

MUSC2802

Aig]WGhX]cHYUWX]b[

HYfa "H\fYY" ## "&\$%-

7ci fgY[·]Cj Yfj]Yk

GHJZZ:7cbHJWh:8YHJ]`g`

7cbj Ybcfg

BUa Y	9a U]`	5j U]`UV]`] lm	@cWUh]cb	D\cbY
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7ci fgY 8YHJ] g

7 fYX]hDc]bhg *

Gi a a UfmcZh\Y 7ci fgY

Gi VYWh5fYU. ? ge[U

A i g]Wghi X]c 'h/UWX]b[']g'h\Y'[UhYk Um'Wci fgY 'Zcf'h\Y'A i g]WDYXU[c[mdUh\k Um]b'h\Y'6UWXY`cf'cZA i g]Wdfc[fUa a Y"'H\Y'Wci fgY']g'ZcW gYX'cb']bglfi a YblU`'UbX'j cWU`'hYUWX]b['UbX''YUfb]b[']b'h\Y'df]j UhY'ghi X]c gYhjb["'Mci 'k]``a U_Y'Zcfa U``cVgYfj UhJcbg'cZYl dYfhhYUWXYfg'Uhk cf_']b'h\Y'ghi X]c"'DUfU``Y``hc'h\]gž mci 'k]``[U]b'dfUWf]WU``YI dYf]YbWY'VmhYUWX]b['UbX'dfYgYbh]b['UgdYWg'cZmci f'gdYW]U`]ghi]bglfi a YblU``UbX j cWU``g_]``gž'k]h\'dYYf'Yj U`i UhJcb'UbX'gi ddcfh/mci 'k]```U`gc '`YUfb'hc'UddfU]gYž'Yj U`i UhY'UbX'Udd`mUgdYWfg cZdfcZYgg]cbU``UbX'fYgYUfWX'_bck 'YX[Y'Ug'mci 'XYj Y`cd'mci ']bX]j]Xi U``UddfcUWXYg'hc'dfUWf)WY"

7 ci fgY @YUfb]b[Ci hMca Yg

- %" 9I d`U]b`UbX`XYa cbghfUhY`h\Y`k cf_]b[g`cZh\Y]f`gdYW]U`]ghi]bghfi a Ybhg`cf`j c]WYž'UbX`U`fUb[Y`cZ Z bXUa YbhU``a i g]WU``WcbWYdhg"
- &" FYWc[b]gY`UbX`Wf]hJWU``miUggYgg`U`j Uf]YhmcZUddfcUWXYg`hc`h\Y`hYUWX]b[`UbX``YUfb]b[`cZaig]W h\fci[\`fYZYWf]cb`cb`h\Y]f`ckb`YIdYf]YbWYždfcZYgg]cbU```]hYfUhifY`UbX`cVgYfj Uh]cbg`cZghiX]c `Yaacba"
- ' " I bXYfgHJbXžYj U`i UHY'UbX'Udd`miUgdYWrg`cZW ffYbhfYgYUfWX`ZcW gYX`cb`]bgHi a YbHJ``UbX'j cWJ``YUfb]b[`UbX'HYUWX]b["

HYUWX]b[GhfUhY[]Yg

Ghi XYbhgʻik]```VYʻ[]j Ybʻ`YWhi fYʻ]bdi hicbʻ]ggi Ygʻg\UfYXʻVmUʻfUb[YʻcZgdYWJU`]ghi]bgffi a Ybhgʻcfʻj c]WYž]bWi X]b[ʻfYʻYj UbhʻUWci ghJWdfcdYfhJYgža YW.Ub]WU``Z bWJcbgžʻUbXʻZ bXUa YbhU``a i g]WU``WcbWldhg''`H\YgY k]```VYʻWca dʻYa YbhYXʻVmghi XYbhidfYgYbhUhJcbgʻ]bʻ`UVcfUhcfmgYhJb[gž']bʻk\]W\`h\Ymk]```JbhfcXi WYž XYa cbghfUhYžX]gWi ggʻUbXʻUbgk Yfʻei YghJcbgʻUVci hih\Y]fʻck bʻ]bghfi a Ybhg''`Ghi XYbhgʻk]```dUfhJWJdUhYʻ]b k cf_g\cdgžVm[]j]b[ˈ]bX]j]Xi U```YggcbgʻhcʻUbch\Yfʻghi XYbhcbʻh\Y]fʻgdYWJU`]ghi]bghfi a Ybhcfʻj c]WYžk]h\ dYYfʻUbXʻhi hcfʻZYXVUUW_"``H\Ymk]```U`gc`\Uj Yʻh\Yʻcddcfhi b]hmhcʻ`YUXʻh\YʻWUggʻ[fci dʻVmhYUW\]b[ʻUʻg_]``'`@YWhi fYʻ]bdi hik]``\Yàd hcʻdfYdUfYʻghi XYbhgʻZcfʻcVgYfj UhJcbgʻcZYI dYfhhi hcfgžVm]bhfcXi WJb[ʻ]ggi YgʻUbX a Yh\cXgʻXfUk bʻZfca 'dfcZYgg]cbU``UbXʻfYgYUfW\`]hYfUhi fY''

. . .

5ggYgga Ybh

5b'Ugg][baYbhVf]YZk]``UddYUf'cb'accXY'ZcfYUW\'UggYggaYbhHUg_"

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5ggYgga YbhˈhJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[CihWca Yg 5ggYggYX
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Dcfh7c`]c	*\$1	Bch5dd`]WUV`Y	%ž&

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GHJfh:XUhY.

@Yb[h. &ž\$\$\$ k cfXg

8YHJ]`g. '9ggUmf&\$\$\$`k cfXgŁ`VUgYX`cb`Zcfa U``cVgYfj Uhjcbg`UbX`Wf]h]WU``UddfU]gU``cZ]bghfi a YbhU```Yggcbg []j Yb`]b`h\Y`gdYW]U`]ghZJY`X`cZYUWX`ghi XYbh'``H\Y`YggUmk]``VY`UWWca dUb]YX`Vmh\Y`cVgYfj Uhjcb`gWXYXi `YXYj]gYX`gdYWJZWU``mZcf`h\Y`di fdcgY`cZh\Y`YI YfWjgY"``K f]hHYb`fYdcfhdfcj]XYX"`

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b gYhh]b[. Hi]g Ugg][ba Ybh]g gi Va]hhYX hi fci [in Hi fb]h]b UbX ghi XYbhg Xc bchgYY Hi fb]h]b g]a] \begin{tabular}{ll} IhYX hi fci [in Hi fb]h]b UbX ghi XYbhg Xc bchgYY Hi fb]h]b g]a] \begin{tabular}{ll} IhYX hi fci [in Hi fb]h]b UbX ghi XYbhg Xc bchgYY Hi fb]h]b g]a] \begin{tabular}{ll} IhYX hi fci [in Hi fb]h]b UbX ghi XYbhg Xc bchgYY Hi fb]h]b g]a] \begin{tabular}{ll} IhYX hi fci [in Hi fb]h]b UbX ghi XYbhg Xc bchgYY Hi fb]h]b g]a] \begin{tabular}{ll} IhYX hi fci [in Hi fb]h]b UbX ghi XYbhg Xc bchgYY Hi fb]h]b g]a] \begin{tabular}{ll} IhYX hi fci [in Hi fb]h]b UbX ghi XYbhg Xc bchgYY Hi fb]h]b g]a] \begin{tabular}{ll} IhYX hi fci [in Hi fb]h]b UbX ghi XYbhg Xc bchgYY Hi fb]h]b g]a] \begin{tabular}{ll} IhYX hi fci [in Hi fb]h]b UbX ghi XYbhg Xc bchgYY Hi fb]h]b g]a] \begin{tabular}{ll} IhYX hi fci [in Hi fb]h]b UbX ghi XYbhg XC bchgYY Hi fb]h]b g]a] \begin{tabular}{ll} IhYX hi fci [in Hi fb]h]b UbX ghi XYbhg XYbh$

