

ARTS3750

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HYfa 'C bY''##'&\$%

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5fY`mci `]bhYfYghYX`]b`i bXYfghUbX]b[`UbX`hUW_`]b[`]a dcfhUbhYbj]fcba YbhXYj Y`cda YbhWkU``Yb[Yg`]b`h\Y &%ghW/bhi fm3·8 c mci 'h\]b_'U'Wca a i b]lm!VUgYX'UddfcUW\ hc XYj Y`cda YbhUbX'Ybj]fcba Ybha UbU[Ya Ybh XY`]j Yfg'a cfY'gi ghU]bUV`Y`UbX`Yei]hUV`Y`ci hWca Yg3'H\Y'Wci fgY`]bhYffc[UhYg'h\YgY`UbX'a Ubma cfY Wf/h/WU``ei Ygh/cbg'gi ffci bX/b['th\Y'Wca a i b]mtVUgYX'UddfcUVX 'tc'XYj Y`cda YbhUbX'Ybj]fcba YbhU` a UbU[Ya Ybh''H\Y'Wci fgY'Yl dcgYg'mci 'hc'U'gcd\]gh]WUhYX'i bXYfghUbX]b['cZh\Y'W ffYbhXYj Y'cda YbhUbX Ybj]fcba Ybh]ggi Yg UZZYW]b['h\Y k Y``!VY]b['cZ`cWU``Wca a i b]h]Yg]b 5i glfU`]U UbX XYj Y`cd]b['Wci blf]Yg" GdYWZWU``mžh\Y`WcifqY`dfcj ZYq'mci k]h\'h\Y`bYWYqqUfmh\YcfYhWU``VUW_[fcibX'Uq'kY```Uq'U'qYhcZWUqY qh X]Yq'bc'i bXYfqHbX'UbX'HJW `Y'fYU'!k cf'X'Ybj]fcba YbHJ`'UbX'XYj Y`cda YbhVXU``Yb[Yq"'H\fci [\'h\]q Wci fgYzmci k]``\Uj Y'Ub'cddcffi b]mm'c']bhY[fUhY'gcWJU'zYWcbca]WUbX'Ybj]fcba YbhU`'UgdYWg'cZ XYj Y`cda YbhUbX'Ybj]fcba YbhU``]gqi Ygž'UbX'h\Y'WbWYdhq'UfY'X]gW ggYX'h\fci [\'h\Y'`Ybg'cZgi ghU]bUV`Y XY; Y`cda YbhZfca 'U'gcWYhU`dYfgdYWlj Y"'=XYUg'UbX]ggi Yg'UfY'dfYgYbhYX'Ug'WcbhYghYX'YbhJhYg'VYWUi gY cZh\Y'a i `h|d`]W|mcZX]j YfqYžUbX`Uh'h|a YqžX]Ua Yhf]WU``mcddcqYX`qhU_Y\c`XYf`]bhYfYqhq"'H\Y'UddfcUWX`cZ h\Y`Wci fgYžh\YfYZcfYžYgW\Yk g`h\Y`"cbY`f][\hik Um" UddfcUW\ hc`XYj Y`cda YbhUbX`Ybj]fcba Ybh a UbU[Ya Ybh]b ZJj ci f`cZUb UddfcUVX VUgYX cb i bXYfglUbX]b['h\Y`X]j YfgY UbX Wca d`Yl 'gcWJYhU` WcbhYl hg`]b`k \]W\`XYj Y`cda YbhUbX`Ybj]fcba YbhdfcZYgg]cbU`g`cdYfUhY"'8]ZZYfYbhUddfcUW\Yg`hc a Ubu[]b['h\Y'Ybj]fcba YbhUbX'XYj Y'cda Ybhdfc'YWg'UbX'dfc[fUa g'UfY']`i glfUhYX'h\fci [\ci hVma YUbg cZfY`Yj UbhWUgY`gh X]Yg"'Mci k]``VY`UV`Y`hc`Yl d`cfY`k\Yh\Yf'UbX`\ck X]ZZYfYbhdc`]WJYg'UbX`dfUWJWYg'UfY k cf_|b['fbf'bchk cf_|b[Łž'UbX'\ck 'dc`]WmfYgdcbgYg'WJb'VY'a UXY'a cfY'Yei]lUV'Y'UbX'gi glU]bUV'Y'Uhlh\Y Wcaaib]mi`YjY`"

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- &" 7 f]h]WU``mUbU`mgY`gca Y`Wca a i b]lm!VUgYX`XYj Y`cda Ybh'UbX`Ybj]fcba Ybh'U``a UbU[Ya Ybh'dc`]WjYg i g]b[`WUgY`ghi X]Ygž'UbX`]XYbh]ZmdchYbh]U``cdh]cbg`hc`UXXfYgg`W\U``Yb[Yg`ZUW]b[`h\Y``cWU` Wca a i b]hYq
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H\Y`Wći fgY`]g`XYg][bYX`]b`U`k Umih\UhiYbWći fU[Yg`mci `hc`hU_Y`U'XYYd`UddfcUW\'hc``YUfb]b["'H\Y`Wći fgY WćbhYbhUbX`h\Y`UXcdh]cb`cZh\Y`hYUW\]b[`ghfUhY[]Yg`UfY`ghi XYbh\W`bhfYX"'H\Y``YUfb]b[`ci hWća Yg`k]```VY UW\]Yj YX`h\fci [\`U'VU`UbWYX`Wća V]bUh]cb`cZhYUW\]b[`UbX``YUfb]b[`UWh]j]h]Yg`gi W\`Ug`gYa]bUfgžfYUX]b[ž WUgg`X]gW gg]cbž[fci d`k cf_ž'cfU``dfYgYbhUh]cbž'UbX`cb`]bY`UWh]j]h]Yg"`8i f]b[`h\Y`gYa YghYfž'mci `UfY YbWći fU[YX`rc`]bhYfUWhk]h\`mci f'dYYfg`UbX`h\Y``YWh fYf`h\fci [\`j Uf]ci g`a YUbgž'gi W\`Ug`h\Y`Wći fgY'g A ccX`Y`g]hY`UbX`Ya U]`g"`GYa]bUfg`UfY`WćbWY]j YX`cZUg`Ub`]bhYfUWhj Y``YUfb]b[`UWhjj]hmk\YfY`mci `WUb`Ug_

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 $H\YfY']g'BC'bYYX'hc'ZcW'g'cb'cbY'dUfr]W'`Uf'YI Ua d'Y'ffh]g']g'Zcf'rh Y'bYI h'Ugg][ba Ybh'''< ck Yj Yfž'mci Wci 'X'XfUk 'cb'U'fUb[Y'cZgYWcfU`YI Ua d'Ygž'UbU`mg]g'UbX']bg][\hg'hc'\][\`][\hcddcfhi b]r]Yg'UbX W\U``Yb[Yg'cZYei]hm]b'fY'Yj UbhXYW'bhfU`]gYX'BFA 'dc']WfYg'UbX'dfUWf]Wfg''$

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8 YHJ]`g. `Gh XYbhg`UfY`fYei]fYX`hc`k f]hY`U`WUgY`gh XmfYdcfhcZ%) \$\$`k cfXg"`±i]bj c`j Yg`h\Y`UbU`mg]g`cZU WUgY`gh XmfY`UhYX`hc`UbmcbY`cZh\Y`h\Ya Yg. `Wca a i b]hmdUfh]W]dUh]cbž`XYVW`bhfU`]gUh]cbž`Yei]hmž`ZccX gYW`f]hmž`X]gUghYfg`UbX`i fVUb]gUh]cbž`]b`U`Wci bhfmfcf`U`fY[]cbŁ"`5 ggYgga YbhWf]hYf]U`UbX`gHUbXUfXg`UfY WYUf`mci h]bYX`]b`h\Y`Wci fgY`ci h]bY`UbX`X]gW ggYX`k]h\`h\Y`gh XYbhg`Uhh\Y`VY[]bb]b[`cZh\Y`gYa YghYf" 7 UgY`gh XmfYdcfhk]``VY`a Uf_YX`UbX`ZYYXVUW_dfcj]XYX`cb`]bY"

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XYj Y`cda YbhUbX`Ybj]fcba YbhU``a UbU[Ya Ybh]ggi Y`k]h\]b`U`[Yc[fUd\]WD``UbX`dc`]WmWcbhYl hcZU
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- &" 7`]a UhY`W\Ub[Y`UXUdhUh]cb`UbX`gcW]U``^ gh]WY.`h\Y`WUgY`ghi XmcZW]a UhY`W\Ub[Y`UXUdhUh]cb`dc`]Wh]b`U`XYjY`cd]b[`Wci bhfmfcf`]b`5i ghfU`]UŁ

Mci 'UfY'fYei]fYX'hc'Vf]YZmX]gW gg'h\Y'fY'Yj UbhWcbWdhi U`']XYUg'VYZcfY'mci 'ZcW g'cb'h\Y'gdYWJZ]Wdc']Wn UbX'dfUWJWD`'UgdYWg'cZh\Y'WUgY"'Mci 'Zcfa i `UhY'U'gYhcZU]a g#cV'YWJj Yg'UbX'glfi Wi fY'h\Y'fYdcfh UWWcfX]b['hc'h\Y']XYUg'mci 'k]``'dfYgYbh''Mci 'a Um`]_Y'hc'Yl dUbX'fbchhc'fYdYUh':h\Y']XYUg'UbX'Uf[i a Ybhg dfYgYbh'X']b'h\Y'5gg][ba Ybh%YggUm''Mci 'UfY'UXj]gYX'hc'Zc``ck 'h\Y'Zc``ck]b['ghYdg']b'd'Ubb]b['mci f dc`]Wh#d'Ubb]b['UbU'mg]g'fYdcfhik]h\'h\Y'Zc``ck]b['gi [[YghYX'ci h]bY.'

