Y`" žFcVYfhK YVghYf'6i]`X]b[ff %(Ł

D\cbY. f\$&Ł'-',) '(,) *

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GW.cc``cZh,Y'5ftg'UbX'AYX]U'k ci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck``YX[Y'th,Y'HfUX]tjcbU``7i gtcX]Ubg\cong hY'6YXY[U'``fPYbg]b[tcb'WJa di g\cong \cong \cong \cong mUbX'5fti/'8Yg][b'7Ua di gYg\cong \cong \con$

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

H\]g`k cf_g\cd!VUgYX`Wci fgY`[]j Yg`mci `h\Y`cddcfhi b]hm\tc`XYj Y`cd`U`a U`cf`WfYUh]j Y`k cf_j]b`Z]\M]cbž dcYhfmž'a Ya c]fž'd`Umcf`g\MYYbk f]h]b["'7`UggYg`UfY`g\fhi \Mi fYX`\c`Ybgi fY`U`gi ddcfh]j Y`k cf_g\cd Ybj]fcba YbhZcf`mci `\c`XYj Y`cd`U`a U`cf`k cf_\hc`Z]bU``XfUZng\U[Y"`H\Y`\\circ i fgY`k]``\Y`d`mci `\cbY`mci fg_]``g`]b`h\Y`UfYUg`cZYX]h]b[ž'\cdmYX]h]b[ž'fYUX]b[`\Wf]h]\WJ``mž[]j]b[`dYYf`ZYYXVU_UbX`]b`bUj][Uh]b[]bXi g\fmcddcfhi b]h]Yg"

Mci k]``VY`fYei]fYX`hc`gi Va]hU'dfcYWhdfcdcgU``bYUf`h\Y`ghUfhcZh\Y`Wci fgY"

7 ci fgY @YUfb]b[Ci hWca Yg

%" D`UbžXfUZhUbX`YX]hU`a U'cf'd]YWY`cZWfYUhjj Y`k f]hjb[" &" 7 f]hJWU``mUddfU]gY`h\Y]f`ck b`UbX`ch\YfgfWfYUhjj Y`k f]hjb[" " 8 Yj Y`cd`UbX`fYZ|bY`Ub`]bXi ghfm!gdYWJZWghUhYa YbhUVci hih\Y`k cf_"

HYUWX]b['GhfUhY[]Yq'

+?@J :FLIJ< N@CC 9< ; <C@MkI<; N?FCCP FEC@E< 8J 8 9C<E; <; C<8IE@E> I<8C K@D< 8E; 8JFE:?IFEFLJ :FDGFE<EKJ 'C<8J< :?<:B \$FF; C< #I #LIK?

H\Y`k cf_g\cd`Zcfa Uhdfcj]XYg`U`_Ymcddcfhi b]lmiZcf`ghi XYbhg`hc`XYj Y`cd`h\Y`_]bX`cZ]bhYbg]j Y`UbX`dfUWf)WU``YX]h]b[`UbX`dYYf`ZYYXVUW_`bYYXYX`hc`\Y`d`Vi]`X`dfc`YWg`h\fci [\`hc`Wca d`Yh]cb"`\\gc` Wcblf]Vi hYg`hc`h\Y`Wci fqYfq`dfUWf\WU``]bXi ghfmZcW`g"

5ggYgga Ybh

H\]g`]g`U`k cf_g\cd!VUgYX`Wci fgY`UbX`]g`bchfYWcfXYX"`5 hhYbXUbWf`]g`Wca di `gcfm'`-Za cfY`h\Ub`cbY`WUgg]g`a]ggYX`U`a YX]WU``Wffh]Z]WUhY`]g`fYei]fYX"`5 `fYUgcbUV`Y`UhhYa dha i ghVY`a UXY`hc`Zi ``mWca d`YhY`U`` UggYgga Ybhgžch\Yfk]gY`U`ZU]``[fUXY`a UmVY`Uk UfXYX"

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hJg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
±b]h]U``DfcYWhDfcdcgU``	&\$I	\$%#\$+#&\$&\$`%%\$\$`DA	%
DYYf': YYXVUW_'5ggYgga Ybh	&) ।	%\$#\$, #&\$&\$`%%\$\$`DA	&
A U'cf'K cf_))।	%\$#\$, #&\$&\$`%%\$\$`DA	% Z

5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% ₺]hJU` Dfc YWhDfcdcgU`

GHJfh:XUhY.

8YHJ]`g.

₽X]j]Xi U`

%) \$\$ k cfXg

 $H\g`UggYgga\Ybh]g`Wca\df]gYX`cZhk\c`Wca\dcbYbhg.`fWc`U`%$$$!k\cfX`dfcdcgU``Zcf`U`a\U'cf`d]YWY`cZWYUh]j\Y`k\f]h]b["`H\]g`Wca\dcbYbh]g`k\cfh\`%) \ \ `5B8`f&c`U') $$!k\cfX`k\f]hYffg`ghUhYa\Ybhci\h]b]b[d\]`cgcd\mUbX#cf`dfUWh]WYg`cZWca\dcg]h]cb`]bZcfa\]b[`h\Y`dfc<math>\Upsilon$ Wh`H\]g`Wca\dcbYbh]g`k\cfh\`) \ "

: YYXVUW_'j]U'@A G

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b gYhf]b[."H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\"Hi fb]h]b "UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a]"Uf]hmfYdcfhg" \\ \end{tabular}$

5ggYgga Ybh& DYYf: YYXVUW_5ggYgga Ybh

GHJfhXUhY.

8 YHJ]`g.

±0X]j]Xi U`

&) \$ k cfXg

Ghi XYbhgik]``dfcj]XY'Vf]YZWcbghfi Whj Yik f]hhYbiZYYXVUW_YUWXikYY_hcighi XYbhgidfYgYbh]b[ihXY]fikcf_]bi

Wci fgY UbX gi Va]hUb YI hYbXYX f&) \$!k cfXŁ'j Yfg]cb cZ]h'

: YYXVUW_'j]U'@A G

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ $H \ fb \end{tabular} $$ fo \end{tabular} $$ Hi & fb \end{tabular} $$ fo \end{tabular} $$ fo \end{tabular} $$ fo \end{tabular} $$ Hi & fb \end{tabular} $$ fo \en$

5ggYgga Ybh' A U'cf K cf_

GhUfh'XUhY.

8 YW] g.

₽X]j]Xi U`

') \$\$ "k cfXg" hchU"

A U'cf'k cf_'cZdfcgY'fl \$\$\$'k cfXgŁ'CF'U'gWf]dhft%) 'dU[YgŁz'XYj Y`cdYX'Zfca 'h\Y'dfcdcgU'''

H\]g`Ugg][ba YbhitUg_`Wa df]gYg`tk c`Wa dcbYbhg.`ff\&`U`a U\afik cf_`cZdfcgY`fl \$\$\$`k cfXg\CF `U`g\f]dhf\cZ %) `dU[Yg\z\XYj Y`cdYX`\Z\afa 'h\Y`dfcdcgU`"`H\]g`Wa dcbYbh]g`k cfh\`() | `5B8`f\\\U') \$\$!k cfX`\Z\afa U`]bXi ghfmgdY\\\Z\WghUhYa Ybhicf`d]h\X\\\Z\dfYgYbh]b[`h\Y`k cf_`hc`U`dfcgdY\\]j Y`YX]hcf"`H\]g`\\\afa dcbYbh]g`k cfh\\%\$I "`

: YYXVUW_'j]U'@A G

Fygci fWyg

DfYgWf]VYX F Ygci fWYg

5j U]`UV`Y 'j]U A ccX`Y

FYWca a YbXYX FYgci fWYg

5j U]`UV`Y`j]U'A ccX`Y

7ci fgY '9j U`i Uh]cb 'UbX '8Yj Y`cda Ybh

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

#mci 'YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dhJb['tc'gi Va]hmci f'Ugg][ba Ybht\fci [\'Hi fb]hJbž'd'YUgY
hY'Yd\cbY'9| hYfbU'Gi ddcfhcb'-',) '''' %cf'Ya U]''h\Ya 'cb'Yl hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft\"'#c'i'#c'i'
UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\ 'Hi fb]hJb'mci 'a Um'Udd'm\Zcf'Ub'Yl hYbg]cb\ZVi hmci
a i ghfYhU]b'mci f'hJ\W_Yhbi a VYf'\Zfca '9| hYfbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fY'Yj UbhXcW a Ybhgthc
]bWi XY'Ug'Yj]XYbW''hc'gi ddcfhmci f'Yl hYbg]cb'Udd']\WUh]cb'''#c''' Zmci 'Ya U]''9| hYfbU''Gi ddcfhmci 'k]'`

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@&9'k]``\Y`d'mci 'a U_Y'h\Y'a cghcZmci f'gh X]Yg'UhI BGK "'\\hd.##qi V'YWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUXYa]W±bZcfa Uhjcb

 $: \ cf `YggYbh]U``ghi \ XYbh]bZcfa \ Uh]cb `fY`Uh]b[\ `hc.$