5ggYgga Ybh& Dcfh7c`]c

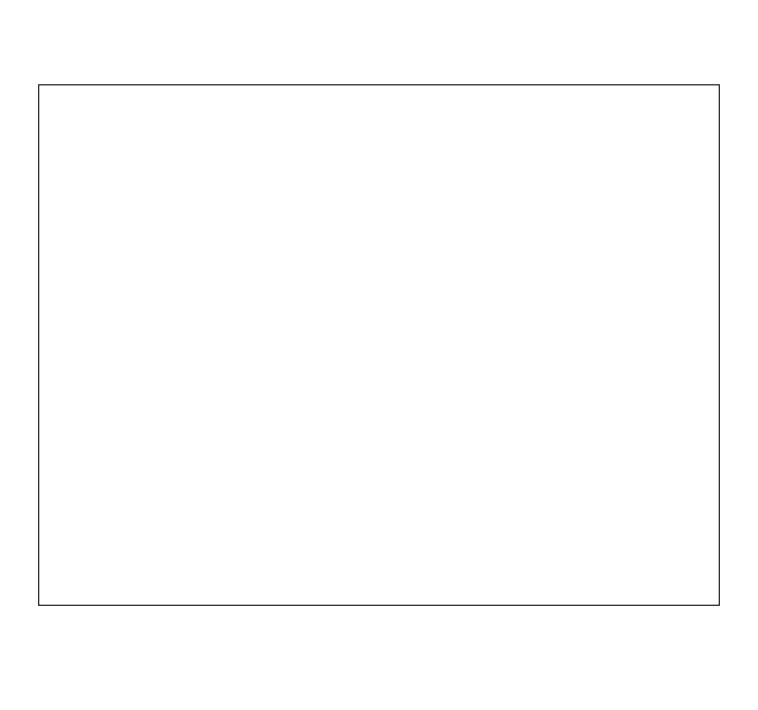
GhUfh'XUhY.

@Yb[h\." 1 * \$\$\$ k cfXg

 $8 YHJ]^c_{3} H^{YY'd}^{YW}_{3}^{Y'U}^{Y'd}^{YW}_{3}^{Y'U}^{Y'd}^{Y'U}^{Y'd}^{Y'U}^{Y'd}^{Y'U}^{Y'd}^{Y'U}^{Y'd}^{Y'U}^{Y'd}^{Y'U}^{Y'd}^{Y'U}^{Y'd}^{Y'U}^{Y'd}^{Y'd}^{Y'U}^{Y'd}$

5XX]h]cbU`XYhU]`g.

GYY'7ci fgY'gWXYXi `Y'Zcf'Xi Y'XUhYg"



FYgci fWfg

DfYgWf]VYX F Ygci fWYg

H\Y`Wci fgY`hYI h`]g`6i fk Y``ž`? "`f&\$%&E`Gh X]c!VUgYX`]bghfi a YbhU```YUfb]b["``: Ufb\Ua .``5g\[UhY"

FYWca a YbXYX FYgci fWYg

@]VfUfmigci fWrgik]``VYia UbU[YXih\fci [\ih\Yi@Y[UbhcigmghYa icbih\YiWci fgYia ccXiYig]hY"

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

: cfa U``Yj U`i Uhjcbg`k]```VY`gci [\h'Uhh\\Y`YbX`cZgYa YghYfž`UbX`]bZcfa U``ZYYXVUW_'UbX`ei Yghjcbg`k]```VY k Y`Wca Y`UhUbmhja Y"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