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&"'H\Y'[YbYfU``WcbhYl hcZh\Y'fY[]cb'UbX'h\Y'h\Ya Y'cZmci f`W\c]\W''!'dfcj]XY'[YbYfU``WcbhYl hi U`]bZcfa Uh]cbž'hfYbXgž']ggi Yg'fY`UhYX'hc'h\Y'h\Ya Y'UbX'fY[]cbg'UbX'h\Y'bUhi fY'cZgi ghU]bUV`Y'XYj Y`cda Ybh W\U``Yb[Y'fr &\$\$`k cfXgŁ

]bHYfZUWYž]bWi X]b['Ub'UbU'mg]g'cZ'W\U``Yb[Yg'UbX'cddcffi b]h]Yg'WfYUhYX'Vmh\Y'dc`]WfiZcf'ZU]f'UbX Yei]hUV'Y'XYj Y`cda Ybhici hWca Yg'ff' \$\$'k cfXgŁ"

) "'**7cbWi g]cb**'!']XYbhJZmh\Y'a U]b'dc]bhg'UVci h\ck 'h\Y'gWJYbWY!dc`]Wm']bhYfZUWY'\Ug'cVWV ffYX']b'h\Y h\Ya Y'UbX'h\Y'WUgY'h\Uh'mci '\Uj Y'W\cgYbž'UbX'k\Uh'`Yggcbg'WUb'VY'`YUfbh'Zfca 'h\Y'WUgY#\Ya Y'ff %\$\$ k cfXgŁ

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8 YHU]`g. `Ghi XYbhg`k]```VY`fYei]fYX`hc`k cf_`]b`U`[fcid'h\fci[\cihh\Y`gYa YghYf`UbX`XY`]j Yf`U`[fcid dfYgYbhUh]cb`]b`h\Y`WUgg"`5ggYgga Ybh`Wf]hYf]U`UbX`ghUbXUfXg`UfY`WYUf`mcih]bYX`]b`h\Y`WcifgY`cih]bYUbX`X]gWiggYX`k]h\`h\Y`ghi XYbhg`Uhh\Y`VY[]bb]b[`cZh\Y`gYa YghYf"`A Uf_`UbX`ZYYXVUW_`cb`dfYgYbhUh]cb k]``VY`dfcj]XYX`cb`UbX`UZhYf`h\Y`dfYgYbhUh]cb"`

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H\]q UqqYqqa Ybh'HUq_'\Uq'h\fYY'Y`Ya Ybhq.'

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5 hhyb XUb WY F Yei] fYa Ybhg

Ghi XYbhgʻufY Yl dYWYX hcʻVYʻfY[i `ufʻubX`di bWi uʻʻ]bʻuhYbXubWʻuhu\``WuggYgʻ]bʻh\YʻWci fgYgʻ]bʻk\]W\\h\YmufY`Ybfc``YX"'5```Udd`]WuhjcbgʻZcfYl Ya dhjcbʻZca `UhhYbXubWʻuhWuggYgʻcZubm_]bX`a i ghVY`a uXY`]b k f]hjb['hcʻh\Yʻ7 ci fgYʻ5i h\cf]mmʻ5ʻghi XYbhk\cʻUhYbXgʻ`Yggʻh\UbʻY][\mdYf`WYbhcZh\Y`WuggYgʻk]h\]bʻU Wci fgY`a UmVYʻfYZ gYX`UʻZjbU``UggYgga Ybh'`Mci `UfY`fYgdcbg]V`YʻZcf`_YYd]b['hfuW_czmci fʻUhhYbXubWY`UbX WcbhUwhjb['mci fʻWci fgY`Wcbj Ybcf']a a YX]UhY`m]Zmci 'UfY`WcbWYfbYX`UVci hmci fʻUhhYbXUbWY`fYWcfX`UbX']hg]a dUWhcb'mci fʻUV]`]mmhc`Wca d`YhY`mci fʻWci fgY`gi WWyggZ``m'

