

Output Growth and Inflation Across Space and Time

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OECD Ordering of Countries

- **Our first task is to use the OECD data base to form country GDP volumes. We will use the OECD ordering of countries, which is as follows:**

1= Australia

2= Austria

3= Belgium

4= Canada

5= Chile

6= Czech Republic

7= Denmark

8= Estonia

9= Finland

10= France

11= Germany

12= Greece

13= Hungary

14= Iceland

15= Ireland

OECD Ordering of Countries (cont)

- 16= Israel**
- 17= Italy**
- 18= Japan**
- 19= Korea**
- 20= Luxembourg**
- 21= Mexico**
- 22= Netherlands**
- 23= New Zealand**
- 24= Norway**
- 25= Poland**
- 26= Portugal**
- 27= Slovak Republic**
- 28= Slovenia**
- 29= Spain**
- 30= Sweden**
- 31= Switzerland**
- 32= Turkey**
- 33= United Kingdom**
- 34= United States.**

Country Domestic Price and Quantity Levels P_n^t and Q_n^t

- The **country values for nominal GDP** in the national currencies for the years 2000-2012 can be obtained from the OECD electronic data base, OECD.Stat.
- Convert these estimates into billions and denote the estimate for country n in year t by V_n^t The **corresponding volume estimates** can be obtained from OECD.Stat TableB1-GE: Gross domestic product (GDP); National currency, constant prices, national base year, millions, annual data.
- Convert these estimates into billions and denote these volumes (or quantities) by Q_n^t for $n = 1, \dots, 34$ and $t = 2000, \dots, 2012$.
- The **corresponding country price level** for country n in year t is defined as $P_n^t = V_n^t / Q_n^t$ for $n = 1, \dots, 34$ and $t = 2000, \dots, 2012$

Definition of OECD Aggregate GDP Inflation

- Since the country volumes Q_n^t are measured in domestic currency units (which are not comparable across countries), we need to convert the domestic nominal values of GDP into **common currency units** using the average exchange rates for each year.
- In principle, the numeraire country could be any of the 34 OECD countries but it seems reasonable to **choose the largest country** as the numeraire country.
- The OECD has conveniently done this for us, converting each country's nominal GDP into US dollars at the average market exchange rates for the given year. Convert these estimates into billions and denote the

Definition of OECD Aggregate GDP Inflation (cont)

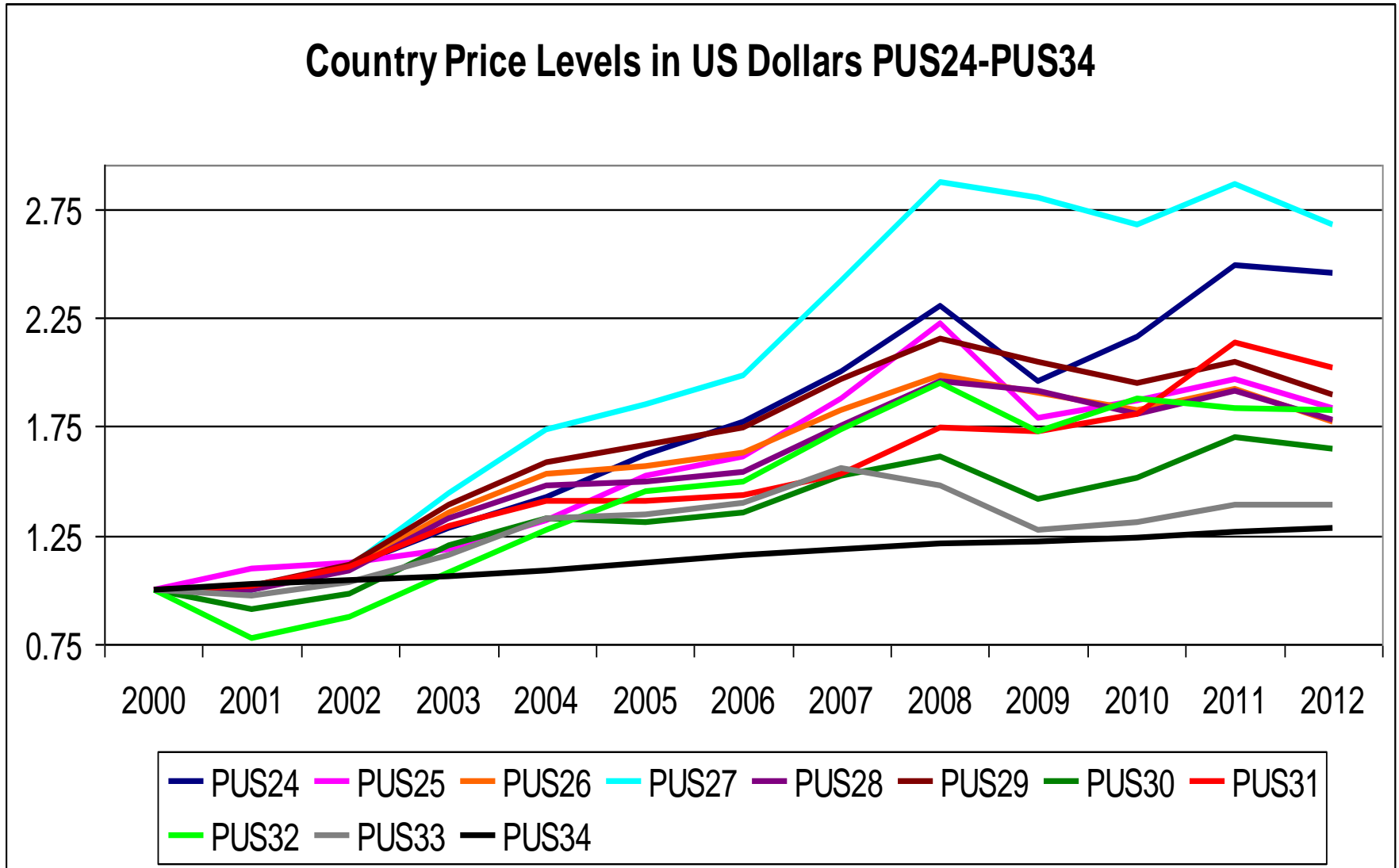
- The year t , country n US dollar price level for GDP, p_n^t , is