#mci 'YbWci bhYf'U'dfcV'Ya 'k \Yb'UhYa dhJb['rc'gi Va]hmci f'Ugg][ba YbhNhfci [\'Hi fb]hJbž'd'YUgY
hY'Yd\cbY'9I hYfbU'Gi ddcfhcb'-',) '' ' ' %cfYa U] 'h\Ya 'cb'YI hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'f1 *) 'XUmg'U'mYUft'' #c'ii
UfY'i bUV'Y 'rc'gi Va]hmci f'Ugg][ba YbhXi Y 'rc'U'ZJi 'hk]h\ 'Hi fb]hJb 'mci 'a Um'Udd'mZcf'Ub'YI hYbg]cbž'Vi hmci
a i ghfYhJ]b 'mci f'hJW_Yhbi a VYf'Zfca '9I hYfbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fY'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbWY 'rc'gi ddcfhmci f'YI hYbg]cb'Udd']WUh]cb'''#mci 'Ya U]''9I hYfbU''Gi ddcfhmci 'k]'`
Ui hca UhJWJ'`mfYWY]j Y'U'hJW_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'``bYYX'hc'gdYWJZJWJ'`mUg_'Zcf'cbY''
Hi fb]hJb'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hYf''

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]``'VY'gHJhYX'cb'mci f'\\ci fgY'g'A ccX`Y'g]hY'k]h\ U`hYfbUh]j Y'gi Va]gg]cb'XYhJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va]hUgq][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±i\WbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiW\YUh]b[ihciU\W]XYbhU`m\Wcdm]b[iZfca iUigci f\Wik]h\ci hiU\w_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b[WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[. '7\Ub[]b['U'ZYk'k cfXg'UbX'd\fUgYg'k\]Y 'a cghmfYhU]b]b['h\Y'cf][]bU`ghi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU`žUbX']bZcfa Uh]cb'k]h\ci hUW_bck`YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k\YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'UbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk'k\c`Yžk]h\ci h'Uddfcdf]UhY'fYZYfYbW]b['"

7 c``i g]cb.'k cf_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhiUbmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_ž'ghYU']b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d'Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[`gcifWrg`k\]W\`\UjY`bchVYYb`fYUXžk]h\cihUW_bck`YX[]b[`h\Y`´gYWcbXUfm´gcifWrZfca`k\]W\`_bck`YX[Y`cZh\Ya`\Ug`VYYb`cVhU]bYX"

8 i d`]\WUh]cb`ffgY`Zd`U[]\Uf]ga "Ł gi Va]\h]b[`mci f`ck b`k cf_ž]b`k \c`Y`cf`]b`dUfhžk \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\hYX`Zcf`Ubch\Yf`UggYgga Ybhcf\Wci fgY`UhI BGK `cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWnfYZYfYbWlb['dfUWnWyg.

• DUfUd\fug]b[z'gi a a Uf]q]b[z'YggUmk f]hb['UbX'ha Y'a UbU[Ya Ybh

5 WUXYa]W = b Zcfa Uh cb ·

: cf`YggYbhJU``gh XYbh`]bZcfa UhJcb`fY`UhJb['hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY`gi Va]gg]cbg`[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK '< YU`h\ 'UbX'GUZYhm'dc`]W[Yg/
- YI Ua]bUhcb dfcWXi fYg/
- gdYWJU``Wcbg]XYfUHjcb`]b`H\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybhi fY/
- gh XYbhYei]mUbX X]gUV]`]m/
- UbX'ch\Yf'YggYbhJU`'UWUXYa]W']bZcfa UhJcbž'gYY

\hdg.#k k k "Uffg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa]W]bZcfa Uf]cb#dfchcWc`g![i]XY`]bYg#

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a U[Y'7fYX]h

Gc`c'DYfZcfa UbWY'A U_]b['&\$%

7F = 7CG

7F = 7CG Dfcj]XYf 7cXY. \$\$\$-,;