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9 dagb e ? aaV WfSe] , 'gi Va]gg]cb cZU'' \$\$ k cfXg gi a a Ufmcb A ccX Y 'g X]gW gg]cb Zcfi a 'Vm %") - da 'cZ&* '5df]"": U] i fY 'hc gi Va]hih]g gi a a Ufm VmXi Y XUhY k]``fYWY]j Y bc a Uf_ Zcf h Y [fci d fci h cZ) ı Ł"

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i bXYf[fUXi UhY``Yj Y`žghi XYbhg`UfY`YI dYWYX`hc`fYUX`U`k]XY`fUb[Y`cZUfr]WYg`h\UhiUfY`fY`UhYX`hc`h\Y
hcd]Wg"``: cf`WUgg`X]gW`gg]cb`UbX`UggYgga Ybhdi fdcgYgž']hk]```VY`Uggi a YX`h\Uhimci `\Uj Y`Wf]r]WU``mfYUX`Uh
U`a]b]a i a `h\Y`dfYgWf]VYX`UbX`gca Y`gi dd`Ya YbhUfmfYUX]b[g"`D`YUgY`fYUX`h\YgY`Ufr]WYg`VYZcfY

? UhYgžF "žDUff]gžH" 'UbX @Y]gYfck]mž5" 'f&\$\$) ŁžK \Uh']g 'gi ghU]bUV`Y 'XYj Y`cda Ybhž7` h[da` _ W fž'j c`" (+ž bc" ' ž'dd" -! &%"

A YVfUhi ž'A "'f\% - , \text{\figure} Gi ghU]\text{\text{bUV}}]\text{\text{m'UbX'WcbW'dhi U'}} \text{\text{fy} [Yk \text{\final} 7 \ h[\text{da'} \ _ W f\s^{\chi_{\chi}} \ b\text{SUf 3} \text{eeWee} \ W f\text{DWh[W} \text{\text{\text{\text{Z'}}} \ \text{\text{Z'}} \ \text{\text{dd''}(-' '-') &\s\"} \\

6Yf_YžD"'f&\$\$&\zerog'Gi ghU]bUV`Y`8Yj Y`cda YbhC ZYf'U`BYk '8]fYWf]cb'Zcf'D`Ubb]b[3'7\U``Yb[Yg'Zcf'h\Y Hk Ybml:]fgh7Ybh fmz'<agd S^aXB'S``[`Y>[fW&SfgdWzj c`"'%+z'bc"'%z'dd"'&%' *"'

6 [eUgee[a],

 $8]YgYbXcfZ^cA "`UbX`< Ua]'hcbz^c7"'f%-+\z'H\Y`9Wc`c[]WU``mGi ghU]bUV`Y`8Yj Y`cda YbhDfcWfgg`]b`5i ghfU`]Uz = b`8]YgYbXcfZ^cA "`UbX`< Ua]'hcbz^c7"'flYXg"\z': g_S` 7Ua'aYkl: g_S` 7Ua`a_kz^5``Yb`UbX`I bk]bz^Gh' @YcbUfXgz^dd"`&,)!'$%"$

GmXbYmK UhYffl zvzz9bj]fcba YbhD`Ub`&\$\!\+.`GmXbYmK UhYfgcVYWhj YgzhUf[Yhg'UbX'UWhjcbghca UbU[Y'UbX']a dfcj Y'Ybj]fcba YbhU`dYfzfa UbW\"

* 'A UfW. ''''' HF 5; 98MC: 'H<9'7CA A CBG

EW_[`Sd

< UfX]bž; "'f%- *, ŁžH\Y'HfU[YXmcZh\Y'7 ca a cbgžEU[W'UW' j c`"'%* &ž'dd"''&(' !\&(, "

G\fYgh\Už?"'?"'/ `C \Už<"'f&\$%+\Z'H\YcfYh\WJ``UXj Ub\Wg']b`Wcaaib]m\VUgYX`bUhifU``fYgcif\WaaubU[YaYbh`Cghfca'UbX`VYmcbXž±b`G\]j U_ch]ž; "ž`DfUX\Ubž'l "ž`UbX`<Y`a]ž<*\Z'&\$%+\Z`D\WW\[`Yab\Cghfca'UbX`VYmcbXž±b`G\]j U_ch]ž; "ž`DfUX\Ubž'l "ž`UbX`<Y`a]ž<*\Z'&\$%+\Z`D\WW\[`Yab\Cghfca'\B\S\S_[UeaX@Sfgc\S^D\WeagdU\We?S`SY\W_Wf[`3e[S"`Jc`iaY`\Z'9`gYj]\Yfz`l? Z'dd"%`'-`(\$"

6 [eUgee[a],

5bh]bcf]ž7"'UbX'6fUmž8"'6"'f&\$\$) Łž7ca a i b]lmiZcfYghYbhYfdf]gYg'Ug'YbhfYdfYbYi f]U``Z]fa g. '9Wcbca]WUbX']bgh]hi h]cbU``dYfgdYWf]j Yg'Zfca 'A Yl]Wcžl adV 6WhWab_ W fl j c'"' ' ž'bc"'- ž'dd"'%) &-!%) (' "

.

% 'A UfWX '********8979BHF5@€5H€B

.

EW_[Sd

 $F \]Vchz^->"7"z^-5"^-5 \ [fuk u^-ubx^-5"^-A "^@ufgcb^-f&$$^+\ Lz^F \ YWbhfu^]n]b[^-k \]^Y \ XYWbhfu^]n]b[^-.'<ck buh]cbu^-[^-cj \ Yfba \ Ybhg^-fYUddfcdf]UhY^-ZcfYghfYgci fWgz^-I adV 6 WhWab_ W fl^-j c^-"'' (z^-bc''-wz^-dd'' % *(!%, *")$

 $G\fYgh\U\check{z}?""?"'f\&\$\&\&\check{z}fflck\UfXg'9bj\]fcba\YbhU`'9ei\]hm]b``B\YdU`YgY'7ca\ a\ i\ b]hm:\ cfYghfmfiz=b`:\ fYXYf]$\underline{W}_{.}8" ; \ cfXcb'/~; \ fY[\ cfm?"':\ fYY`UbX`ffYX"\Ez';`fWd\Sf[a`S^7`h[da`_W fS^<gef[UW\5a_bWf[`Y\5'S[_eS`V]Wb]bWb]hWbZ=@A\`Di\V`]$WbhJcbgz'<\YfhZcfXg\]fYž'I\? z`dd"'-+-\%%"'$

..

6 [eUgee[a],

; U``Ya cfYž'7 "ž'A "'8]'; fY[cf]cž'A "'A cY`]cbc`UbX`A "'6fcW_\Ui g'f&\$%) Łž'HfUbgUWf]cb`Wcghgž'dck Yfž'UbX a i `h]!`Yj Y``ZcfYgh'[cj YfbUbW']b'=bXcbYg]Už'7 Ua 'aY[US^7 Ua` a_ [Uežj c`"'\%(ž'dd"'\%*, !\%+- "

GU\]XYžA "žG":Gi dfUha Ubž5":A Ufmi X]žM!!G":?]a 'UbX'@";]YggYb'f&\$%*\Ez8YWYblfU`]gUh]cb'dc`]WmUg fYWYblfU`]gUh]cb'glfUhY[m'ZcfYgha UbU[Ya Ybhi b]hg'UbX'Wca a i b]hm'ZcfYglfmi]b'\DXcbYg]Už;`fWd Sf[a`S^8adWefdk DWh]W ž'j c`"% ž'bc"'%Z'dd"'+,!-)"

.

&\$^A UfW\\....7 C A A I B ≠IM\.D5F H=7 ±D5H±C B

.

EW_[Sd

 $5\,fbghY]b\check{z}G'''f\%-*- \check{z}5\,``UXXYf'`cZW]h]nYb'`dUfh]W]dUh]cb\check{z}'<agd`S^aX3_Wd[US`;`ef[fgfWaXB'S``[`Y\check{z}j'c`'''])\check{z}bc''(\check{z}'dd'''&\%'!\&\&("))$

5 [fUk U`ž5"'UbX';]Vgcbž7"'f1%--- Łž9bWkUbha Ybh'UbX'8]gYbWkUbha Ybh'H\Y'F c`Y'cZ7 ca a i b]hm']b B Uh' fU`F Yqci fWY'7 cbqYfj Uh'cbžl adV 6 WhWab_ W fl j c`"&+ž'bc"(ždd"'* &-!*(-"

G\fYgh\Už?""?""UbX`A WA Ubi gžD"`f&\$\$, ŁžH\Y`dc`]h]Wg`cZWca a i b]hmdUfh]WjdUh]cb`]b`bUhi fU``fYgci fWY a UbU[Ya Ybhž`Yggcbg`Zfca `Wca a i b]hmZcfYghfm]b`BYdU`ž3gefc\${\sigma} \%` 8adWefckž'j c`"+\%zbc"&\zdd"\%')!\%(*"

8 fYgg`Yfž'K "ž'6i gWXYfž'6"ž'GWXccbž'A "ž'6fcW_]b[hcbž'8"ž'< UnYgž'H"ž'? i ``ž'7 "ž'A W7 Uft\mž'>"ž'UbX'G\fYgt\Už ? "'? "'f&\$%\$\Lz': fca '<cdY'hc'7 f]g]g`UbX'6UW_'5[U]b3'5'7 f]h]\W`; YbYU`c[mcZt\Y'; `cVU`'7 6BFA B UffUh]j Yž'7`h[da` _ W fS^5a`eWthSf[a` ž'j c`"' +ž'bc"%ž'dd"') -%) " .

6 [eUgee[a`,

6YbbYhřB"'>"'UbX'D"'8YUfXYb'f&\$%(\text{\m`cWU`dYcd`Y'Xc'bchgi ddcfh'WcbgYfj Uhjcb.'Wcaaib]hm dYfWYdhjcbg'cZaUf]bY'dfchYWhYX'UfYU'`]j Y`]\ccX']adUWgž'[cj YfbUbWY'UbX'aUbU[YaYbh]bH\U]`UbXž'? Sq` WBa'{Ukžj c`ž ((ž'dd"'%\$+!'%*"

 $\label{eq:huaub} HUa\ Ub[\ \check{z}'G\check{z}'DUi\ XY`\check{z}'?\ ''G\fYgh\ U\check{z}'?\ ''?''f\&\$\%(\ \check{z}':\ Ya\]b]nUh]cb'cZU[\ f]W\ 'hi\ fY'UbX']hg']a\ d`]WUh]cbg'Zcf'ZccX gYW f]hm]b'BYdU'\check{z}'<agd\ S^aX8adWefS` V >[hW[ZaaV"'jc`''%\&\check{z}'bc''''\&\ddot{z}''dd'''\&\$-'' \&"$

; UfhUi `Už'< "`B "ž'5"`B]Y\cZUbX'@"`J]ggYf`f&\$%\$\Z: Ya]b]gUh]cb`cZ5[f]W`h fY`Ug`Ub`9ZZYWhcZAU`Y`Ciha][fUh]cb.`I bYIdYWhYX`CihWcaYg`Zfca`>\UdU'8]ghf]Wb`9UghYfb`BYdU`z';`fWd Sf[a`S^<agd S^aX \\#\f\M\Jg\J\J\S\fa\Ga\J\J\S\G\J\M\b\U\M\g\z']c`"`) z`bc" \&z`dd"`) *) '-') ++"`

: $fUbWc\check{z}>"`7"\check{z}\&\$\&"; ``cVU```UbX`[fUVV]b[``UbX`hfUYWcf]Yg`cZU[fUf]Ub`VXUb[Y. `5`dfY`]a \]bUfm UbU`mg]g\check{z}<agd` S^aXSYcSd[S` UZS` YWzdd"'&z'bc"' & z'dd"'' (!) - "$

%55df]````A 5@565F'<958@5B8'B'GM8B9M5F'<958@5B8'B'GM8B9M5F'<958@5B8

%+'5df]`.'''G@ AG'5B8'IF65B'DCJ9FHM

.

EW [Sd

6i Hu`Už`B "ž'J UbF ccmYbž'A "`UbX`DUhY`ž`F "`f&\$%\$\{Z`=a dfcj YX`\YU`h\`ci hWca Yg`]b`i fVUb`g`i a g`h\fci [\]bZfUghfi Wi fY`i d[fUX]b[ž`E aU[S^E U[W UWS` V ? WW[U[` Wz'j c`"+%z'dd"-')!-(\$"'

A UI k Y``ž5"/ `G\fYgh\Už?"'?"'f&\$%) Łž'A]WfcZ]bUbWY `UbX'dcj Yfhm'U``Yj]Uh]cb']b'i fVUb` \pm x]U. `@Yggcbg'Zfca Zci f'a]WfcZ]bUbWY `cf[Ub]nUh]cbg'ž' \pm `G\fYgh\UžWf S'Zf&\$%) Łž'; `U'ge[hWGdTS` [I Sf[a`, DWfZ[`] [`Y BSdf]U[bSf[a` I B'S`` [`Y S` V Ba']UkI F ci hYX[Yž'@cbXcb'UbX'BYk 'Mcf_ž'dd'''\&\$-\% *"

; cj Yfba YbhcZ±x]U'fl žVžfl G`i a 'XYj Y`cda Ybhdc`]Wm&\$%\$ž@k k k "]bX]UYbj]fcba YbhdcfhU`"cf["]b#Ydcfhg! XcW a Ybhg#g`i a !XYj Y`cda Ybhdc`]Whl&\$%\$Q

.

6 [eUgee[a`.'

7 \c]žB"'f&\$%* \tžA Y\fc'A Ub]`U'\h\fci [\'h\Y'[Yb\f]Z]\W\f]cb``Ybg.'8]gdUf]\f\Yg']b'i f\VUb'd`\Ubb]\b['\UbX\X]gd`\U\Wa Yb\f]\g_g"'Gd\TS` E\fg\V\\text{\mathebasis} c`\"') '\zbc\"''\zbc\"'' \zeta\d\"') ++!) - &

&('5df]`.8 €5 GH9 F G

EW [Sd

G\fYgh\Už?""?"žYhU``f\$" #* \Z`8]gUghYf`^1 gh]\W`]b`BYdU`'g`YUfh\ei U_Y`fY\cj Yfmž'\bhYfbUh]cbU``>ci fbU``cZ 8]gUghYf`F]g_`FYXi \\f\]cb`fcb`]bY`Uj U]`UV`Y\!"

 $5 \text{ fmU} \tilde{z}$? "'F "'f&\$%($\tilde{z} = h \tilde{y} =$

- :]g\Yfz̃F "'f&\$\$%±z̃9I dYf]YbWgz̃7\U``Yb[Ygz̃UbX`DfcgdYWg`]b`7c``UVcfUhj Y`A UbU[Ya YbhcZDfchYWYX 5fYUg. '5b`+bhYfbUhjcbU``DYfgdYWf] Yz̃+b`6i W_ž@ci]gY`6"ž7\Uf`Yg`7"; Y]g`Yfz̃>c\b`GWXY`\Ug`UbX`9j U K c``YbVYf['fl/XgŁ'4[a^aY[US^6[hWbe[fk, 4S^S` U[` Y;` flwdWefe FZdagYZ 3VSbf[hW5a^STadSf[hW? S` SYW_ W fz̄7F7'DfYgg. '6cWU'F Uhcbz̃: `cf]XU"
- :]g\YfžF "'f&\$%\$\z'8Yj c`i h]cb'cf'DYfg]ghYbWY'cZGhUhY'7cbhfc`ž'\=b'FZWBa'{f[Ue aX6WUW fc&{| Sf[a`, @Sfgc&^DWeagdUW? S`SYW_Wf[`3e[S flYXg"K]hHUmUdU_ž'7"'UbX'J UbXYf[YYghz'D"\z'7\]Ub['A U]ž'A Y_cb['DfYgg"
- 9 S[ZSł Dž/\$" " %fi5fY A]``Ybb]i a '; cU`gʻcZDcj YfhmF YXi Whjcbʻl gYZi `3`A j XadV 6 WhWab_ W f E fgV[WežJ c`" ' %žBc%ždd"') '--', ("
- ; `Uggcb'>"`WfS'z'fl%-(tz';` fcaVgUf[a` fa'7` h[ca`_ W fS^;_ bSUf 3eeWee_ W f, Bd[`U[b'We S` V BcaUWWgdWetBcaUWeetBcaUWeetBcaUWS` V BcaebWUfez'I 7@DfYggz'@cbXcb"'fgY`YWYYX'W\UdhYfgt
- ; i bXYfž'A "'f&\$\$*\Z'Gi gHJ]bUV]`]\miD`Ubb]b['g'GUj]b['; fUW\'cf'FcUX'\tc'DYfX]\f]cb3'<agd S^aXB'S``['Y 7 VgUSf[a` S`V DW\eW\$dJZ\z'j c`"\&*\z'dd"\&\;!&&\"
- < Ufj YmžB"'f%--, Łž7`h[da`_WifS^;_bSUf3eeWee_Wif, BdaUWWgdWellBdSUf[UWIS`V BdaebWUfe [3gefdS/[StClZcfX'lb]]Yfg]miDfYggžAY`VcifbY"'fbYYWWYX'W.UdhYfgŁ
- ±bYgž'>"'UbX'6cc\Yfž'8"'f&\$\$(tžFYZfUa]b['DiV']WDUff]WdUh]cb.'GtfUhY[]Yg'Zcf'th\Y'&%gh7Ybhifmž'B'\$``[YFZWack BcSUf[UWz'jc'") z'bc"'(z'dd"'(%-('*"
- @UbYž'A "''f&\$\$) Łž'Di V`]WDUffjWjdUfjcb']b'D`Ubb]b[.'Ub']bhY``YWfi U``\]ghcfmž3gefc\${\S` 9 WaYc\$bZWdž'j c`"'' * ž bc"'' ž'dd"'&, '!&--"
- @UbYžA "'f&\$\$*\z'H\Y'fc`Y'cZd`Ubb]b[']b`UV\\]Yj]b[']bX][Ybci g``UbX'^\ gh]WY'UbX'Wca a i b]hmi[cU`gž>S` V GeWBa^{Ukž'j c`"\&' ž'dd"' ,) -' ("
- @ck žB"'UbX'; `YYgcbž6"'f%-, Łž9bj]fcba YbHJ`'>i gh]WY.'8]ghf]Vi h]b['9bj]fcba YbHJ`'Ei U`]hmž\oldow`ck žB" UbX'; `YYgcbž6"ž<gef[UW EaU[Wfk S` V @SfgdW 3` Wj b^adSf[a` aXba^ff[US^WJa^aYkžFci hYX[Yž@cbXcb UbX'BYk 'Mcf_ždd"'%\$&!% &"
- A UbbVYf[\check{z} A "'UbX'K]\`Vcf[\check{z} 9"'f&\$\$, \check{t} 7 ca a i b]\Whj Y'D`Ubb]b['-': f]YbX'cf': cY3'C VgHU\YYg'UbX C ddcfhi b]h]Yg'Zcf'=a d`Ya Ybh]b['Gi ghU]bUV`Y'8 Yj Y`cda Ybh@c\W\`mz`E gefS[ST\W6\Wh\Wab_\W fz'j c`''\%* \check{z} dd" ')-(' "'
- A W8 Yfa chrž A "'f&\$\$- Łž @c WUh]b['VYbY Z]hg. '8 YW]g]cb! gdUW gž fYgci fW 'UWW gg 'UbX 'Yei]hm]b 'I G 'Wca a i b]hm VUgYX 'ZcfYghfnž 9 Wa Xacg_ ł j c `" (\$ Z dd " & (-&) "
- A cff]gcbžH"'f&\$\$+ŁžA i `hjgWU`Uf`; cj YfbUbW'`UbX`F Y[]cbU``9bj]fcba YbhU``A UbU[Ya Ybh]b`5i ghfU`]Už` EbSUWS` V Ba`ffkžij c`"'%%zbc"'' ž'dd"'&&+-&(%"
- C glfca \ddot{z} '9" fl% \$£'9 a hWd [`Y fZW5 a _ _ a`e, FZW7 ha \dot{q} f[a` aX;`ef[fgf[a`e Xad5 a \dot{q} WUf[hW3 Uf[a`" 7 Ua Vf]X[Y" b]j Yfg]lmiDfYggž7 Ua Vf]X[Y"
- DUbX]hžF "'UbX'6Yj]'UWei Už9"'f&\$%\ZGCW]U``< YhYfc[YbY]hmUbX'7ca a i b]hm: cfYghfmDfcWfggYg.
 FYZYWIcbq'Zfca : cfYqhI gYfg'cZ8\UX]b['8]qhf]\WzBYdU`zE_ S^ZeUS^W8adWefdkzj c`"'\%\z'bc"\z'dd"'-+-\%""

