

ARTS2877

HYW.bc`c[]Ygž'7i`h fYž

7 m

7 ci fgY Cj Yfj]Yk

`GhUZZ`7cbhUWh`8YhU]`g`

7cbj Ybcfg

BUa Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
? Ulf]bU'@ci]gY		K YXbYgXUmg %! %		B 5
A ccfY		cf:Vm:Uddc]bha Ybh fd:YUgY:Ya U]:'hc	%%&5	
		a U_Y`Ub		
		Uddc]bha YbhŁ		

GWXcc`7cbHJWh ±bZcfa Uh]cb

GWkcc``cZGcWJU``GWJYbWYg

Fcca '%) -

A cfj Yb 6fck b 7 &\$

Ya U]`. 'gcgg4 i bgk "YXi "Ui

d\cbY. '\$&'-',) '% \$+

7ci fgY[.]8YHJ]`g

cZ\Y`d]b['ghi XYbhg'XYj Y`cd'h\Y]f'ck b'j]Yk g"'@YWhi fYg'Wcbg]ghicZU'j Uf]YhmicZ]bhYfUWhjj Y'UWhjj]h]Yg'h\Uh]bWi XY'X]gWi gg]cb'cZh\Y'fYUX]b[g'UbX'WUgg!k]XY'XYVUhYg"		

5ggYgga Ybh

 $B\ chY'\ h\ Uhih\ Y':\ f]XUm']b'k\ YY_'-']g';\ ccX':\ f]XUm'!\ 'UbX'\ 0\ YbW'\ Qh\ Y'WUggYg'\ Zfca''h\]g'XUmk\]``'VY'\ hU_]b['d`UW'\ cb'K\ YXbYgXUmcZK\ YY_''''fGcGGŁ''$

5ggYgga YbhHUg_g

5ggYgga Ybh'hJg_	K Y][\h	8i Y [.] 8UhY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
Hi hcf]U`ZUWj]hUh]cb'	&\$I	Bch5dd`]WUV`Y	' <u>ž(</u> ž) ž* ž+ž,
K cf_Vcc_	(\$1	&, #\$(#&\$% `\$).\$\$`DA) <u>ž</u> * ž. ž-
9ggUm	(\$1	% #\$(#&\$% `\$(.\$\$`DA	* Ž+

5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% Hi hcf]U`ZUW[]HJh]cb

GhUfhXUhY. Bch5dd`]WUV`Y

8 YHU]`g. ; fci d`ZJW]`]HUH]cbg`cZ[fci dg`fUb[]b[`Zfca `(!+'ghi XYbhgžhc`VY`\Y`X`]b`hi hcf]U`g`k YY_g`(`hc`%&" : UW]`]HuH]cbg`k]```VY`%)!&\$`a]bi hYg`]b``Yb[h\"``Ghi XYbhg`k]```fYWV]j Y`k f]HhYb`ZYYXVUW_f]ZfYei YghYXŁ'UbX`Ubi a Yf]WU``[fUXY`k]h\]b`hk c`k YY_g`cZZJW]`]HUH]b["`H\Y`ZYYXVUW_g\YYh#fi Vf]Wk]```VY`Uj U]`UV`Y`hc`ghi XYbhg Uhh\Y`gHUfhcZh\Y`Wci fgY`gc`h\Uhh\YmWUb`k cf_hck UfXg`gdYW]Z]YX`gHUbXUfXg

5XX]h]cbU`XYhU]`g.

Ga U```fYUX]b['[fci dg'k]``'VY'Zcfa YX']b'h\Y'WUgg']b'k YY_'&"

9UWX 'fYUX]b['[fci d'k]```\Uj Y'U'[fci d'`YUXYf'k \c']g'fYgdcbg]V`Y'Zcf'Wc``YWNjb['WcbHUWn']bZcfa Uhjcb'cZ
[fci d'a Ya VYfgž'Ybgi f]b['th\UhYUWX'[fci d'a Ya VYf']g'WYUf'Ug'hc'th\Y]f'Ugg][bYX'HUg_ž'cf[Ub]g]b[
a YYhjb[gž_YYd]b['Ub'UhhYbXUbWY'`]gh'Zcf'Yj Yfm'a YYhjb[ž'UbX'bchjZmjb['th\Y'Wci fgY'Wcbj Ybcf'cZUbm
WcbWYfbg'cf'[f]Yj UbWYg'Ug'gccb'Ug'dcgq]V'Y"

9 UWX '[fci d'k]```VY`Wc``YWlji Y`mfYgdcbg]V`Y`Zcf`ZUWJ`]hUh]b['U'&\$!a]bi hY`gYWl]cb']b`cbY`cZh\Y`gYa]bUf`]b k YY_g'' 'hc`+ž`ZcW g]b['cb'h\Y`fYUX]b[g'h\Uhk YfY`Ugg][bYX"

; fci d'a Ya VYfg'UfY'YbWci fU[YX'hc'[Yhhc'_bck 'cbY'Ubch\Yf'UbX'YI d'cfY'h\Y'X]ZZYfYbh'hmdYg'cZ

X]gW]d']bUfm_bck `YX[Y'UbX'g_]``g'h\Uh'YUW\'a Ya VYf'Vf]b[g'hc'h\Y'[fci d"; fci d'a Ya VYfg'WJb'gi ddcfh

YUW\'ch\Yfz'XfUk]b['cb'h\Y]f'fYgdYWf]j Y'ghfYb[h\gz'k\Yb'dfYdUf]b['h\Y'fYUX]b[g'Zcf'h\Y'ZJW]']hJhjcb"

: UW]]hJhcfg'UfY'YI dYWfYX'hc'dfYdUfY'UWf]j]hJYg'UbX'i gY'h\YgY'UWf]j]hJYg'hc'ZJW]']hJhY'X]gW gg]cb']b'h\Y

hi hcf]u`"'H\YgY`UWhji]hjYg`UfY`]bhYbXYX`hc`[YbYfUhY`WfYUhji Y`h\]b_]b[ž`UWhji Y`X]gWi gg]cbž`UbX`ghja i `UhY X]u`c[i Y`]b`h\Y`WUggfcca "'; fci dg`UfY`YbWci fU[YX`hc`XYj Y`cd`X]gWi gg]cb`ei Yghjcbg`UbX`i gY`a YX]U`h\Uh h\Ymh\]b_`k]```\Y`d`h\Y`WUgg"

: UWj`]hUhjcb`[fci dg`k]```i gY`h\Y`gYa]bUf`]b`Zjfghik YY_g`cZhYfa `hc`VfU]bghcfa `h\Y]f`ZJWj`]hUhjcb`d`Ubg"

F Ya Ya VYf. 'h\Y`U]a `cZh\Y`ZJWj`]hUhjcb`]g`bchihc`X]gd`Umimci f`ck b`_bck`YX[Y`Vi hihc`\Y`d`h\Y`k \c`Y`WUgg
XYj Y`cd`Ub

]bZcfa YX:UddfYV]Uhjcb:cZh\Y:WcbWrdhg:UbX:h\YcfYh]WU:Uf[i a Ybhg:Uf]g]b[iZfca:h\Uhk YY_'g:X]gWigg]cb"

H\Y:ZUW];]hUhjcb:k]```VY:Yj U`i UhYX:]b`hYfa g`cZ\ck:k Y```]hYb[U[Yg`k]h\:h\Y`k YY_'g`fYUX]b[gž'UbX:\ck
\Y`dZ\``]h]g`hc:h\Y``YUfb]b[icZch\Yf`ghi XYbhg":\\\\]g`]a dcfhUbhihc:[]j Y`mci f`Ui X]YbWi'h\Y`cddcfhi b]hmhc:a U_Y
h\Y]f`Wcbhf]Vi h]cb"

9UWX [fci d']g'Yl dYWMYX'hc'a YYh'ci hg]XY'WUgg'\ci fg'hc'cf[Ub]gY']hg'ZJWJ]hUh]cbg"; fci dg'k]``\Uj Y'hc

VfU]bghcfa 'cb'h\Y']XYUg']b'h\Y'fYUX]b[gž'UbX'h\Yb'd'Ub'h\Y'cj YfU``ZJWJ]hUh]cb'UbX'XYj Y`cd'U'gi]hUV`Y

X]j]g]cb'cZ`UVci f"'9j Yfm[fci d'a Ya VYf'a i gh'Wcbhf]Vi hY'hc'Vch\'h\YgY'a YYh]b[g'UbX'h\Y'WUgg'ZJWJ]hUh]cb"

FY\YUfgY'mci f'ZJWJ]hUh]cbg']b'dYfgcbž'hc'[Yh'U'gYbgY'cZ\ck 'k Y``h\Ymk]``k cf_'Zcf'h\Y'Ui X]YbWf.

 $\label{thm:condition} $$W_YW_h^h$ Uhimci fiel Yghlcbg^ufY^vch^ubgk YfUV^Y^ubX^ghla i `Uhlb[~z^ubX^h$ Uhih\Y^dUfhg^cZh\Y ZUW]^hUhlcb^k cf_k Y^*^hc[Yh\Yf'']$

A Uf_]b['Wf]hYf]U. 'Wf]h]WU``UbU`mg]g`cZh\Y`hYI hgžXYdh\`cZYb[U[Ya Ybhik]h\`WcbWfdhg`]b`h\Y`k YY_'g fYUX]b[g`UbX``YWh fYgž]bXYdYbXYbhifYgYUfW\žWfYUh]j]hmž'UbX`g_]`Z` ``X]gW gg]cb`ZUW]`]hUh]cb

5ggYgga Ybh& K cf_Vcc_

GHJfh'XUhY.

8 YHJ]`g. 'H\Y`K cf_Vcc_'Wcbg]ghg`cZfYZYWf]cbg`cb`h\Y`k YY_'g`fYUX]b[g`hc`VY`YbhYfYX`df]cf`hc`h\Y``YWfi fYž Ug`k Y```Ug`fYZYWf]cbg`UZhYf`h\Y``YWfi fY"`5ddfcl]a UhY`m' \$\$`-`(\$\$`k cfXg`Yj Yfmk YY_"'Ghi XYbhg`k]```fYWY]j Y k f]hhYb`ZYYXVUW_f]ZfYei YghYXŁ'UbX`U`bi a Yf]WU``[fUXY`k]h\]b`h\fYY`k YY_g`cZgi Va]gg]cb"'H\Y`ZYYXVUW_g\YYh#fi Vf]Wk]```VY`Uj U]`UV`Y`hc`ghi XYbhg`Uh'h\Y`ghUfficZh\Y`Wci fgY`gc`h\Uh'h\YmWUb`k cf_'hck UfXg gdYW]Z]YX`ghUbXUfXg"

5XX]h]cbU`XYhU]`g.

: fca 'K YY_'&'hc'K YY_'- 'f]bWi g]j Ytz'gh XYbhg'Wca d'YhY'' \$\$'k cfXg'YUWX'k YY_

D`YUgY`Vf]b[`h\YgY`XfUZrg`hc`hi hcf]U`g`YUW\kYY_`Zcf`X]gWigg]cb"`H\Y`XfUZrg`k]```]bWi XY`U`dfY!`YWhifY Wa dcbYbhfYZYWhb[`cb`h\Y`kYY_fg`fYUX]b[gžUbX`U`dcgh'YWhifY`Wa dcbYbh'`: cf`h\Y`dcgh'YWhifY

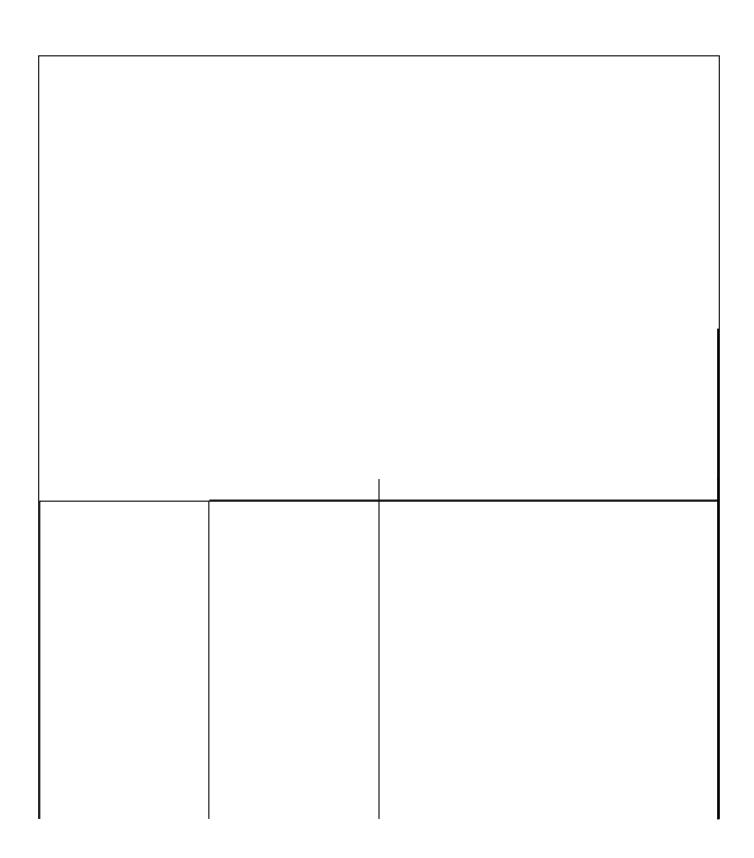
 $\label{thm:condition} We a debYbhžighi XYbhgia i ghk f]hY fYZYWh]cbg Zc``ck]b['h\Y``YWh fY 'hc`fYZYWhcb`\ck 'h\Y]f'h\ci [\hg`UVci h h\Y`fYUX]b[g`g\]ZhYX k]h\ 'h\Y]f'Yb[U[Ya Ybhk]h\ 'h\Y`WUgg`X]gWi gg]cb" Gca Y`ghi XYbhg`a Umk f]hY 'h\Y`Vi `_ch\Y]f'Ybhf]Yg`VYZcfY``YWh fYžk \]Y`ch\Yfg`a Um\Uj Y`a cfY'hc`k f]hY`UZhYf'h\Y]f'dUfh]W]dUh]cb`]b'h\Y``YWh fY"$

5 hith Y 'YbX'cZth Y 'hYfa žighi XYbhg'a i ghiWkccgY 'th Y]f'VYghiZji Y ik YY_gfik cfth 'cZYbhf]Yg'fa Ul]a i a 'hchU'ik cfX

&F][cif.'UhhYbh]cb'hc'XYhU]'ž'dig\]b['dc]bhg'Ug'ZJf'Ug'dcgg]V'Yž'XfUk]b['cih]ad']WUh]cbg"

'=\bXYdYbXYbhfYgYUfW.'8fUk]b['cb']XYUg'Zfca'VYmcbX'h\Y'WcifgY'fYUXYf"

('8Yj Y`cda YbhcZWYUf'Uf[ia Ybhg'k]h\'U'ghfcb['gYbgY'cZghfi WhifY"



		a YUb3'7cb7fcbh]b['h\Y' <mxfu""; f]z[h\'fyj]yk"<br="">9X]h]cb'')"</mxfu"";>
 K_YY_''%)	Hi hcf]U` @YWhi fY	5fh'UbX' < YU`]b[
		F YUX]b[.
		<pre>< c``UbXz'5``]gcb"'&\$%) "``BUh fU``X]gUghYfz'ffUi a U UbX'UWfj]ga `]b'h\Y'UfhcZHU_Ua]bY'HUXUgi "'H\Y 5g]U!DUWZW>ci fbU`.'>UdUb': cW g'% 'ff*Ł'%'*"</pre>
	Hi lcf]U`	0
K YY_'%\$.'&&'5df]`'!'&, 5df]`	@YWhifY	BchY'h\Un'h\Y': f]XUm']b'kYY_'-']g'; ccX': f]XUm!'UbX QYbWYQh\Y'WUggYg'Zfca 'h\]g'XUm'k]``VY'HU_]b[d`UWY'cb'KYXbYgXUmcZKYY_'%"'fGcGGŁ"
		K]bbYfž'@Ub[Xcb"'%, \$"'"8c'5ffjZJWfg' <ujy Dc`]fjWg3"'8UYXU'ig"'%\$%"'GdYWJU''⊕ggiY.'AcXYfb</ujy
		HYWW.bc`c[m'DfcV`Y ‰

* + '%| * "

FYgci fWyg

DfYgWf]VYX F Ygci fWyg

 $D^YUgY^gYY^h Y^Wci fgY^k YVg]hY^Zcf^]bZcfa Uh]cb^UVci hfYUX]b[g^UfY^Uj U]^UV^Y^cb^h Y^Wci fgY k YVg]hY^U$

.

FYWca a YbXYX FYgci fWyg

•

H\Y'I b]j Yfg]lmicZBYk 'Gci h\ 'K U'Yg']g'Wca a]lthYX'rc'UXj UbW]b['h\Y'a YbtU``\YU't\ 'UbX'k Y``!VY]b['cZ]ltg ghi XYbhg"'-₹mci 'cf'gca YcbY'mci '_bck ']g'ZYY`]b['cj Yfk \Y`a YXž'XYdfYggYXž'UbX#cf']b'bYYX'cZgi ddcfhž gYfj]WYg'UfY'Uj U]`UV'Y''\Ci 'WUb``YUfb'a cfY'UVci hit\Y'VfcUX'fUb[Y'cZWcbZ]XYbt]U``a YbtU``\YU't\ 'gYfj]\W'g Uj U]`UV'Y'cb'WJa di g"'-₺Zcfa Ut]cb']g'Uj U]`UV'Y'Uh\\thd.#k k k "\YU't\gYfj]\W'g"i bgk "YXi "Ui #\/cbt\Ybt\#a YbtU'!\\YU't\

.

7 ci fgY '9j U'i Uhcb 'UbX '8 Yj Y 'cda Ybh

@f]YZmci h]bY`\ck 'ghi XYbhZYYXVUW_flVch\ 'Zcfa U``UbX`]bZcfa U`Ecb'h\Y`Wci fgY`k]```VY`[Uh\YfYXž\ck ']h k]```VY`UbU`mgYX`UbX`\ck ']hk]```VY`UWYX`i dcb'hc']a dfcj Y'h\Y'ghi XYbh`YUfb]b['YI dYf]YbWY": cf'YI Ua d`Yž mci 'a][\hX]gW gg`k \Uhk Ug`]XYbh]Z]YX`]b`dUghZYYXVUW_'UbX`\ck 'h\]g`Wci fgY`k Ug`W\Ub[YX`hc`UXXfYgg`h\Y]qqi Y"Q

.

Gh XYbhYj U`i Uhj Y`ZYYXVUW_`]g`[Uh\YfYX`dYf]cX]WU``mi g]b[ž'Ua cb[`ch\Yf`a YUbgž'I BGK fij a m9I dYf]YbWl dfcWlgg": +bZcfa U``ZYYXVUW_`UbX`WUgg![YbYfUhYX`ZYYXVUW_`UfY`U`gc`]a dcfhUbh''Gh XYbhZYYXVUW_`]g`hU_Yb gYf]ci g`mž'UbX`]a dfcj Ya Ybhg`k]``VY`a UXY`hc`h\Y`Wci fgY`VUgYX`]b`dUfhcb`gi W\`ZYYXVUW_"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-',) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k]``

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'\lefi\Wb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UVW]XYbhU`mWcdm]b['Zfca `U`gci f\W' k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidgdu[]Uf]ga]bhch\YZc``ck]b[WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUX Ya] W = D Zcfa Uh] cb ·

: cf`YggYbhJU``gh XYbh]bZcfa UhJcb`fY`UhJb['hc.

fYei YghgʻZcfʻYl hYbg]cb/ `UhYʻgi Va]gg]cbgʻ[i]XY`]bYg/ fYj]Yk ʻcZa Uf_g/ I BGK ʻ< YUʻh\ 'UbXʻGUZYhmdc`]WJYg/