

ARTS2876

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7 fYX]hDc]bhg**

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9UW.cZi g`\Ug`U`gY`ZUbX`Ub`]XYbh]hmVmj]fhi Y`cZVY]b[`\i a Ub"'`6i hXc`ch\Yf``]j]b['VY]b[g`\Uj Y`U`gY`Z3 8c`ch\Yf``]j]b['VY]b[g`\Uj Y`gcW]Yhm]b`h\Y`gUa Y`k Um\i a Ubg`Xc3``\\b`h\]g`Wti fgYž'mci `k]```Wtbg]XYf`gca Y hfUX]h]cbU``Uggi a dh]cbg`cZgY`Z\ccX`fY"["ž'h\Y`WUdUW]hmZcf`fYUgcbž`gdYYW\ž'UbX`a Ya cfmtZfca 'U`fUb[Y`cZ X]ZZYfYbhgcW]c`c[]WU``dYfgdYWfj Yg"'`Mci `k]```Wtbg]XYf`\ck `fYWfbh]bhYfX]gW]d`]bUfmk cf_'W\U``Yb[Yg`gca Y cZh\YgY`VUg]WUggi a dh]cbg"'`Mci `k]``YI Ua]bY`Uf[i a Ybhg`Zcf`UbX`U[U]bghYl hYbX]b[`gY`Z\ccX`hc`ch\Yf

Hi hcf]U`g. 'Hi hcf]U`g'k]```VY'i gYX'hc'cf[Ub]nY'ga U```[fci dg'hc'YgHUV`]g\ 'U'Wc``UVcfUhjj Y'k cf_]b[
Ybj]fcba Ybhk \YfY'ghi XYbhg'Wb'`YUfb'Zfca 'cbY'Ubch\Yf"'`Ga U```[fci dg'k]```VY'cf[Ub]nYX']b'h\Y'Z]fgh
hi hcf]U``]b'K YY__&'cf'' "'`-\(\frac{1}{4}\)'h\Y'gYWcbX\\U'ZcZh\Y'gYa YghYfz'[fci dg'k]```ZUW]]hUhY'Ub']b!WUgg'UWfji]hmXi f]b[
hi hcf]U``ha Y"''; fci dg'k]```VY'YI dYWhYX'hc'a YYhhc'X]gW gg'h\Y'fYUX]b[g'UbX'dfYdUfY'Zcf'h\Y]f'hi hcf]U`
ZUW]]hUh]cb'ci hg]XY'WUgg'ha Y"

FYUX]b['E i Ygh]cbg. FYUX]b['ei Ygh]cbg'k]``'VY'Uj U]`UV`Y'cb!`]bY'hc'Ugg]gh'ghi XYbhg'''H\Ymk]``[]j Yghi XYbhg'U'dc]bh'cZfYZYfYbW'Zcf'X]gW gg]cb']b'h\Y'hi hcf]U`"

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8YHJ]`g. K cf_Vcc_Ghi XYbhg`k]```dfYdUfY`k f]hhYb`Ybhf]Yg`cZ%l&`dU[Yg`cb`h\Y`Ugg][bYX`fYUX]b[g"`H\Y`k cf_k]```VY`gi Va]hhYX`UbX`UggYggYX`dYf]cX]WU``mh\fci [\`h\Y`hYfa "': YYXVUW_.``K f]hhYb`Wca a Ybhg`UbX`[fUXYX U[U]bgh'U`Wf]hYf]U`g\YYh'`

5XX]h]cbU`XYhJ]`g.

Mci 'k]```gi Va]hmci f`k cf_Vcc_Ybhf]Yg`Zcf`K YY_g`%UbX`&`]b`K YY_'' 'Zcf`[fUXYX`ZYYXVUW_"

Mci 'k]```gi Va]hYblf]Yg'Zcf'K YY_g'' 'kc', ']b'K YY_'- 'Zcf'[fUXYX'ZYYXVUW_''

 $\label{thm:continuous} \begin{tabular}{ll} His plant of the lighthfully a property of the ligh$

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8 YHJ]`g. `Ghi XYbhg`k]```XYg][b`Ub`]b!WUgg`UWij]lmf&\$`a]bŁh\UhVf]b[g`]bhc`fY`]YZUb`]a dcftUbhUgdYWicZh\Yh'I hi bXYf`Wcbg]XYfUhjcb"'H\Ymk]```VY`UggYggYX`cb`h\Y]f`UV]`]lmhc`k cf_'Wc``UVcfUhj Y`mUbX`hc`YI Ua]bY`h\Y dfUWj\WJ`]a d`]WUhjcbg`cZh\Y`fYUX]b['Zcf`Yj YfmXUm`]ZY"'Ghi XYbhg`k]```VY`fYei]fYX`hc`dUffj\MjdUhY`]b`cbY hi hcf]U``[fci d`ZUWj`]hUhjcb`]b`h\Y`gYa YghYf"'9UW\`[fci d`k]```fYWf]j Y`U`Wc``YWfj Y`[fUXY"; fUXYX`U[U]bgh Wf]hYf]U`g\YYhŽ`k f]hhYb`Wca a Ybhg`UbX`j YfVU``ZYXVUW_cb`h\Y`XUm'

5XX]h]cbU`XYhU]`g.