F]WXUfXgcbžH"'f&\$\$) Łž9bj]fcba YbhU``UggYgga YbhUbX'd`Ubb]b['h\Ycfm'Zci f'g\cfhighcf]Yg'UVci hdck Yfža i `h]d`Y'fUh]cbU`]hmž'UbX'Yh\]Wgž'7` h[da` _ W fS^;_ bSUf 3eeWee_ W f D Wh[W ž'j c`"&) ž'dd"' (%-' *) "

FcV]bgž@"'UbX'8cjYfgžG"'f&\$\$+ŁžBFA 'FY[]cbg']b'5i ghfU`]U. 'h\Y''<UjYg''UbX'h\Y''<UjY'Bchg'ž' 9 WaYc\$bZ[US^D WeW\$dJZž'j c`"'() ž'bc"'' ž'dd"'&+' -&-\$"

FcV]bgcbž>"'f&\$\$(E'Gei Uf]b['h\Y'W]fWY3'Gca Y'h\ci [\hg'cb'Gi ghJ]bUV`Y'XYj Y`cda Ybhž'9Wc`c[]\WJ`9\\chica]\\dignormathci [\hg'cb'Gi ghJ]bUV`Y'XYj Y`cda Ybhž'9\\chica]\\dignormathci [\hg'cb'Gi ghJ]bUV`Y'XYj Y`cda Ybhž'9\\chica]\\dignormathci [\hg'cb'Gi ghJ]bUV`Y'XYj Y`cda Ybhž'9\\chica []\\WJ`

G\fYgh\Už?""UbX'8YYž>"'f&\$\$, ŁžD`Ubb]b['Zcf'gi ghU]bUV`Y'XYj Y`cda Ybh' ₺j Ygh][Uh]b['Ybj]fcba Ybh'XYj Y`cda Ybh'XYW]g]cb'a U_]b[']b'GmXbYm'g'A U`UVUf' < YUX`UbXž'6[S'âYgWe [`GdTS`B'S``[Y, Fai SdVe E gefS[STWDWY[a`e

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-',) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k]``

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'=hWUb`hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhU``mWcdm]b['Zfca `U`gci fW'k]h\ci hUW_bck `YX[Ya Ybh'

 $I BGK `[fci dg'd`U[]Uf]ga `]bhc'h\Y'Zc``ck]b[`WUhY[cf]Yg.$

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi V'YWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY