

POLS5100

=blYfbUhjcbU``F Y`Uhjcbg'=blYfbg\]d

HYfa "C bY" ## "&\$&\$

7ci fgY⁻Cj Yfj]Yk

`GhUZZ`7cbhUWh`8YhU]`g`

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWJhjcb	D\cbY
A JWKUY``DYHYfg	3 0	%/Ja!	F cca '% &ž @Yj Y`'%ž A cfj Yfb 6fck b	-',) % ''

GWXcc`7cbHJWh′±bZcfa Uh]cb

GWkcc``cZGcWJU``GWJYbWYg

Fcca '%) -

A cfj Yb 6fck b 7 &\$

Ya U]`. 'gcgg4 i bgk "YXi "Ui

d\cbY. '\$&'-',) '% \$+

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

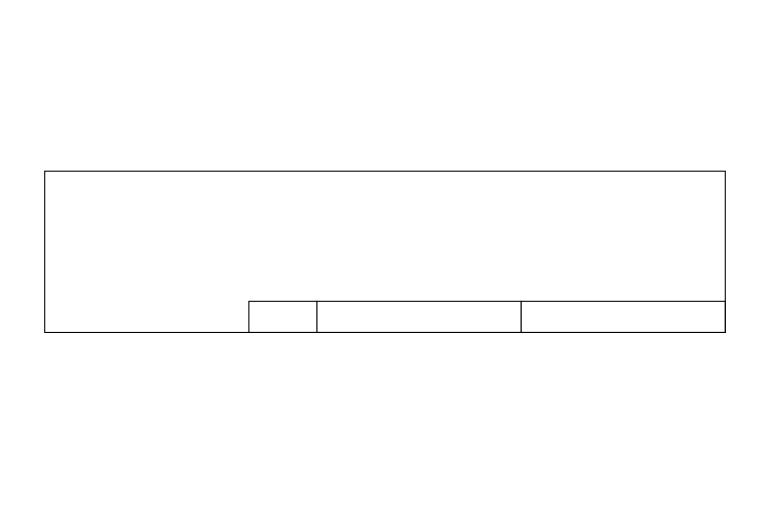
Gi a a UfmcZh\Y 7ci fgY

7 ci fgY @YUfb]b[Ci hWca Yg

- %" 8 Ya cbglfUhY 'Udd`]YX`]bXYdYbXYbhidfcV`Ya 'gc`j]b['UV]`]hm`]b`h\Y`k cf_d`UWY`h\fci [\`h\Y`gi WW\ggZ ` Wca d`Yh]cb`cZh\Y`]bhYfbq\]d`UbX`h\Y`fYgYUfW\`dfcYWffqLfYei]fYX`Vmmci f`U[YbWh'
- &" 7 ca a i b]\WhY \WYUf`m\UbX \YZZY\Wfj Y\m]b \Vch\ k f]\HYb \UbX \cfU \Zcfa "
- '"8Yd`cmih\YcfYh]WU``]bg][\hg`Zfca`h\Y`X]gW]d`]bY`hc`fYZYWhWf]h]WU``micb`dc`]Wha_U_]b[`UbX#cf cf[Ub]gUh]cbU``dfcWYggYg"
- ("5fh]W 'Uh' Ub Uk UfYbYgg cZWUfYYf cdh]cbg UbX dcgg]V]]h]Yg

HYUWX]b[GhfUhY[]Yg

#b 'fk]g'Wci fgY 'ghi XYbhg'k]``Yb[U[Y'k]hk 'Vchk 'fk Y'dfUWf]WU``UbX'fk YcfYh]WU``Wca dcbYbhg'cZhk Y]f' X]gW]d\]bY UbX'k]hk 'fk Y' Udd\]Wh]cb'cZhk YgY 'Wca dcbYbhg'ci hg]XY'i b]j Yfg]mh''Ghi XYbhg'UfY 'Yl dYWhYX'hc Yb[U[Y'k]hk hk Y]f'Yl dYf]YbWyg'hk fci [\kcf_g\cdg'Xi f]b['fk Y'gYa YghYf'dfcj]X]b['fk Y'cddcfhi b]mmZcf'fYZYWfj Y ZYXVUW_UbX'Yl WkUb[Y'cZYl dYf]YbWY VYłk YYb'dYYfg'''Ghi XYbhg'UfY 'U`gc'Yl dYWhYX'hc'dfYgYbh hk Y]f'Yl dYf]YbWyg']b'hk Y'Zcfa UhcZU'Wf]hJWD`'fYZYWfjcb'UbX'U'XYhU]'YX'dfc YWhfYdcfh''Ghi XYbhg'UfY YbWci fU[YX'hc'WcbhYl hi U`]gY'hk Y]f'X]gW]d`]bY!fY`UhYX'hk YcfYhJWD`'UbX'a Yhk cXc`c[]WD`'_bck `YX[Y']b'hk Y''fYU` k cfX''cZfYgYUfWk 'UbX'dc`]Wma U_]b['fk fci [\hk Y']bhYfbg\]d'UbX'hk fci [\hk Y']dfc YWhfYdcfh''H\Y k cf_g\cdg'dfcj]XY'Ub']bWi g]j Y'Zcfi a 'Zcf'hk Y'X]j YfgY'Yl dYf]YbWyg'cZghi XYbhg'hc 'VY'g\UfYX'UbX X]gW ggYXž'UbX'Zcf'hk YgY'hc 'VY'`]b_YX'VUW_'hc'hk Y'_Ymhk Ya Ygž'hk Ycf]Ygž'UbX'a Yhk cXg']b'hk Y'X]gW]d`]bY'cZ di V`]Wdc`]WmUbX#cf']bhYfbUhjcbU`'fY`Uhjcbg'' Dfc YWhfYdcfhg'k]``'fYZYWhhk Y'fUb[Y'cZWcbhYl hg'UbX'a UhYf]U`hk UhX]ZZYfYbhghi XYbhg'Yl dYf]YbWYX'Xi f]b['fk Y]f']bhYfbg\]dg''



H\Y'U]a 'cZh\]g'Ugg][ba Ybh]g'hc'[i]XY'mci ']b'h\Y'Wc'`YWjcb'cZYj]XYbW'Zcfmci f'YggUm'Mci 'bYYX'hc dfcj]XY'h\Y'UWUXYa]WwccfX]bUhcfik]h\'U'fYdcfhigi a a UfmcZh\Y'hUg_g'mci '\Uj Y'VYYb'gYh]b'h\Y'WcbhYl hcZh\Y'\cghcf[Ub]nUhjcb'g'k cf_"'H\Y'fYdcfhigi a a Ufmg\ci `X']bX]WUhY'fUŁh\Y'hUg_fgŁ'Ugg][bYX'hc'mci /flVŁ'h\Y cj YfU``'dc`]WmZfUa Yk cf_'cZh\Y'cf[Ub]nUhjcb/fWŁ'Ub'Yj U'i Uhjcb'cZmci f'dfc[fYgg'hc'XUhY/UbX'fXŁ'Ub UggYgga YbhcZh\Y'k cf_'fYei]fYX'hc'Wa d'YhY'mci f']bhYfbg\]d"'H\]g'Ugg][ba Ybh]g'Wcbghfi WhYx'gc'h\Uh gh XYbhg'UfY'[i]XYX']b'gYhhjb['h\Y]f'ck b'fYdcfhei Yghjcb'UbX'h\YcfYhWD`ZfUa Yk cf_'k\]W\'XYa cbghfUhYg Ub'i bXYfghUbX]b['cZh\Y'fYei]fYa Ybhg'cZh\Y'Z]bU``hUg_'gYhZcf'h\]g'Wci fgY"'9b[U[]b[']b'h\]g'UggYgga Ybh g\ci 'X']bX]WUhY'U`Yj Y``cZYZZYWhj Y'Wca a i b]WWhJcb'cZh\Y']b_'VYhk YYb'h\Y'`]hYfUhi fY'gYhZcf'h\]g'Wci fgY UbX'h\Y'] cWUhjcbU`'d'UWYa YbhYl dYf]YbW''U`gc'dfcj]X]b['U'ghfi Whi fYX'd'Ub'Zcf'h\Y'Uf[i a YbhUhj Y'fYdcfh'

Hi fb]hjb'gYhjb[.'H\]g'Ugg][ba Ybh]g'gi Va]hYX'h\fci [\'Hi fb]hjb'UbX'ghi XYbhg'Xc'bchgYY'Hi fb]hjb %6 g]a]'Uf]hmfYdcfrg"

5ggYgga Ybh' . Cf[Ub]gUh]cbU` 9ggUm

GhUfh'XUhY.

8 YHJ]`g. "\$\$\$`k cfXg"`I dcb`Wca d`YhJcb`cZh\Y`]bhYfbg\]d`gh`XYbhg`UfY`fYei]fYX`hc`gi Va]h`Ub`YggUmi\Uh dfcj]XYg`U`Wf]hJWJ``Yj U`i UhJcb`cZh\Y`]bhYfbg\]d`YI dYf]YbWY`h\fci [\"U`ZcW g`cb`h\Y`\cghcf[Ub]nUhJcb"`H\]g]g`bcha YUbh'hc`VY`U`Wf]hJei Y`cZh\Y`\cghcf[Ub]nUhJcb`Vi h`U`XYa cbghfUhJcb`cZk \Uh\Ug`VYYb``YUfbh'UVci h h\Y`cf[Ub]nUhJcb`fUbX`k cf_]b[`]b`h\Uh'UfYU`cZdi V`]Wdc`]WhZh\fci [\`h\Y`]bhYfbg\]d`YI dYf]YbWY": YYXVUW_k]``VY`]b`h\Y`Zcfa `cZk f]hYb`Wca a Ybhg`Vmh\Y`Wci fgY`Wcbj Ybcf"H\]g`]g`h\Y`Z]bU``Ugg][ba YbhcZh\Y`Wci fgY

5XX]h]cbU`XYhU]`g.

H\]g`Ugg][ba Ybh`]g`Wcbghfi Wh/X`gc`h\Uhighi XYbhg`UfY`Yb[U[YX`]b`]bXYdYbXYbhUbX`fYZYWhj Y``YUfb]b["`5g U`ghUfh]b[`dc]bhmci `a Um`Wcbg]XYf`h\Y`Zc``ck]b[`ei Ygh]cbg`]b`hYfa g`cZghfi Wh f]b[`mci f`YggUm

: fca 'Ub'cf[Ub]gUh]cbU`dYfgdYWh]j Y.

ck\UhXcYgh\Y\cghcf[Ub]gUh]cbXc3

c'\ck']g'h\Y'\cghcf[Ub]gUh]cb'cf[Ub]gYX'hc'UV\\]YjY']hg'cV'YWf]jYg3

: fca UZ bWhcb]b[cZa UbU[Ya YbhdYfgdYWh] Y.

c'\ck'UfY'dfcV`Yag'hUW_YX'k]h\]b'h\Y'\cghcf[Ub]gUh]cb3

c k \mUfY 'h\YmdfcV'Ya g k]h\]b 'h\Y \cghcf[Ub]gUh]cb3

: fca 'U'_bck `YX[Y'UbX'dck Yf'dYfgdYWnjj Y.'

F Ygci fWyg

DfYgWf]VYX F Ygci fWyg

 $\label{thm:continuous} $$H\Y'I b]j Yfg]mi]g'UV'Y'hc'ZUW]`]hUhY'gi WWYggZ ``Udd`]WUbhg `]bhYfbg\]dg'k]h\'h\Y'fYgYUfW\!VUgYX UXj cWU\mB; C'>i V]`YY'5i ghfU`]U'Zfca 'U'WUa di g!VUgYX ``cWUh]cb"Gh XYbhg'a Um'U`gc]bXYdYbXYbhm`cWUhY'ch\Yf'\cghcf[Ub]nUh]cbg'UbX'cVhU]b'U'dYfa]gg]cb'Zfca 'h\Y'A \neq 'Wcbj Ybcf'hc \Uj Y']bhYfbg\]d'h\YfY"$

FYUX]b['UbX'fYgci fWfg

H\YfY`]g`bc`gYhfYUXYf`Zcf`h\]g`Wci fgY"`<ck Yj Yf`h\Y`Zc``ck]b[`]g`U``]ghcZfYWca a YbXYX`fYUX]b[g`UbX fYgci fW\g`h\Uhk]``\Y`d`mci `Xi f]b[`mci f`]bhYfbg\]d`UbX`k]h\`mci f`k f]hhYb`UggYgga Ybhg`UbX`cfU` dfYgYbhUh]cbg"`±ik]``VY`YI hfYa Y`mX]Z]W\`hhc`Wca d`YhY`mci f`UggYgga Ybhg`k]h\ci hWcbgi `h]b[`U`bi a VYf`cZ h\Y`fYUX]b[g`cb`h\]g``]gh''

H\YfY']g'U'`chicZYI WY``Ybhi`]hYfUhi fY'cb'h\Y'a U_]b['cZdi V']Wdc`]Whi]b'5i glfU']Už']b'ch\Yf'Wci blf]Ygž'UbX'Uh h\Y'['cVU```Yj Y`"'H\]g``]ghi]b'bchYI \Ui ghj Yž']b'ZJWhi]hVUfY`mgWfUhW\Yg'h\Y'gi fZJWY'"\Mci 'UfY'YbWci fU[YX'hci gY'h\YgY'gci fW'g'Ug'U'ghJh]b['dc]bhUbX'UXX'mci f'ck b'gci fWg'hc'h\]g``]ghh\fci [\ci hmci f']bhYfbg\]d" H\Y'Zc``ck]b['`]ghi]bWi XYg'fYWbhVcc_g'k f]hYb']b'h\]g'UfYU'Ug'k Y```Ug'gca Y'WUgg]WVcc_g'h\Uh\Uj Y XYZ]bYX'h\Y'UbU`mg]g'cZdi V`]Wdc`]Whi'H\Y'Vcc_g'mci 'W\ccgY'hc'fYUX'k]``XYdYbX'cb'h\Y'ZcW'g'cZh\Y U[YbWhk\\YfY'mci 'UfY'Xc]b['mci f']bhYfbg\]d'UbX'mci f'ck b']bhYfYghg"'=kk]``dfcVUV`mU`gc'XYdYbX'cb k\Yh\Yf'h\Y]f'cdYfUh]cbg'UfY'`cWJ'ž'bUhJcbU'ž'cf'['cVU'"A cghcZh\Y'Vcc_g`]ghYX'\YfY'UfY'YX]hYX'k cf_g'k]h\X]ZZYfYbhWJgY'ghi X]Yg']b'YUW\WUdhYf'k\]W\a U_Yg'h\Ya 'j Yfmi d'hc'XUhY'UbX'a YUbg'h\YmWcj Yf'U'j Ugh fUb[Y'cZdi V']Wdc`]Whi]ggi Yg"'F Ya Ya VYfž'h\Y'Vyghd`UW''hc'Z]bX'dc']WhiXcW a Ybhg']g'h\Y']bghJhi h]cbž XYdUfha Ybhz'cf'cf[Ub]gUh]cb'h\UhdfcXi WYX'h\Ya !'gc'h\Uhg\ci X'Uh\Umg'VY'mci f'Z]fghghcd"

+b 'mci f'Yl d`cfUh]cb 'cZh\Y'di V`]Wdc`]Wn`]hYfUhi fYžmci 'k]```Z]bX'h\Y``]VfUfmik YVg]hY'Ub']bj U`i UV`Y'fYgci fW"'`\hd.#k k k "`]VfUfm'i bgk "YXi "Ui

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

5 WUXYa | W < cbYqlmUbX D`U[]Uf]qa

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'\lefi\Wb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UVW]XYbhU`mWcdm]b['Zfca `U`gci f\W' k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b[WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfc YWlgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb.'k cf_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhiUbmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_ž'ghYU']b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d'Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[ˈgci fWYg'k\]WX`\Uj Y`bchVYYb`fYUXžk]h\ci hUW_bck`YX[]b[`h\Y`″gYWcbXUfm″gci fWY`Zfca 'k\]WX`_bck`YX[Y`cZh\Ya`\Ug'VYYb`cVhU]bYX"

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'Uh'l BGK "'\\hd.##gi V'YWfi |XYg"`]VfUfm'i bgk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUXYa]W±bZcfa Uhjcb

: cf'YggYbhJU`gh XYbh]bZcfa UhJcb'fY`UhJb['hc.

• fYei YghgʻZcfʻYl hYbg]cb/