Hi hcf]U``[fci d`ZUWj`]hUh]cbg`k]```VY`\Y`X`Xi f]b['K YY_g')!- ']bWi g]j Y"

Hi fb]h]b gYh]b[. H\]g]g bchU Hi fb]h]b Ugg][ba Ybh

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@Yb[h\.'%)\$\$ k cfXg

8YHJ]`g. FYZYWIJ Y'YggUmGhi XYbhg`k]``kf]hY`Ub'YggUmfN) \$\$`kcfXgŁ'h\Uh']XYbhJZ]Yg`%\&`Wf]hJWU```YUfb]b[acaYbhg`h\YmYldYf]YbWYX']b`h\Y`WcifgY"'H\YgY'YldYf]YbWYg'k]``VY'X]gW ggYX'Yld`]Whm]b`fY`Uhjcb`hcWcifgY'WcbhYbh']b`cfXYf`hc`XYacbghfUhY`Ub`ibXYfghUbX]b[`cZ_YmWcbWYdhg"H\]g`]g`h\Y`Z]bU``UggYggaYbh']bh\Y`WcifgY"'

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' \end{tab$

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 $\hbox{Ghi XYbhg'UfY'ghfcb[`mYbWc'i fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk'``YWhi fY'fYWcfX]b[g''] } \\$

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	Hi hcf]U`	; fcid: UWj]hUnjcbg:VY[]b:h\]g:k:YY_"
K YY_'*.^&) ^A UfWX^!^' % A UfWX	@YWi fY	H\Y'5b]a U`UbX'h\Y'GY`Z!'8YgWUfhYgž 8Yff]XU'fA UfW\'&) Ł
	FYUX]b[8 YgWJffYgžF YbY f‰,, Ł'8]gWci fgY Cb H\Y A Yh\cX - DUfhJ 'Dd" (\$! (* '± 6 WeUSdWe, E WWJfWW BZ['aeabZ[US^I dff]` Ye fHfUbg" >c\b 7 chjb[\Ua 5bX F cVYfhGhcch\cZŁ" 7 Ua Vf]X[Y. 7 Ua Vf]X[Y I b]j Yfg]miDfYgg"
		8 Yff]XUž>UWei Ygʻf&\$\$(ŁʻʻJ]c`YbWYʻ5[U]bgh5b]a U`g']bʻ8adl ZSfFa_adai, 3 6 [S'aYgWʻGHJbZcfX. GHJbZcfXʻI b]j Yfg]lmiDfYggždd"*&!+*"
	Hi hcf]U`	; fcid ZUWj]hUhjcbg WcbhjbiY"
	Cb`]bY'5Wh]j]hm	GYY A ccX Y Zcf XYHJ] g"
K_YY_'+.'%'5df]`'!'+'5df]`	@YWi fY	A cfU`]hmUbX`h\Y`GY`Z'!`8Y`K UU``f5df]``%
	FYUX]b[8 Y K UU`ž: fUbgʻf&\$\$' Ł'A cfU`]lmiUbX`h\Y`GcW]U` +bgh]bWig. 7 cbh]bi]lmik]h\ h\Y`Ch\Yf`Df]a UhYg'`FZW FS``Wd>WUfgdWe a`: g_S`HS^gWež'8Y`]j YfYX`Uh Df]bWYhcb`l b]j Yfg]lmž'Bcj Ya VYf'%!&\$ž'&\$\$' "`Dd" %%!%('"
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K YY_', .', '5df]`!'%('5df]`	· ·	H]a Y'UbX'h\Y'GY'Z!'6Yf[gcb'fbdf]`', Ł
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	C b`]bY'5Wh]j]hm	GYY'A ccX`Y'Zcf'XYhJ]`g"
K YY_''%) '5df]`'!'&% 5df]`	@YWi fY	H\]b_]b['Ug'5b]a U`g'! '8Yff]XU'f5df]`'%) Ł
	FYUX]b[8 Yff]XUz >UWei Yg f0% - +0&\$\$(Ł'H\Y 5b]a U`H\Uh H\YfYZcfY =5a fA cfY Hc': c`ck Ł' = A Uth\Yk 7 U`UfWc 5bX DYtYf 5 thYftcb f9 Xg"Ł'3` [_ S^ BZ['aeabZk" @cbXcb. 7 cbt]bi i a 'DfYgg" 91 WYfdtg Dd" '%%! &-"
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	5ggYgga Ybh	Mci f'k.cf_Vcc_'Ybhf]Yg'Zcf'K YY_g''!, ']bWi g]j Y'UfY Xi Y']b'K YY_'-"
K_YY_'%\$.'&&'5df]`!'&,	C b`]bY'5Wfj]hm	Bc"YWnifY'cf'hihcf]U`g'h\]g'kYY_'VYWUigY'5df]`'&&h\

