

EDST5144

H\YcfYh]WU`8ca U]bg']b'J]gi U`5fhg'9Xi WUh]cb

GYa YghYf'Hk c''#'&\$%

7ci fgY^{*}Cj Yfj]Yk

GHUZZ:7cbHUWh8YHJ]`g`

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWJhjcb	D\cbY
? UfYb A UfUg		d`YUgY'Ya U]`'Zcf'Ub Uddc]bha Ybh	%*∵ ccXgY`` 6i]`X]b[- ' &, &) (&

GWXcc`7cbHJWhi±bZcfa Uhjcb

GWXcc``cZ9Xi WUHjcb 5fhg`UbX`GcWjU``GWjYbWYg

@Yj Y``%z'>c\b'; ccXgY```6i]`X]b['fl &\$Ł

Н.

 $ghfi \ \ Whif YX fYgYUfWX fUWhj \]hjYg UbX fyZyI \]iXi \ U``fYZYWhjcb`k \]WX k \]```dfcj \]XY ghi \ XYbhg`k \]h\ `cddcfhi b]hjYg`hc Yb [\ U[\ Y`XYYd`mž`X]U`c [\]WU\`mž`UbX fYZYI]j \ Y`mk \]h\ 'h\ Y`hcd]Wg`UbX'h\ Y]f`Udd`]WUhjcb''$

5i ghfu`]Ub DfcZYgg]cbU`; fUXi UhY HYUWX]b[GhUbXUfXg

GHJbXUfX	8 YgWf]dh]cb
%"&""	9I dUbX'i bXYfgHUbX]b['cZ\ck 'ghi XYbhg''YUfb'i g]b['fYgYUfWX'UbX'k cf_d`UWY_bck 'YX[Y"
%") "&	8 Yj Y`cd`HYUWX]b[`'UWNjj]HjYg`H\UH`]bWcfdcfUHY`X]ZZYfYbHjUHYX`gHfUHY[]Yg`hc`a YYHH\Y gdYW]ZJW`YUfb]b[`'bYYXg`cZgH`XYbhg`'UWfcgg`H\Y`Z]```fUb[Y`cZUV]`]HjYg"
&"%"	Gi ddcfhWc``YU[i Yg'i g]b['W ffYbhUbX'Wca dfY\Ybg]j Y'_bck `YX[Y'cZWcbHYbhUbX hYUWX]b['ghfUhY[]Yg'hc'XYj Y`cd'UbX']a d`Ya YbhYb[U[]b['`YUfb]b['UbX'hYUWX]b[dfc[fUa g"
&"&"'	91\]V]h]bbcj Uhjj Y`dfUWjWY`]b`h\Y`gY`YWjcb`UbX`cf[Ub]gUhjcb`cZWcbhYbhUbX`XY`]j YfmcZ `YUfb]b[`UbX`hYUW\]b[`dfc[fUa g"
&"' "'	Gi ddcfhWc``YU[i Yg'hc'd`Ub'UbX']a d`Ya Ybh`YUfb]b['UbX'hYUW\]b['dfc[fUa g'i g]b[WcbhYa dcfUfm_bck `YX[Y'UbX'i bXYfghUbX]b['cZW'ff]W'i a ž'UggYgga YbhUbX'fYdcfh]b[fYei]fYa Ybhg"
' "&"'	K cf_'k]h\ 'Wc``YU[i Yg'hc'd`Ubž'Yj U`i UhY'UbX'a cX]Zm`YUfb]b['UbX'hYUW\]b['dfc[fUa g'hc WfYUhY'dfcXi Wfj Y``YUfb]b['Ybj]fcba Ybhg'h\UhYb[U[Y'U``'gh XYbhg"
1 111 111	Gi ddcfhWc``YU[i Yg'hc'gY`YWhUbX'Udd`mYZZYWhjj Y'hYUWX]b['ghfUhY[]Yg'hc'XYj Y`cd _bck `YX[Yžg_]``gždfcV`Ya 'gc`j]b['UbX'Wf]hjWU``UbX'WfYUhjj Y'h\]b_]b[

5ggYgga Ybh

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hUg_	K Y][\h	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX	5 +HG@'DfcZYgg]cbU`; fUXi UhY HYUWX]b['GHJbXUfXg'5ggYggYX	8iY 8UhY
HUg_'%') \$1	%ž&	&"%" ž&"&" ž&" "	&(#\$,#& \$% \$)% 5A
HUg_'&'fl = 85@L') \$1	% <u>z</u> & <u>z</u>	%"&"' ½%") "&ž&"%"' ž&"&"' ž&"' "' ž "&"' ž "' "'	%-#%\$#& \$% \$).\$\$ DA

D'YUgY fYZYf hc 3gefcS/[S` BdaXWee[a`S^EfS` VSdVe XadFWSUZWde cb h\Y dfYj]ci g'dU[Y

5ggYgga Ybh8YhJ]`g

5ggYgga Ybh'% 'HUg_'%

GhUfh'XUhY. '&' #\$+#&\$% '%&.\$\$ 5A

8YHJ]`g. `HUg__`%'` \$\$\$`k cfXg"`9I d`U]b`fk c`fh.YcfYfjWJ``Xca U]bg`UbX`fh.Y]f`]a d`]\WJfjcbg`Zcf`j]gi U``Uffg dYXU[c[]\WJ``dfUWf\Wf"Gfi XYbfg`k]```fYWf]j Y`k f]fhYb`ZYYXVUW_`%\$`Vi q]bYgg`XUmg`UZH\f`gi Va]gg]cb"

5XX]h]cbU`XYhU]`g.

 $\label{eq:continuous} 5\,ggYgga\ Ybh'\% \ F\ YgdcbgY \ dudYf. \ <\ ck\ \ 'Xc'hk\ c'cZh\ Y'h\ YcfYh]WU``ZfUa\ Yk\ cf_g`Wcbhf]Vi\ hY'hc'X]ZYfYbh i bXYfghUbX]b[\ gž'a\ YUb]b[\ 'UbX'j\ U`i\ Y']b'Ufh3 \\ HUq_.$

fUŁGY`YWitk c'cZthY'thYcfYtjWU``Xca U]bg'cZdfUWjWY'Ug'Yl d'U]bYX']b'A cXi `Yg'5!: "'A U_Y'Yl d`]Wjh\ck WcbWYdrg'UbX'fYZYfYbWYg'`]b_'hc[YthYf'Ug'U'bYtk cf_'cZfY`Uh]cbg'UbX'a YUb]b[g'k]th\]b'YUW\'cZth\Y gY`YWhYX'Xca U]bg": cf'Yl Ua d`Yž'VYtk YYb'th\Y'Ufftk cf_'UbX'Ufftjghž'Ufftk cf_'UbX']a]tUh]cb'cZth\Y'k cf'Xž Ufftk cf_'UbX']bZi YbWY'cZth\Y'gd]f]ti U'ž'Ufftk cf_'UbX']bgtjhi tjcbgž'Ufftk cf_'UbX']a U[]bUtjcbž'UfftjghiUbX]bgtjhi tjcb'YtW'±Wi XY'Yl d`]WjhfYZYfYbWY'tc'g]l 'fYei]fYX'fYUX]b[g'UbX'gY`YWfYX'fYWca a YbXYX'fYUX]b[g'Zca 'th]g'Wci fgY'Ug'Uddfcdf]UthY"

fVŁ'H 'cZh\Y

8YHJ]`g. `HUg_`&`fl +B 5@£`' \$\$\$`k cfXg"`8Yj Y`cd`U`GHJ[Y`(!) `dfc[fUa `]bZcfa YX`VmH\Y`H\YcfYh]WJ`Xca U]bgGh XYbhg`k]```fYWY]j Y`k f]hhYb`ZYYXVUW_`%\$`Vi g]bYgg`XUmg`UZHYf`gi Va]gg]cb"

5XX]h]cbU`XYhU]`g.

 $5ggYggaYbh\&`Dfc[fUaidfcdcgU`.`5dd`m]b['h\YcfYh]WU``XcaU]bg'cZdfUWfW'hc'dfc[fUaa]b[']b'GhU[Y'('/)']giU``5fhg$

HUg_.

8 Y j Y`cd`UbX`^\ gh]Zm`U`dfcdcgU``Zcf`U`dfc[fUa `h\UhiVi]`Xg`cb`cbY`UbX#cf`hk c`cZh\Y`h\YcfYh]WU``ZfUa Yk cf_g Ug`Ub`Yd]ghYa]Whcc``gi]hYX`hc`ghi XYbhg'`Wc[b]h]j Y`UV]`]h]Yg`UhdUfh]W``Uf`U[Yg#ghU[Yg`cZgW\cc`]b[`fY[GhU[Y`) L''`G\ck `\ck `mci `k]```UXUdhmci f`hYUW\]b[`gmmY`UbX`X]ZZYfYbh]UhY`WcbhYbh`]b`fY`Uh]cb hc`h\Y`h\YcfYh]WU``Xca U]b`gY`YW#YX"

 $7 \cosh[XYf'h Y'] a \ d'[WUh] \cosh[Zcf'Ufha U_] b [\ 'UbX'Wf] h] WU ``UbX' \ [ghcf] WU ``gh X] Y g'UbX' a \ U_Y' f YZ f Y b W 'hc' g d Y W] Z] WU Ufh] g hg Y UbX' h Y] f' k cf_g''$

Mci 'a Umi gY'cbY'cZh\Y'h\YcfYh]WU``Xca U]bg'gY`YWYX`Zfca '5ggYgga Ybh%'UbX'cbY'ch\Yf'Zfca 'A cXi `Yg'; ! ? 'h\Uh'mci '\Uj Y'bchmYhUXXfYggYX"'-\bWi XY'fYZYfYbWYg'h\fci [\ci h\Zfca 'h\Y'Wci fgY'fYUX]b[g'UbX'ch\Yfg gi]hYX'hc'mci f'di fdcgYg'i g]b['h\Y'5D5'ghmY"

: i ft\ Yf XYlU] g cb h\ Y Zcfa Uhk]``VY dfcj]XYX]b WUgg#cb A ccX`Y"

I BGK 'G7<CC@'C: '98I 75H€B'FI 6F = 7 # 998657? 'G<99H' 98GH) %((

5ggYgga Ybh'HUg_. 'HUg_'%

GdYWJZJW7 fJHYfJU	"flŁ———	——2fŽŁ
bXYfghJbX`cZh\Y`eiYgh]cb`cf`]ggiY`UbX`h\Y`_YmWcbWYdhg bjc`jYX		
i bXYfgfUbX]b['cZf\Y'fUg_'UbX'W\c]WY'cZgY`YWYX h\YcfYh]WU`ZfUa Yk cf_g		
088 °√w."		

GdYW] ZJW7 f]HYf]U	¨flŁ—		 2fŽŁ
jcWUVi`UfmigYžgdY``]b[ždibWniUh]cb'UbX'kcfX'`Yb[h\			
; YbYfU`Wca a Ybhg#YWca a YbXUh]cbg'Zcf'bYl h'h]a Y.			

@YWhifYf

FYWcaa YbXYX. ' "#&\$'f!@'DG'7F'8B'<8E' k Y][\h]b['.'&\$i

GdYWJZJW7 f]HYf]U	¨flŁ—	 	2fŽŁ
jcWUVi`UfmigYžgdY``]b[ždibWniUh]cb`UbX`kcfX``Yb[h\			
; YbYfU`Wca a Ybhg#fYWca a YbXUh]cbg'Zcf'bYl h'h]a Y.			

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FYWcaa YbXYX. ' "#&\$'f!@'DG'7F'8B'<8E' k Y][\h]b['.'&\$i

"B6. h\]g'h\\g']b'h\Y'j Uf]ci g'Vcl Yg'UbX'XYg][bYX'bc'dfcj]XY'ZYYXVUW_bc'gh XYbhg/h\Ym'UfY'bch[]j Yb Yei U``k Y][\h']b'XYhYfa]b]b['h\Y'fYWca a YbXYX'[fUXY''8YdYbX]b['cb'h\Y'bUh fY'cZ'h\Y'UggYgga Ybh'H\g_ž YWh fYfg'a Um'U`gc'WcbhYl hi U`]nY'UbX#cf'Ua YbX'h\YgY'gdYWJZ]WWf]hYf]U''' H\Y'fYWca a YbXYX'[fUXY']g hYbh'Hhj Y'cb`mžgi Va YWh'bc'gh'UbXUfX]gUhjcb'dfcWYggYg'UbX'Uddfcj U``Vmh\Y'GW\cc``cZ'9Xi WUhjcb@YUfb]b['UbX'HYUW\]b['7ca a]hhYY"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

 $Gh\ XYbhg\ 'UfY\ 'YI\ dYWhYX\ 'hc\ 'di\ hh\ Y]f\ 'bUa\ Yg\ 'UbX\ 'gh\ XYbhbi\ a\ VYfg\ 'cb\ 'Yj\ YfmdU[\ Y\ 'cZh\ Y]f\ 'Ugg][\ ba\ Ybhg\ ''$