US Dollar Price Levels; OECD and Countries 1-11

US Dollar Price Levels; Countries 12-23



US Dollar Price Levels; Countries 24-34



Aggregate Measures of OECD GDP Growth and Inflation:

First Approach

- We are now in a position to calculate **aggregate OECD real output** and the **corresponding OECD price level** for the years 2000-2012 using the (US dollar) price and (domestic) volume data, p_n^t and q_n^t , as inputs into the **Fisher chained index number formula**.
- Denote the chained Fisher aggregate OECD volume level for year t by Q^t and the corresponding US dollar year t price level by P^t for $t = 2000, \dots, 2012$. For $t = 2001, \dots, 2012$, define the year t OECD **Approach 1 volume growth rate** t and the corresponding OECD **US dollar inflation rate** t in percentage points as follows:

$$(10) \quad ^t = 100[(Q^t/Q^{t-1}) - 1] ;$$

$$(11) \quad ^t = 100[(P^t/P^{t-1}) - 1] .$$

Aggregate Measures of OECD GDP Growth and Inflation: First Approach (cont)

- The chained **Fisher OECD aggregate price and volume levels**, P^t and Q^t , for the years 2000-2012 are listed in Table 1 along with the corresponding percentage point annual growth rates, t and t , for the years 2001-2012.
- For comparison purposes, we also calculated the **aggregate OECD chained Laspeyres and Paasche indexes** over the same period. The resulting Laspeyres and Paasche price levels, P_L^t and P_P^t , are also listed in Table

Table 1: OECD Annual Aggregate Volumes Q^t and Price Levels in US Dollars P^t and Percentage Point Changes

Year t	Q^t	P^t	P_L^t	P_P^t	ΔQ^t	ΔP^t	ΔP_{EU}^t	P_{EU}^t	P_{ICP}^t
2000	26694.3	1.0000	1.0000	1.0000				1.0000	1.0000
2001	27022.9	0.9794	0.9793	0.9795	1.23	-2.06	0.84	1.0084	1.0301
2002	27432.9	1.0081	1.0079	1.0082	1.52	2.92	-2.14	0.9868	1.0549
2003	28007.3	1.1079	1.1079	1.1078	2.09	9.90	-8.35	0.9044	1.0796
2004	28896.6	1.1933	1.1934	1.1932	3.18	7.71	-2.10	0.8854	1.1067
2005	29670.9	1.2271	1.2271	1.2272	2.68	2.84	2.68	0.9091	1.1326
2006	30566.7	1.2560	1.2558	1.2561	3.02	2.35	1.46	0.9224	1.1610
2007	31374.2	1.3392	1.3388	1.3396	2.64	6.63	-2.27	0.9015	1.1893
2008	31410.0	1.4109	1.4104	1.4114	0.11	5.36	-1.56	0.8874	1.2173
2009	30267.1	1.3729	1.3726	1.3732	-3.64	-2.69	2.60	0.9105	1.2305
2010	31138.6	1.4013	1.4007	1.4019	2.88	2.07	7.06	0.9748	1.2477
2011	31688.5	1.4776	1.4770	1.4782	1.77	5.44	0.46	0.9793	1.2697
2012	32162.6	1.4534	1.4525	1.4543	1.50	-1.63	6.42	1.0422	1.2888

Discussion of Table 1

- The sample average growth rate for OECD real GDP was 3.18% per year. The sample average OECD inflation rate (measured in US dollars at market exchange rates) was 3.24% per year.
- It can be seen that there was only one year where OECD real growth was negative: 2009 (- 3.64%).
- What is somewhat surprising is that there were **3 years where OECD inflation was negative** when measured in US dollars at market exchange rates: 2001 (2.06%), 2009 (2.69%) and 2012 (1.63%). The deflation for 2012 is particularly surprising, given the fairly loose monetary policies across the OECD region in recent years.
- As mentioned before, the choice of Laspeyres, Paasche or Fisher aggregate GDP and price level indexes does not matter much for this application.

More Discussion of Table 1

- The principles used to construct our OECD aggregate measures of real GDP, Q^t , are the same principles used to construct country wide estimates of real GDP within a country.
- The country

And Even More Discussion of Table 1

- There is one difference in our suggested method for constructing OECD real GDP as opposed to methods used to construct national estimates of real GDP: in order to construct OECD real GDP, **we needed to convert national values of GDP into a common currency using annual average market exchange rates.**
- We chose to make this conversion using US dollars as the numeraire currency. In principle, we could have chosen the numeraire currency to be the currency of any one of the 34 member countries.
- What would happen if we chose another currency to be the numeraire currency? The unit of measurement would change, but the overall **OECD growth rates for real GDP would remain the same**; i.e., the π^t listed in Table 1 *would not change* but the inflation measures π^t would change; i.e., **they are numeraire dependent.** [Compare π^t with π_{EU}^t].

OECD Growth and Inflation Measurement Using Annual PPP Information: **Approach 2**

- For many purposes, it is useful to be able to compare the GDP of one country with the GDP of another country in **comparable units of measurement**.
- Thus the OECD (in close cooperation with Eurostat) produces an annual series of price indexes (or PPPs) that enable one to compare the real GDP of member countries with each other.
- The relevant table of PPPs for the 34 countries can be found in OECD.Stat, Table 4: PPPs and Exchange Rates; PPPGDP; Purchasing Power Parities for GDP; National currency per US dollar; Annual; 2000-2012.
- Our second approach to measuring OECD volumes and inflation uses these PPPs along with national growth rates.

Country Shares of Annual OECD Real Output

- Recall that the country n **nominal value of GDP** in year t in **domestic currency** $g_{n,t}$ $T_{n,t}$ $g_{n,t} = G_n(t) T_{n,t}$ TJETB

Harmonization Principles

- Our suggested solution to the problem of harmonizing national growth rates of GDP with the country shares of OECD aggregate real GDP rests on two principles:

The resulting harmonized estimates of country volumes must be consistent with the real annual cross country volume shares s_n^t listed in Table 2;

OECD aggregate real GDP growth must be equal to the rates of aggregate growth generated by our recommended Fisher indexes F^t defined by (16) below.

- Using the above two principles, the comparable across time and space country GDP volumes will be uniquely determined (up to a scalar units of measurement factor).

Aggregate OECD Volume Growth: Laspeyres

- The *year t growth factor for country*

Aggregate OECD Volume Growth: Paasche and Fisher

- The counterpart to the Laspeyres type formula defined by (14) is the following *Paasche type formula* (which applies the Laspeyres formula but reverses the role of time):

$$(15) \quad P^t = \left[\sum_{n=1}^{34} S_n^t (Q_n^t / Q_n^{t-1}) \right]^{1/2}; \quad t = 2001, \dots, 2012.$$

- Since both indexes have the same logical foundation, it seems best to take a symmetric average of the two indexes, which leads to the following *Fisher type formula for OECD volume growth* for year t:

$$(16) \quad F^t = \left[L^t P^t \right]^{1/2}; \quad t = 2001, \dots, 2012.$$

- The annual Fisher chain links defined by (16) are our preferred estimates for OECD volume growth using Approach 2.

Aggregate OECD Volume Growth: Approach 2 Indexes

- The growth factors (or chain link indexes) defined by (14)-(16) can be multiplied together to generate OECD aggregate volume levels. The **growth factors** can also be transformed into **growth rates**, L^t , P^t and F^t (in percentage points), by using the following definitions for $t = 2001, \dots, 2012$:

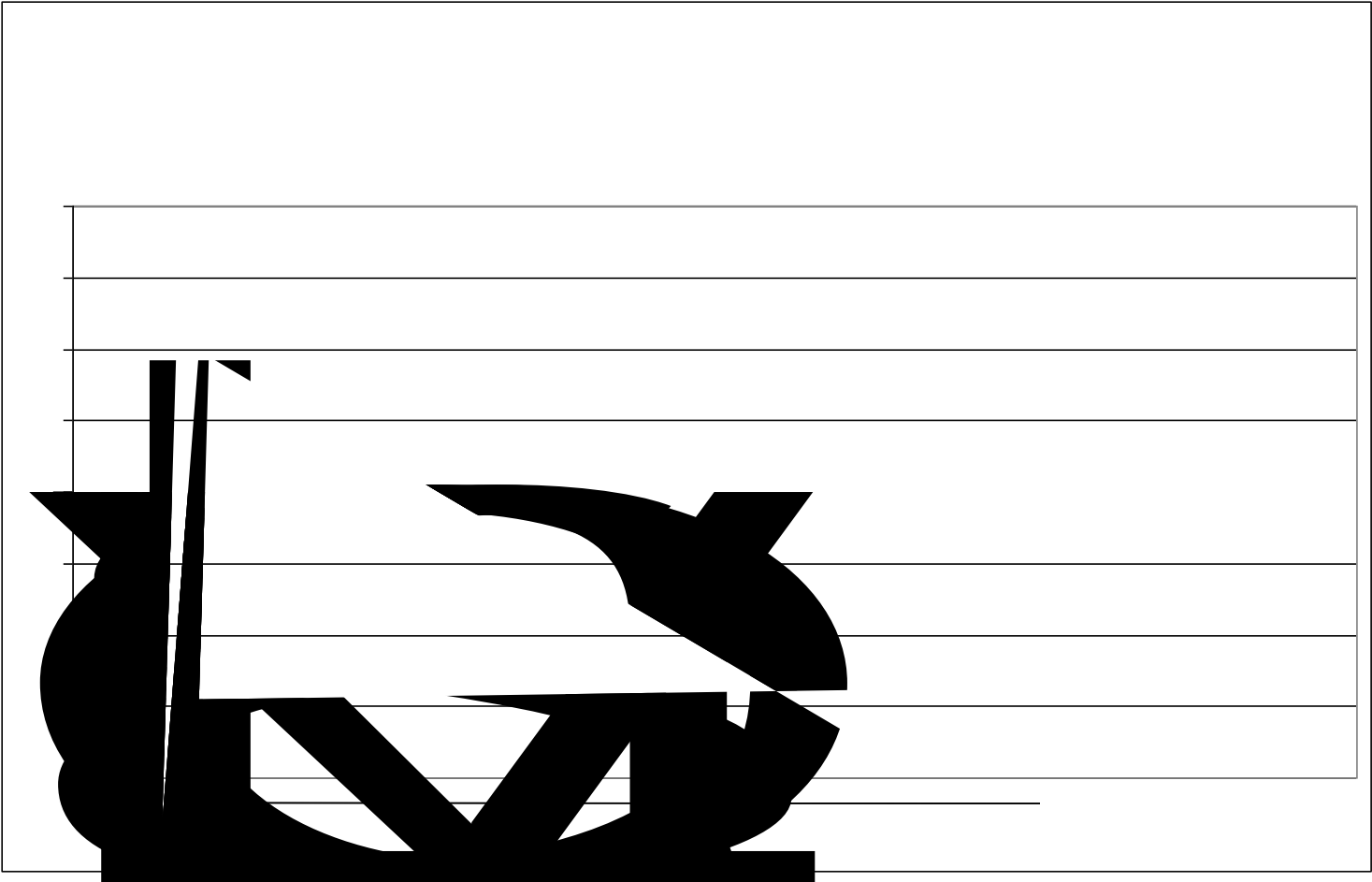
$$(17) \quad L^t = 100 \left[\frac{L^t}{L^{t-1}} - 1 \right]; \quad P^t = 100 \left[\frac{P^t}{P^{t-1}} - 1 \right]; \quad F^t = 100 \left[\frac{F^t}{F^{t-1}} - 1 \right].$$

- The **Approach 2** annual **OECD volume growth measures** defined by (17) as well as our earlier **Approach 1 US dollar weighted measures** t are listed in Table 3 which is reproduced on the next slide.

Alternative Aggregate OECD Volume Growth Measures

• Year t	L^t	P^t	F^t	t
• 2001	1.2908	1.2964	1.2936	1.2313
• 2002	1.6832	1.6767	1.6799	1.5171
• 2003	2.1670	2.1610	2.1640	2.0937
• 2004	3.3269	3.3331	3.3300	3.1753
• 2005	2.8318	2.8311	2.8314	2.6796
• 2006	3.1525	3.1592	3.1558	3.0193
• 2007	2.7065	2.7074	2.7070	2.6415
• 2008	0.1912	225.89	Tm[(2.)-3(70)-4(65)]1 1115	Tm.048 Tc[(200

Alternative Aggregate OECD Volume Growth Measures



Aggregate OECD Inflation Indexes: Approach 2

- The OECD real output shares, s_n^t defined by (13), can be used as weights for national GDP the national currency GDP price deflator for country n in year t was defined as P_n^t . Recall also that (14)-(16) defined OECD Laspeyres, Paasche and Fisher volume link volume indexes, L^t , P^t

Aggregate OECD Inflation Indexes: Approach 2

- These chain link indexes can be multiplied together to generate the corresponding OECD aggregate price levels. The inflation growth factors can also be transformed into *growth rates*, L^t , P^t and F^t in percentage points, by using the following definitions for $t = 2001, \dots, 2012$:

$$(19) \quad L^t = 100[L^t - 1]; \quad P^t = 100[P^t - 1]; \quad F^t = 100[F^t - 1].$$

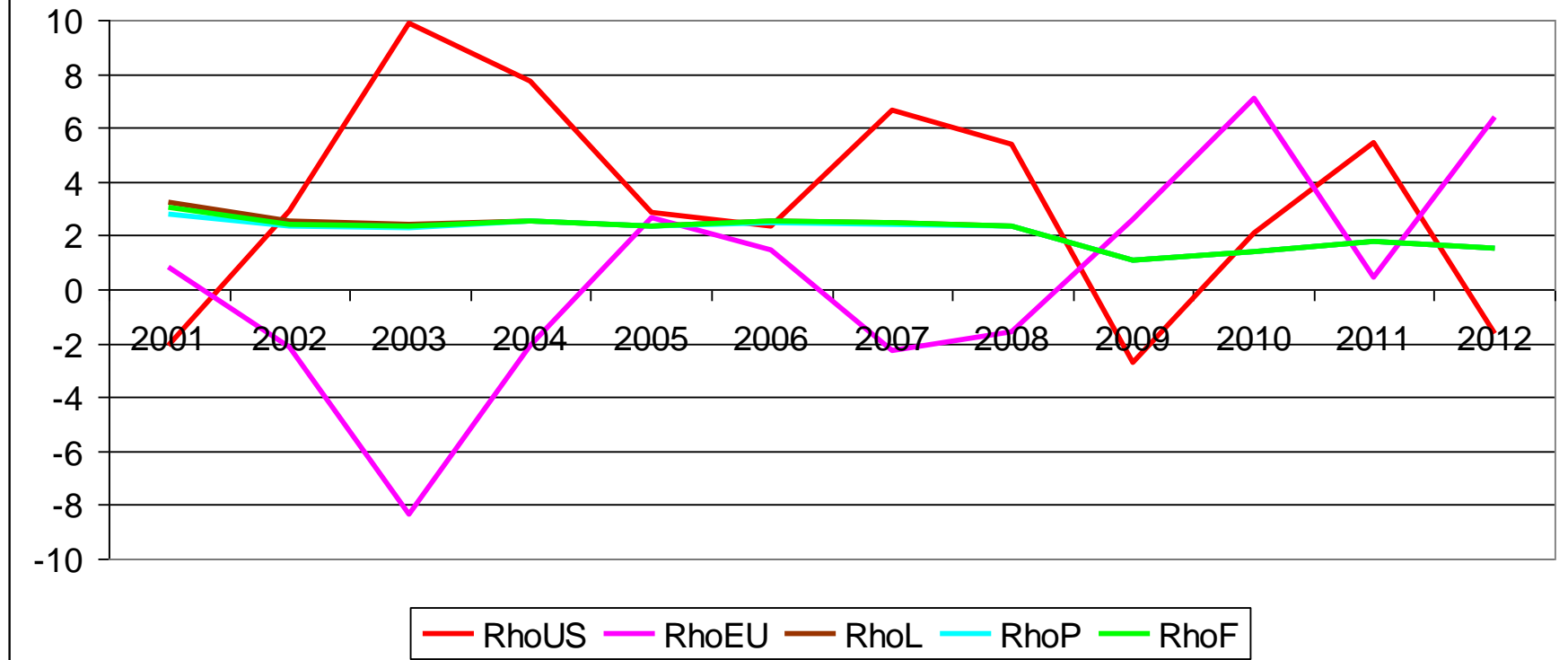
- The above ICP based inflation rates (in percentage points) are listed in the next slide along with the earlier US dollar and Euro based inflation estimates that were derived using US dollar and Euro estimates of country nominal GDP values and country volume estimates, $V^t = V_{US}^t$ and $V^t = V_{EU}^t$

Aggregate OECD Inflation Indexes: Approach 2

Year	US^t	EU^t	L^t	P^t	F^t
2001	-2.06	0.84	3.22	2.80	3.01
2002	2.92	-2.14	2.51	2.31	2.41
2003	9.90	-8.35	2.39	2.29	2.34
2004	7.71	-2.10	2.53	2.50	2.51
2005	2.84	2.68	2.34	2.34	2.34
2006	2.35	1.46	2.51	2.50	2.51
2007	6.63	-2.27	2.45	2.44	2.44
2008	5.36	-1.56	2.36	2.34	2.35
2009	-2.69	2.60	1.09	1.08	1.09
2010	2.07	7.06	1.41	1.39	1.40
2011	5.44	0.46	1.76	1.76	1.76
2012	-1.63	6.42	1.51	1.50	1.50

Aggregate OECD Inflation Indexes: Approach 2

US and EU Exchange Rate Based Inflation Rates and ICP Based Laspeyre, Paasche and Fisher OECD Inflation Rates



New Problem: The Construction of Comparable Across Countries and Time GDP (Harmonized) Volumes

- Use the Fisher chain links defined by (16) to define Q_H^t as follows:

$$(18) \quad Q_H^{2000} = 1 ; \quad Q_H^t = Q_H^{t-1} F^t ; \quad t = 2001, \dots, 2012.$$

- Now use the country shares of OECD real GDP s_n^t listed in Table 2 and the aggregate index Q_H^t to define the following **preliminary harmonized country volumes** for country n in year t , q_{Hn}^t , as follows:

$$(19) \quad q_{Hn}^t = Q_H^t s_n^t ; \quad n = 1, \dots, 34 ; \quad t = 2000, \dots, 2012.$$

- For each year t , $\sum_{n=1}^{34} q_{Hn}^t = \sum_{n=1}^{34} Q_H^t s_n^t = Q_H^t \left(\sum_{n=1}^{34} s_n^t \right) = Q_H^t$ and so the harmonized volumes satisfy the two principles listed on slide 19 above. In principle, the country volumes defined by (19) are independent of country prices and exchange rates. [Consider the one good case.]

The Construction of Comparable Across Countries and Time GDP Volumes (cont)

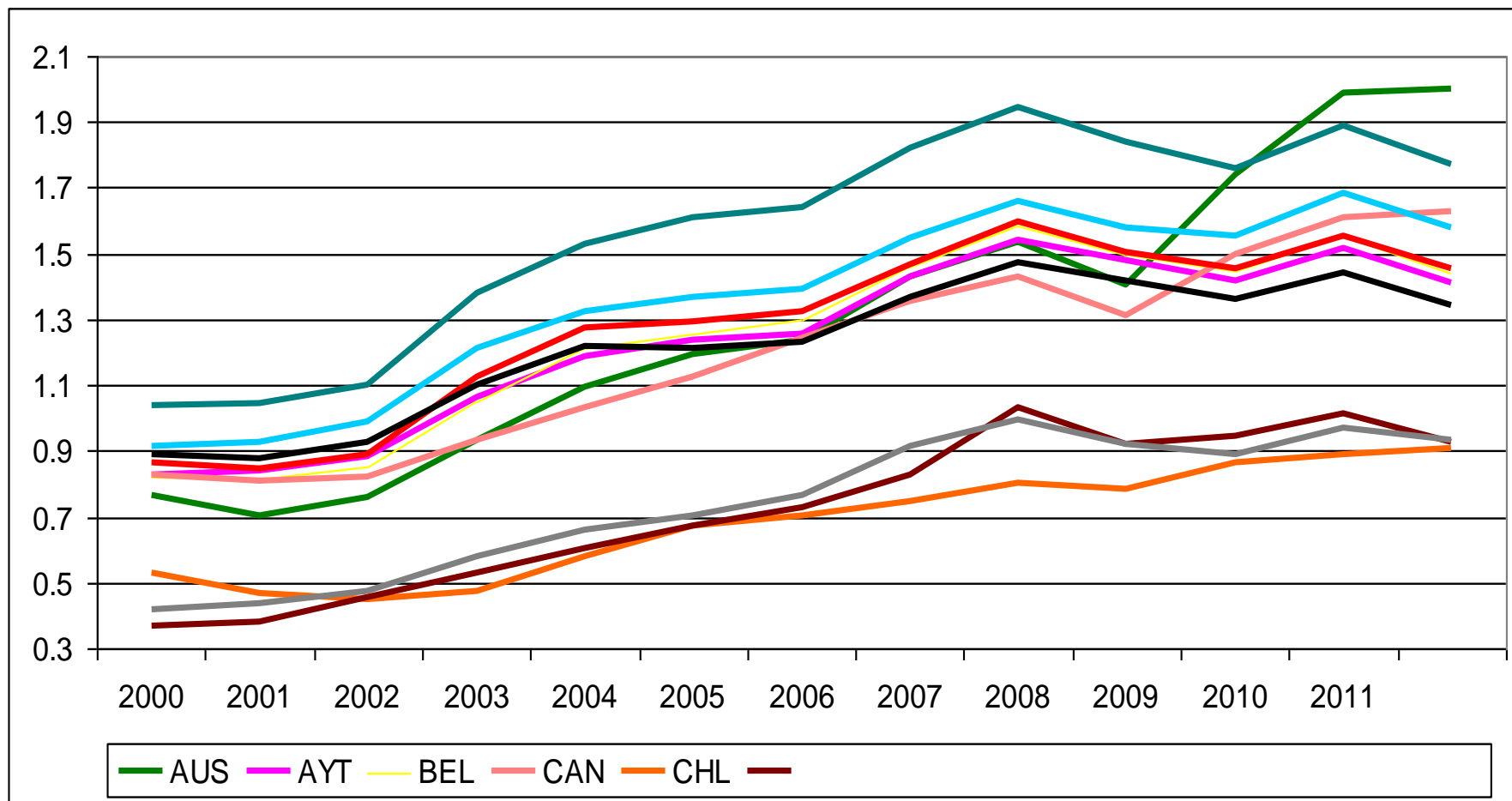
- **Recall that the value of country n's nominal GDP converted**

Discussion of the Country Comparable Prices and Volumes Listed in Tables 4 and 5

- Note that $q_{H34}^{2000} = q_{34}^{2000}$ and $p_{H34}^{2000} = p_{34}^{2000} = 1$ so that country GDP volumes are measured as multiples of a bundle of US GDP produced in the year 2000.
- Thus the price levels in Table 5 measure the US dollar value of constant bundle of GDP that is (in theory) comparable across countries.
- The price levels in Table 5 are comparable across space and time, whereas the price levels p_n^t listed in Table A3 of the Appendix are only comparable across time for each country.
- These comparable US dollar price levels are shown in the next 3 slides.
- They are (imperfect) indicators of a country's competitiveness. [Imperfect because not all commodities are internationally traded plus there are errors in the PPPs].

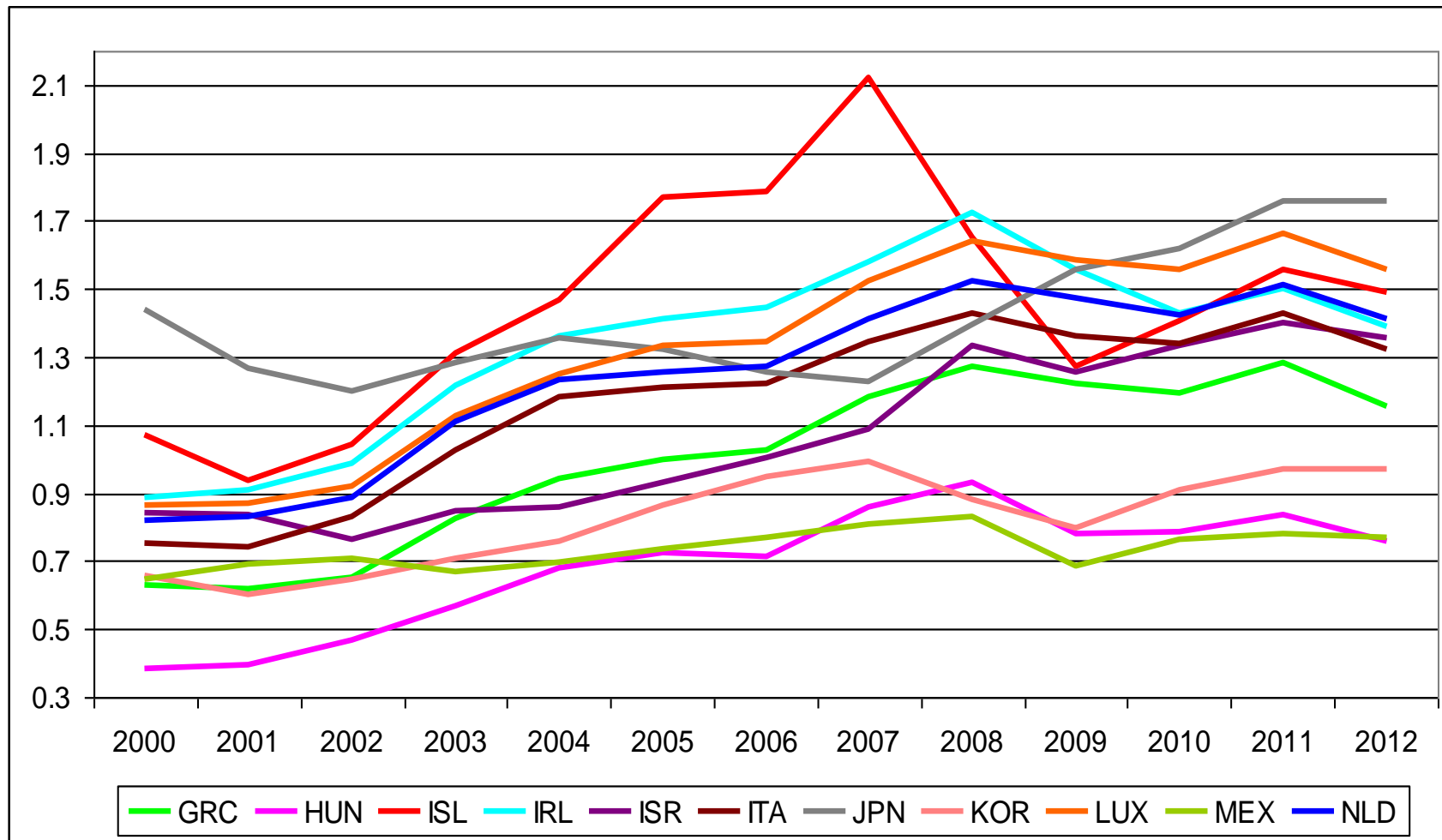
Harmonized OECD Country GDP Price Levels in Comparable US Dollar Units of Measurement p_{Hn}^t

Countries 1-11; US Price Level in 2000 = 1 [Convergence?]



Harmonized OECD Country GDP Price Levels in Comparable US Dollar Units of Measurement p_{Hn}^t

Countries 12-22; US Price Level in 2000 = 1 [Note ISL!]



Harmonized OECD Country GDP Price Levels in Comparable

Discussion of Harmonized GDP Price Levels in US \$

- From Table 5, it can be seen that the countries with the **lowest price levels** (in US dollars) in 2012 are countries 13, 21, 25 and 32 (**Hungary, Mexico, Poland and Turkey**) with price levels in the 0.76 to 0.77 range.
- Countries with relatively **high price levels** in 2012 are countries 1 (**Australia**, $p_{H1}^{2012} = 2.00$), 4 (**Canada**, $p_{H4}^{2012} = 1.62$), 7 (**Denmark**, 1.77), 9 (**Finland**, 1.58), 18 (**Japan**, 1.76), 20 (**Luxembourg**, 1.56), 23 (**New Zealand**, 1.55), 24 (**Norway**, 2.01), 30 (**Sweden**, 1.69) and 31 (**Switzerland**, 1.96).
- Again, these price level estimates are (imperfect) indicators of the competitiveness of the country on international markets, with lower price levels indicating greater competitiveness.
- The Tables 4 and 5 country volumes and price levels are our preferred comparable across time and space estimates.

Section 4. OECD Growth and Inflation Using Country Annual GDP Volume Growth Rates and Base Period Shares of OECD Real GDP

- In this section, we generated comparable country GDP volume estimates for OECD countries covering the period 2000-2012 by using the real GDP country volume shares for 2000, the s_n^{2000} listed in Table 2 above, along with the national growth rates of country real GDP relative to 2000, the Q_n^t/Q_n^{2000} listed in Table A2 of the Appendix.
- This is a typical strategy in forming estimates of real GDP that rely on PPPs that are only produced infrequently: **use the PPP based country shares of “world” GDP for a base period and project these shares forward using national GDP volume growth rates .**
- How different the resulting estimates are from our preferred harmonized volume estimates, q_{Hn}^t , listed in Table 4 above?

Section 4 Results

- If we take each column in Table 5, subtract the corresponding entries in the same column of Table 7 and then take the absolute value of the differences, we find that the *average* absolute difference grows from 0 in 2000 to 9.4 percentage points in 2012.
- The maximum absolute difference grows from 0 in 2000 to 54.0 percentage points in 2012. These are massive differences in price levels, which translate into massive differences in GDP levels.
- The sequence of **average absolute differences** in percentage points over the 13 years is as follows: 0, 0.8, 1.6, 2.0, 2.2, 3.8, 5.2, 6.4, 7.6, 6.8, 8.0, 9.3, 9.4.
- The sequence of **maximum absolute differences** in percentage points over the 13 years is as follows: 0, 2.9, 5.2,

Section 5. OECD Growth and Inflation Using Country Annual GDP Volume Growth Rates and Final Period Shares of OECD Real GDP

- In this section, we generated comparable country GDP volume estimates for OECD countries covering the period 2000-2012 by using the real GDP country volume shares for 2012, the s_n^{2012} listed in Table 2 above, along with the national growth rates of country real GDP relative to 2000, the Q_n^t/Q_n^{2000} listed in Table A2 of the Appendix.
- **This method for forming comparable country GDP volumes is used by the World Bank when the International Comparisons Project produces a new set of PPPs.**
- The methodology is straightforward and follows the approach used in the previous section except that the 2012 country volume shares are used in place of the 2000 shares.

Section 5 Results

- Take each column in Table 5, subtract the corresponding entries in the same column of Table 8 and then take the absolute value of the differences.
- The *average* absolute difference for 2000 over the 34 countries is 6.0 percentage points, which increases to 7.9 percentage points for 2005 and then gradually decreases to 4.2 percentage points in 2012. Over all observations, the *maximum* absolute deviation is 35.6 percentage points.
- The sequence of **average absolute differences** over the 34 countries in percentage points over the 13 years is as follows: 6.0, 6.1, 6.2, 7.0, 7.8, 7.9, 6.2, 6.8, 5.9, 5.4, 5.1, 5.2, 4.2.
- The sequence of **maximum absolute differences** in percentage points over the 13 years is as follows: 24.6, 25.9, 32.3, 35.6, 33.1, 27.1, 20.5, 22.0, 16.0, 24.7, 18.7, 12.6, 6.3.
- **Conclusion:** extrapolation does not work well over 13 years

The Section 6 Methodology

- We will propose an interpolation method that leads to country shares of real GDP that are exactly consistent with the shares s_n^{2000} for the year 2000 and the shares s_n^{2012} for the year 2012.
- Step 1: Construct country measures of real GDP that jump from the year 2000 to the year 2012.
- The *long term growth factor for country n* can be defined as Q_n^{2012}/Q_n^{2000} where Q_n^t is country n's GDP volume in year t.
- Now use these long term growth factors along with the year 2000 country shares of OECD

The Section 6 Methodology (cont)

- The counterpart to the Laspeyres type formula defined by (27) is the following *Paasche type formula* that uses the shares of 2012 and reciprocal long term growth rates:

$$(28) \quad P = \left[\sum_{n=1}^{34} S_n^{2012} (Q_n^{2012}/Q_n^{2000})^{-1} \right]^{-1}; \quad t = 2001, \dots, 2012.$$

- Since both indexes have the same logical foundation, it seems best to take a symmetric average of the two indexes, which leads to the following *Fisher type formula* for *OECD long term volume growth* going from the year 2000 to the year 2012:

$$(29) \quad F = [L \cdot P]^{1/2}; \quad t = 2001, \dots, 2012.$$

- (29) defines a direct comparison of the data of 2000 with the data of 2012 whereas in section 3, we used chained Fisher type indexes to go from 2000 to 2012.
- The chained Fisher index for 2012 relative to 2000 is equal to 1.2203, which is very close to its direct counterpart, 1.2208.

The Section 6 Methodology: Step 2

- Preliminary estimates of country GDP volumes in comparable units for the years 2000 and 2012, q_{In}^{2000} and q_{In}^{2012} (the index I indicates that these are interpolated estimates), are defined as follows:

~~$$(30) \quad q_{In}^{2000} = s_n^{2000}, \quad q_{In}^{2012} = F s_n^{2012}, \quad n = 1, \dots, 24.$$~~

- The volumes defined by (30) will be imposed as constraints on our interpolation scheme. Define the *implied long term growth factor* over the years 2000-2012 for

The Section 6 Methodology: Step 2 (cont)

- These growth factors are not necessarily equal to the **national growth factors** G_n that are implied by the national growth rates defined as:

$$(32) \quad G_n = Q_n^{2012}/Q_n^{2000} ; \quad n = 1, \dots, 34.$$

- Thus for each country n , there is an **“error” factor** or discrepancy, $E_n = g_n/G_n$ between the implied growth rates g_n defined by (31) and the national growth rates between 2000 and 2012, G_n defined by (32).
- We will distribute these errors in a proportional manner and use the resulting adjusted national growth rates to interpolate between the two benchmark observations. Thus define the **country n proportional discrepancy factor**, α_n , as follows:
- (33) $\alpha_n = [g_n/G_n]^{1/12} ; \quad n = 1, \dots, 34.$

Overall Conclusions

The results listed in sections 3-5 show that it is very hazardous for analysts interested in comparative levels of GDP across countries to use national growth rates and a *single*