5df]`]g'U'di V`]W\c`]XUmf9UghYf'AcbXUm2"''±oghYUX'cZ WUgg'h\]g'k YY_ž'd`YUgY'Wcad`YhY'cb`]bY'UWfj]]hm fgYY'AccX`Y'Zcf'XYhJ]`g2"''7`UggYg'k]``'VY fYgWXYXi 'YX'Zcf'5df]`&-'fK YY_'%42"
K YY_'%%'&-'5df]`'!'% A Um	@YWh fY	@]j YgʻcZʻ5b]a Uʻgʻ! '7cYmYYʻf&- '5df]`Ł
	F YUX]b[7 cYmYYž'>"A "'f% Ł''H\Y'D\]`cgcd\Yfg'UbX'f\Y 5b]a U`g'']b'HZWFS`` Wd>WUfgdWe a`: g_S` HS'gWe" 8Y`]j YfYX'Uh'Df]bWYhcb'l b]j Yfg]lmž'C WhcVYf'%) ž % - +"'Dd"%%!%(' "
		? UZ_Už: fUbn'f% +%L'"5 F Ydcfh'hc'Ub'5WUXYa m" Dd"&) \$!&) - ']b'8d\$` =SX] S, FZW5a_ b'MfWEfad[We" BYk'Mcf'GW\cW_Yb'6cc_g
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7 ci fgY 9j U'i Uh]cb UbX 8 Yj Y'cda Ybh

Ghi XYbhiZYXVUW_k]``VY`[Uh\YfYX`h\fci [\'a m9IdYf]YbW'gi fj Ymgž'UbX`mci k]``VY`[]j Yb`cddcfhi b]hjYg h\fci [\ci hih\Y`gYa YghYf`hc`[]j Y`]bZcfa U``ZYYXVUW_"`H\Y``Ughiha Y`=hUi [\hi5F HG&, +* 'I bXYfghUbX]b[`h\Y GY`Zk Ug`]b`&\$%+"`H\]g`Wc\cfhcZghi XYbhg`[Uj Y`dcg]hj Y`ZYYXVUW_cb`h\Y`Wci fgY"

 $6 \text{ UgYX} \cdot \text{cb} \cdot \text{h} \cdot \text{lg} \cdot \text{Z} \cdot \text{Y} \times \text{VUW} \cdot \text{Z} = \text{VI} \cdot \text{Y} \cdot \text{a} \cdot \text{UXY} \cdot \text{h} \cdot \text{Y} \cdot \text{Zc} \cdot \text{ck} \cdot \text{lb} \cdot \text{I} \cdot \text{VX} \cdot \text{Ub} \cdot \text{I} \cdot \text{Y} \cdot \text{Wci} \cdot \text{fgY}.$

 $\%''5ggYgga Ybh'Ghfi WhifY'!'=\Uj Y'a cX]ZYX'h\Y'UggYgga Ybh'ghfi WhifY'hc'Ybgi fY'h\Uh'h\Y'k cf_Vcc_'hUg_']g df]cf]h]gYX''$

&"'5bbci bW'a Ybhg'! '=k]``i d`cUX'j]XYc'gY[a Ybhg'hc'A ccX`Y']b'cfXYf'hc'UXXfYgg']ggi Yg'fY`UhYX'hc UggYgga Ybh'UbX'Wci fgY'Vi g]bYgg"'H\]g'k]``[]j Y'i g'a cfY'hJa Y']b''YWf fYg'hc'X]gW gg'Wci fgY'WcbhYbh'

=j Yfma i W. 'UddfYVJUhY 'h.]g 'ZYYXVUW_ž'UbX ``cc_'Zcfk UfX 'hc']a d`Ya Ybh]b['h. YgY'gi [[Yg]cbg']b '&\$% "'6Ygh k]g\Yg 'Zcf'h. Y 'bYk 'hYfa "

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

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D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh]b[ihciUVW]XYbhU`mWcdm]b[iZfca iUiqci fWYik]h\ci hUW_bck iYX[Ya Ybh'i

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7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY