

PPEC1001

=blfcXi Wjcb hc Dc`]hjWgžD\]`cgcd\m'UbX 9Wcbca]Wg

HYfa 'C bY''##'&\$&\$

7ci fgY[·]Cj Yfj]Yk

GHJZZ:7cbHJWh:8YHJ]`g`

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUh]cb	D\cbY
: Yb[g\]'K i		H\ifgXUmg &.\$\$!(.\$\$da 'UbX'Vm Uddc]bhaYbhg	F a '% ' ž A cfj Yb 6fck b 6`X["	-',)!'*%

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7ci fgY 8YHJ] g

7 fYX]hDc]bhg *

Gi a a UfmcZh\Y 7ci fgY

H\]g`Wci fgY`]bffcXi Wrg`f\Y`X]gW]d`]bYg`cZD\]`cgcd\mžDc`]f]Wg`UbX`9Wcbca]WgžUbX`a cfY`gdYW]Z]WU``mih\Y gmbYf[]Yg`UbX`hYbg]cbg`gi ffci bX]b[`f\Y]f`X]gh]bWf] Y`UddfcUW\Yg`fc`_bck]b[`f\Y`k cfX"'H\Y`a U'cf W\U``Yb[Yg`ZJW]b[`dc`]Wma U_Yfg`fcXUm—'Zfca ']bYei U`]mmfc`[Ycdc`]h]WU``]bghUV]`]mm—'UfY`UhcbW'dc`]h]WU`ZYWcbca]WUbX`d\]`cgcd\]WU`"'< ck 'f\YgY`W\U``Yb[Yg`UfY`i bXYfgfccX`UbX`UXXfYggYX`fYgficb`]XYUg`UbX a Yf\cXg`XYj Y`cdYX`VmZci bXUh]cbU``f\]b_Yfg`]b`D\]`cgcd\mžDc`]h]Wg`UbX`9Wcbca]WgžUbX`YgdYW]U``mVm f\cgY`k\cgy`f\\ci [\hYa VfUWX`f\YgY`f\\fyY`X]gW]d`]bYg"'\d`f\]g'Wci fgYžmci `k]``ghi Xmf\Y']XYUg`UbX a Yf\cXg`cZf\YgY`f\\fyY`X]gW]d`]bYgžUbX`f\YY`X]gW]d`]bYg"'\d`f\]bYg"'\d`f\]g'Wci fgYžmci `k]``ghi Xmf\Y']XYUg`UbX a Yf\cXg`cZf\YgY`f\\fyY`X]gW]d`]bYgžUbX`f\YY`Wf]h]WU``]bhYfUWf]cbg`VYfk YYb`f\Ya ž`]b`cfXYfž'Z]fghmžfc`]XYbf\Zmf\Y]f`WYbffU``fc`Y`]b`WcbfYa dcfUfmdi V`]WXYVUfYgžUbXžgYWcbX`mžfc`UggYgg`f\Y]f`fY`Yj UbWY`fc`W ffYbh dfcV`Ya gžei Ygf]cbg`UbX`W\U``Yb[Yg"'H\fci [\h\Y`UbU`mg]g`cZgdYW]Z]WWUgY`ghi X]Ygžmci `k]``Yb[U[Y k]h\Ya i 'h\Ya i 'h]X]a Ybg]cbU``bUfi fY`cZ[`cVU``W\U``Yb[YgžUbX`k]```Wcbg]XYf`f\\Y`XYg]fUV]`]fmUbX X]Z]W``fmcZa i `h]X]gW]d`]bUfmUddfcUW\Yg`fc`UXXfYgg]b[`f\Ya "

7 ci fgY @YUfb]b['Ci hWca Yg

- %" 8 YgWf]VY 'h\Y _YmZci bXUljcbg'cZd\]cgcd\]\W]`ž'dc`]lj\W]``UbX'Y\\bca]\WUddfcU\\Yg'lc'_bck]b['h\Y k cf`Xž']b'`c\W]``UbX'[`c\U]`\\bhYl l'
- &" 5bU`mgY`dUfh]WV`Uf`WUgY`ghi XmiYl Ua d`Yg`cZd\]`cgcd\]WU`ž'dc`]h]WU``UbX`YWcbca]WUddfcUVX\Ygž UWrcfg`UbX`dfUW]WYgž]b``cWU``UbX`[`cVU``WcbhYl h"

Wei fgY'k]h\"Uddfcdf]UhY'UbX'fY`Yj Ubh]bXYdYbXYbhfYgYUfW\"

H\Y'XYg][b'cZh\Y'hYUW\]b['acXY'UbX'WcbhUWh\cifg'fYZYWhVch\'U'Wcaa]haYbhhc'ZJWY'hc'ZJWY'hYUW\]b[UbX'Yb[U[YaYbhk]h\'jUf]cig'UgdYWg'cZV'YbXYX'`YUfb]b[ž'Ug'cih]bYX']b'h\Y'IBGK'&\$&)'GhfUhY[mH\YaY'5&.'9XiWUhcbU``9IWY``YbWY

5ggYgga Ybh

 $\label{eq:linear_continuous_con$

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5ggYgga Ybh'tUg_	K Y][\h	8iY'8UhY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
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7 UgY Gh XmDUdYf	(\$1	&) #\$(#&\$&\$`\$) .\$\$`DA	&ž(
91 Ua]bUh]cb	(\$1	Bch5dd`]WUV`Y	%Ž Ž(

5ggYgga Ybh'8YhJ]`g

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@Yb[h\.'+)\$

8 YHJ]`g. `Ghi XYbhg`a i ghgi Va]h`U`+) \$`k cfX`V`c[`dcghi`H\]g`UWhj]hm]g`k cfh\`&\$\\ `cZh\Y`Z]bU``[fUXY" Ghi XYbhg`g\ci `X`gY`YWhU'h\Ya Y`Zcf'h\Y]f`V`c[`dcghiVUgYX`cb`fY`Yj Ubh`UbX`WcbhYa dcfUbYci g`W ffYbh UZJJ]fg`UbX#cf'bYk ga YX]U`XYVUhYg`UbX`Udd`mfY`Yj Ubh`WcbWfdhg`]bhfcXi WYX`Vmh\Y``YWhi fYg"`K f]hhYb ZYYXVUW_ž`]bWi X]b[`U`fi Vf]WUbX`bi a Yf]WU``[fUXYž'k]```VY`dfcj]XYX`k]h\]b`%\$`k cf_]b[`XUmg`cZ gi Va]gg]cb"`Ghi XYbhg`k]```VY`dfcj]XYX`k]h\ [i]XY`]bYg`cb`UggYgga YbhYl dYWhJhjcbgž'a UXY`Uj U]`UV`Y h\fci [\`h\Y`Wci fgYfg`A ccX`Y`dU[Y"

5XX]h]cbU`XYhJ]`g.

 $H\g^+) \$!k\ cfX'6`c[\ 'Dcgh']g'YggYbh']U``m'U'g\cfh'fYUW]cb'YggUmcb'fY`Yj\ Ubh'UbX'WcbhYa\ dcfUbYci\ g'W ffYbhUZU]fg'UbX#cf'bYk\ g'a\ YX]U'XYVUhYg'UbX'Udd`]Yg'fY`Yj\ Ubh'WcbW'dhg']bhfcXi\ W'X'Vmh\Y'`YWi\ fYg'Zfca'k\ YY_'%hc')\ "'Nci'a i\ ghYb[\ U[\ Y'k\]h\'Y'YWi\ fY'WcbhYbh'UbX'fYei\]fYX'fYUX]b[\ g''$

H\]g`9ggUm]g`k cfh\`&\$i `cZh\Y`ZbU`[fUXY"'Hc`dfYdUfY`Zcf'h\Y`ZbU`XfUZhžmci `UfY`fYei]fYX`hc`gi Va]h`U g\cfhbchY`cZbc`a cfY`h\Ub`%\$\$`k cfXg`]b`K YY_`(`gc`h\Uhmci `a UmfYWY]j Y`gca Y`gdYW]Z]WZYYXVUW_`Zfca h\Y`Wci fgY`Wcbj Ybcf"`A YUbk\]`Yžmci `WUb`U`gc`i gY`h\Y`A ccX`Y`: cfi a `Z bWf]cb`fi]ghYX`i bXYf`K YY_`) Ł`hc fU]gY`Ubmei Ygh]cbg`UVci h\ck 'hc`k f]hY`h\]g`9ggUm"

Hi fb]h]b gYh]b[. `H\]g Ugg][ba Ybh]g gi Va]hhYX h\fci [\`Hi fb]h]b UbX ghi XYbhg WUb gYY Hi fb]h]b g]a]`Uf]hm fYdcfhg"

5ggYgga Ybh& 7UgY Gh XmDUdYf

GhUfhXUhY. Bch5dd`]WUV`Y

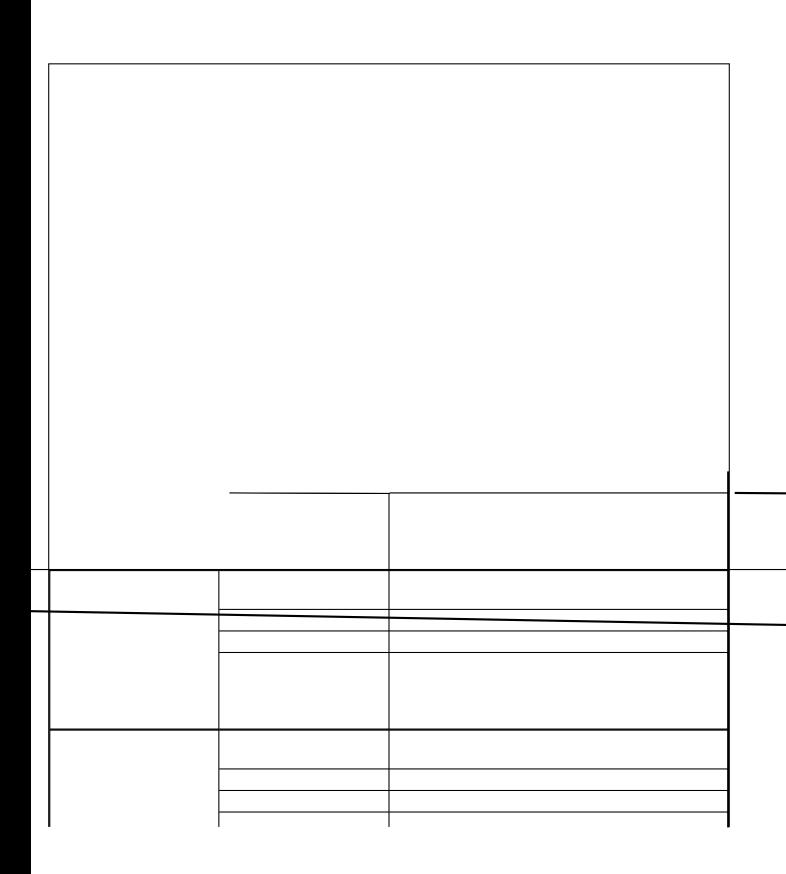
@Yb[h\. &\$\$\$ k cfXg

8 YHJ]`g. `Ghi XYbhg`a i ghigi Va]h`U`&ž\$\$\$`k cfX`WUgY`ghi XmdUdYf"'H\]g`dUdYf`]g`k cfh\`(\$i `cZh\Y`Z]bU` [fUXY"`K YY_g`) `hc`- `cZh\Y`Wci fgY`UfY`XYj chYX`hc`WUgY`ghi XmUbU`mg]g`UbX`ghi XYbhg`g\ci `X`gY`YWh`U`WUgY ghi XmcZh\Y]f`ck b`W\ccg]b[`hc`fYZYWhcb`fY`Yj Ubh\Wci fgY`h\Ya YgžXYVUhYgž`]bhY``YWhi U``\]ghcf]Yg`UbX`h\Y]f dfUWf]\WU``Udd`]\WUh]cb"`K f]hhYb`ZYYXVU\Zž]bWi X]b[`U`fi Vf]\WUbX`bi a Yf]\WU``[fUXYžk]``VY`dfcj]XYX`k]h\]b`%\$ k cf_]b[`XUmg`cZgi Va]gg]cb"`Ghi XYbhg`k]```VY`dfcj]XYX`k]h\[i]XY`]bYg`cb`UggYgga YbhYI dY\WUh]cbgž a UXY`Uj U]`UV`Y`h\fci [\`h\Y`\Ci fgYfg`A ccX`Y`dU[Y"``

5XX]h]cbU``XYhU]`g.

VY 'gi Va]hh/ X 'k]h\ 'mci f '91 Ua 'fYgdcbgYg"

 $\label{eq:higher_high_substitution} \mbox{Hi fb]h]b "gYhh]b[."H\]g"]g"bchU"Hi fb]h]b"Ugg][\ ba\ Ybh$



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DfYgWf]VYX F Ygci fWYg

5```fYei]fYX`fYUX]b[g`fY"["ž`ĉi fbU``Uft]WYgž'Wca a YbHJf]Yg`UbX`Vcc_`WXUdHYfgŁ`UfY`Y]h\Yf``]gHYX`cb`h\Y A ccX`Y`dU[Y`Zcf`DD97%\$\$%cf`WJb`VY`Zci bX`UbX`UWWYggYX`j]U`I BGK '@]VfUfmfg`cb`]bY`WJHJ`c[i Yž Y`ĉi fbU``UbX`YVcc_`gYfj]WYg"

FYWca a YbXYX FYgci fW/g

 $D`YUgY`WXYW\underline{\ '}A\ ccX`Y`Zcf`gi\ [\ [\ YghYX`fYUX]b[\ g`UbX`UXX]h]cbU``fYgci\ fWfg"$

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

Ghi XYbhYj U`i Uhji Y`ZYYXVUW_`]g`[Uh\YfYX`dYf]cX]WJ``mi g]b['h\Y`I b]j Yfg]migʻʻA m9I dYf]YbW''gi fj Ymʻ ±JZcfa U``ZYYXVUW_`]g`U`gc`]a dcfhUbhž`UbX`k Y`Wca Yž'h\fci [\ci hh\Y`gYa YghYf''`Mci f`ZYYXVUW_`]g`i gYX`hc\Y`d`[i]XY`hi hcf]U`gž`YWhi fYg`UbX`hc`\Y`d`mci 'UddfcUW\`mci f`UggYgga Ybhg''`Ghi XYbhZYYXVUW_`]g`hU_Yb gYf]ci g`mUbX`WcbhJbi U``Yb\UbWa Ybhg`UfY`a UXY`hc``YUfb]b['UbX`hYUW\]b['UWNjj]hJYg`cb`h\Y`VUg]g`cZ ghi XYbhZYYXVUW_'''H\]g`]bWi XYg`a U_]b['W\Ub[Yg`hc`h\Y`UggYgga YbhfY[]a Yž'hi hcf]U``UWNjj]hJYgž'k YY_`m fYUX]b[gž'A ccXY`WcbhYbhž'UbX`gc`cb'''H\YfY`]g`U`gc`U`g\cfhA]XhYfa 'fYj]Yk 'gW\YXi `YX`]b`K YY_'+`Zcf ghi XYbhg`hc`fU]gY`h\Y]f`WcbWfbg`UVci h`YUfb]b['UbX`hYUW\]b["

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

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hY'Yd\cbY'9| hYfbU''Gi ddcfhcb'-',) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'''#c'ici
UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbž'Vi hmci
a i ghfYhJ]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW'a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd']WUhJcb'''#mci 'Ya U]''9! hYfbU''Gi ddcfhmci 'k]''
Ui hca UhJWJ'`mfYWf']j Y'U'HJW_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'''bYYX'hc'gdYWJZJWJ'`mUg_'Zcf'cbY''
Hi fb]hJb'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'gfUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mj]U'Y]h\Yf'Hi fb]hjb'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]``'VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\ U`hYfbUhj Y'gi Va]gg]cb'XYHJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va]hUgq][ba Ybha ccXiY

5 WUXYa | W < cbYghmUbX D`U[| Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUiqci fWYik]h\ci hiUW_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya



5 WUXYa]W = b Zcfa Uh cb ·

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- fYei Yghg Zcf Yl hYbg]cb/
- `UhY`gi Va]gg]cbg`[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK '< YU`h\ 'UbX'GUZYhm'dc`]W[Yg/
- YI Ua]bUhcb dfcWXi fYg/
- gdYWJU``Wcbg]XYfUhJcb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybh fY/
- gh XYbhYei]lmUbX X]gUV]`]lm/
- UbX'ch\Yf'YggYbhJU`'UWUXYa]W]bZcfa UhJcbž'gYY

<u>\hdg.#k k k "Ufng"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa]W]bZcfa Uhlcb#dfchcWc`g![i]XY`]bYg#</u>

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a U[Y'7fYX]h

GmbYf[]Yg^]b^Gci bX^&\$%*

7F = 7CG

7 F = 7 C G 'Dfcj]XYf'7 cXY. '\$\$\$-,;