Hi fb]h]b Gi Va]gg]cb

 $\bullet \ \ \, \text{K cf_'gi Va]} \\ \text{lh} \\ \text{X '\%' hc '\% 'XUmg 'UZhYf' h\ Y 'gh]} \\ \text{di ``UhYX 'XYUX`]} \\ \text{bY } \\$

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWfg'cZ9@G9'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "f\hd.#gi VYWf[i]XYg"]VfUfm"i bgk "YXi "Ui #\`]gY#JVci hY`]gY

7ci fgY GW YXi `Y

J]Yk 'WUgg'Ha YHUV'Y

		WcbWYdhi U``XYa UbX`UWfcgg`ghU[Yg`cZ`YUfb]b[`]b J]gi U``5fhg" •`K \Uh`]g`fY`Uh]cbU``5Ygh\Yh]Wg •`7f]h]WU``DfUWf]WY`UbX`fY`Uh]cbU``UYgh\Yh]Wg •`FY`Uh]cbU``dYXU[c[m
K YY_', .'%\$'GYdhYa VYf'! %*'GYdhYa VYf	Cb`]bY'5Whjj]hm	; `f]ĿH\Y`5fmg`Ug`U`kUmcZ_bck]b[f]]ĿH\Y`WcbWdhcZUfhUg`U`dfUWf]WU``UbX`WcbWdhiU` _]bX`cZfYUgcb]b[`UbX`dYXU[c[m]b`j]giU``Ufng
K YY_''%+'GYdhYa VYf'! &' 'GYdhYa VYf	C b`]bY`5W]j]hm	< `f]Ŀ`H\Y`5ffg`Ug`U`Zcfa `cZWcaaib])WUh]cb f]]Ŀ`7cbW'dhg`cZj`]gi U```]hYfUWhž`Ufh'Ug`U`bcb! gdcbhUbYcig``Ub[iU[Y`h\Uh'bYYXg`hc`VY`kf]hhYb`UbX fYUX`UbX`dYXU[c[m]b`j`]giU``Ufhg

FYgci fWyg

DfYgWf]VYX F Ygci fWYg

5```fYUX]b[g'UbX'fYgcifWyg'k]``VY'dcgHYX'cb'h\Y'AccX`Y'g]hY'Zcf'h\]g'WcifgY"

FYWca a YbXYX FYgci fW/g

5```fYUX]b[g'UbX'fYgcifWyg'k]```VY'dcgHYX'cb'H\Y'AccX`Y'g]HY'Zcf'H\]g'WcifgY"

7 ci fgY 9j U'i Uh]cb UbX 8 Yj Y'cda Ybh

F Yj]g]cbg'h\]g'Wci fgY'UfY'a UXY'cb'h\Y'VUg]g'cZghi XYbhg'ZYXVUW_j JU'Ya U]`g'UbX'X]gW gg]cbg'Ug'k Y```Ug h\fci [\'A m9I dYfY]bW'ZYYXVUW_cb'h\Y'Wci fgY"'=\Uj Y'fYgdcbXYX'hc'ZYYXVUW_Zfca 'ghi XYbhg']b'h\Y'dUgh' H\]g'ZYYXVUW_gi [[Yghg'h\Uh[]j Yb'h\Y'\][\'Yj Y`g'cZfYUX]b['a UhYf]U`UbX'h\Y'd\]`gcd\]W'`bUhi fY'cZh\Y a UhYf]U`g']b'h\]g'Wci fgY'h\Uhih\Y]f'dfYZYfbYW']g'hc'U'gY`ZX]fYWhYX'k Umk]h\ZcW g'`YWi fYg']b'k\]W'\Wci fgY WcbhYbhUbX'UddfcUW\Yg'Zcf'Wca d'Yh]b['h\Y'UggYgga Ybhg'UfY'X]gW ggYX'UbX'a cXY``YX"'+b'h\]g'gYa YghYf'' '`YWi fYgž'%'Uhh\Y'VY[]bb]b['ž'%'Uh'U'a]Xdc]bhUbX'Ubch\Yf'df]cf'hc'h\Y'XYj Y`cda YbhcZ5ggYgga Ybh&''UfY gW\YXi 'YX"

=a U[Y'7 fYX]h

H\]g^]a U[Y'k Ug'HJ_Yb'VmH\Y'Wci fgY'Wcbj Ybcf"

7F = 7CG

7F = 7CG Dfc|]XYf 7cXY. \$\$\$-,;