

ARTS2820

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8 YHU]`g. Mci 'k]``gi Va]hU'&) \$\$`k cfX`fYgYUfW.`YggUm'`E i Ygh]cbg`UfY`gYhVmh\Y`Wci fgY`Wcbj Ybcf"`Mci k]``fYWY]j Y`k f]hYb`ZYYXVUW_UbX`U'bi a Yf]WU`[fUXY`k]h\]b`hYb`k cf_]b[`XUmg`cZgi Va]gg]cb"`H\Y`ZYYXVUW_g\YYh#fi Vf]Wk]``VY`Uj U]`UV`Y`hc`mci 'Uhih\Y`ghUfhcZh\Y`Wci fgY`gc`h\Uhimci 'WUb`k cf_hck UfXg`gdYW]Z]YX ghUbXUfXg"`H\]g`]g`h\Y`Z]bU``UggYgga YbhZcf`h\Y`Wci fgY"`

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b"gYh]b" . "H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\ 'Hi fb]h]b"UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a]"Uf]hmfYdcfhg" \\ \end{tabular}$

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5XX]h]cbU`XYhU]`g.

D`YUgY`bchY`h\Uhih\Y`h\Y`kY][\h]b[`[]jYb`]b`h\Y`XgYgWf]dh]cb`UVcjY`cZ&\$i `]g`Ub`Yffcf"`\dig\ci`X`fYUX`%) i YUW\"K\mbch^ghUa YbX`h\Y`&\$i `hc`%) i žmci `Ug_3`9jYf`fYUX`? UZ_U3`

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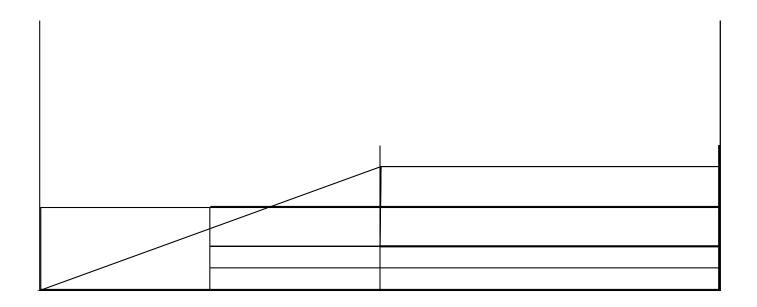
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H\]g^]g^U^bYk 'Wci fgY'UbX']lg^XYj Y`cda Ybhk]```VYbYZ]hZca 'mci f'Wcbglfi Wfj Y'ZYYXVUW_"'Ghi XYbh Yj U`i Uhj Y'ZYYXVUW_]g^[Uh\YfYX'dYf]cX]\WJ``mi g]b[ž'Ua cb['ch\Yf'a YUbgž'I BGK fg'7 ci fgY'UbX'HYUW\]b[9j U`i Uhjcb'UbX`\= dfcj Ya Ybhdfc\Wgg"'\=\Zcfa U``ZYYXVUW_'UbX'\WJgg![YbYfUhYX'ZYYXVUW_'UfY'U`gc]a dcfhUbh''Ghi XYbhZYYXVUW_]g'HU_Yb'gYf]ci g`mž'UbX`]a dfcj Ya Ybhg'k]```VY'a UXY'hc'h\Y'\cifgY'VUgYX']b dUfhcb'gi \W\ZYYXVUW_"'G][b]Z\WJbh\W\Ub[Yg'hc'h\Y'\cifgY'k]```VY'\cap a i b]\WJhYX'hc'gi VgYei Ybh\\chook\cfhg'cZ ghi XYbhg'hU_]b['h\Y'\cifgY''

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a i ghfYhU]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
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7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

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- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@&9'k]``\Y`d'mci 'a U_Y'h\Y'a cghcZmci f'gh X]Yg'UhI BGK "'\\hd.##qi V'YWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY