Decomposing Value Added Growth over Sectors into Explanatory Factors

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in the efficiency of the
output and input prices, technical progress and returns to scale. In order to implement t
decomposition for a sector, an estimate of the sector's cost constrained value added
function for the two periods under consideration is required, which is taken to be the fre
disposal hull of past observations. The problems associated with aggregating over sectors
are also considered. The methodology is illustrated using U.S. data for two sectors over
the years 1960-2014.

Keywords

Measurement of output, input and productivity, value added functions, revenue functions, variable profit functions, duality theory, economic price and quantity indexes, technical progress, total factor productivity, revenue efficiency, aggregation over sectors.

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index of input growth is E^t. Thus define the period t TFP growth rate, TFPG^t, for the production unit as follows:[19]

(35) $\mathrm{TFPG}^t \{ \{[p^t \sim y^t/p^{t\ 1} \sim y^{t\ 1}]/ D^t\}/ E^t = H^t J^t G^t W^t$

$$(40)\ \mathrm{TFP}^t = [p^t\,\tilde{y}^t/p^1\,\tilde{y}^1]/[A^t\,B^t] = C^t\,D^t\,E^t\,T^t\ ; \qquad t = 2,\ldots,T.$$

In the following section, we explain a practical method for obtaining estimates for the cost constrained value added function for a sector.

$$= w \cdot x / \min_s \{w \cdot x^s / p \cdot y^s : s = 1,2,\ldots,t\}$$
$$= w \cdot x / c^t(w,p)$$

where we define the period t nonparametric unit cost function $c^t(w,p)$ as follows:

$$(44)\ c^t(w,p) \equiv \min_s \{w \cdot x^s / p \cdot y^s : s = 1,2,\ldots,t\}.$$

Thus we have an explicit functional form for the unit cost function $c^t(w,p)$ that was defined earlier by (10) above. It can be seen that $c^t(w,p)$ defined by (44) is a linear nondecreasing function of w (and hence is linearly homogeneous and concave in w which is a necessary property for unit cost functions) and is convex and homogeneous of degree minus one in p.

Now we are in a position to apply the decompositions of value added growth (34), of TFP growth (35) and for the level of TFP (40), using the specific functional form for a sector's cost constrained value added function defined by (43). However, with the assumption of constant returns to scale in production, the returns to scale growth factor G^t is identically equal to one and so this factor vanishes from the decompositions of value added and TFP growth defined by (34) and (35) above. The levels return to scale growth factor D^t in (40) is also identically equal to one and hence vanishes from the decomposition (40).

In the following two sections we return to the more general model that was described in Section 3, but we assume that we have constructed cost constrained value added functions for the K sectors. We will study two alternative approaches to the problems associated with aggregating over sectors. For brevity, we refer to the aggregate of the K sectors as the national economy.

5. National Value Added Growth Decompositions: The Sectoral Weighted Average Approach

In this first approach to deriving a decomposition of national value added growth into explanatory components, we simply use weighted averages of the sectoral decompositions.

Suppose we have $K \geq 2$ sectors and we apply the sectoral value added decomposition methodology that was explained in section 3 to each sector. Denote the net output vector, primary input vector and the corresponding price vectors for sector k and period t by y^{kt}, x^{kt}, p^{kt}, w^{kt} for $k = 1,\ldots,K$ and $t = 1,\ldots,T$.[22] Denote the period t nominal value added for sector k by $v^{kt} \equiv p^{kt} \cdot y^{kt}$ for $k = 1,\ldots,K$ and $t = 1,\ldots,T$. Define the sector k decomposition

1,...,K and t = 2,...,T. Then the sectoral counterparts to the decompositions (34), (35) and (40) are as follows for k = 1,...,K and t = 2,...,T:

(45) $v^{kt}/v^{k,t\ 1} = D^{kt}\ E^{kt}\ J^{kt}\ G^{kt}\ H^{kt}\ W^{kt}$;
(46) $TFPG^{kt}\ \{\ [v^{kt}/v^{k,t\ 1}]/\ D^{kt}\ E^{kt} = J^{kt}\ G^{kt}\ H^{kt}\ W^{kt}$;
(47) $TFP^{kt} = [v^{kt}/v^{k,1}]/A^{kt}\ B^{kt} = C^{kt}\ D^{kt}\ E^{kt}\ T^{kt}$.

Define the period t national value added v^t as the sum of the period t sectoral value added for each sector:

(48) $v^t\ \{\ 6_{k=1}^{K}\ v^{kt}$; t = 1,...,T.

Define the period t share of national value added for sector k, s^{kt}, as follows:

(49) $s^{kt}\ \{\ v^{kt}/v^t$; k = 1,...,K; t = 1,...,T.

Using definitions (48) and (49), it is easy to see that we have the following exact decomposition of period t national value added growth into sectoral explanatory components:

(50) v^t

The above decompositions are useful if we want a decomposition of aggregate value added growth into sectoral contributions but they do not lead to simple decompositions into national explanatory factors for value added efficiency, output price effects, input quantity effects and so on.[23] In order to accomplish this task, we resort to the use of approximations to the exact decompositions defined by (50) and (51). Before introducing these approximations, we first define period t national weighted averages of the sectoral explanatory factors. The period t logarithms of these national explanatory factors are defined as follows for t = 2,...,T:

(52) $\ln D^{t x} \{ 6_{k=1}^{K} (1/2)(s^{kt} + s^{k,t\ 1})\ln D^{kt}$;
(53) $\ln E^{t x} \{ 6_{k=1}^{K} (1/2)(s^{kt} + s^{k,t\ 1})\ln E^{kt}$;
(54) $\ln J^{t x} \{ 6_{k=1}^{K} (1/2)(s^{kt} + s^{k,t\ 1})\ln J^{kt}$;
(55) $\ln G^{t x} \{ 6_{k=1}^{K} (1/2)(s^{kt} + s^{k,t\ 1})\ln G^{kt}$;
(56) $\ln H^{t x} \{ 6_{k=1}^{K} (1/2)(s^{kt} + s^{k,t\ 1})\ln H^{kt}$;
(57) $\ln W^{t x} \{ 6_{k=1}^{K} (1/2)(s^{kt} + s^{k,t\ 1})\ln W^{kt}$.

Thus $D^{t x}$ is a weighted geometric average of the period t sectoral value added price indexes D^{kt} where the weight for sector k in period t is $(1/2)(s^{kt} + s^{k,t\ 1})$, the arithmetic ri1 98 2aTJ 0n

The approximate national TFP growth decomposition that corresponds to (62) is the following one:

(63) $\mathrm{TFPG}^{t} \{ [v^{t}/v^{t\ 1}]/ D^{tx} E^{tx} \mid W^{tx}$; $\quad$ t = 2,...,T.

K,t1

K t1622 5056>Tj /TT(R)Tj - (e)Tj 7.8 -0 0 7.t6(,t713.52 Tm (k)Tj 0.504 0 Td

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(69) $H\{e^{t}/e^{t}$ K e 1 $= [\,6_{k=1}^{K}\ V^{kt}\, e^{kt}\,]/[\,6_{k=1}^{K}\ V^{k,t\ 1}\, e^{k,t\ 1}\,]$; t = 2,...,T.

K

Ket2 637.2.52 T,w

We now tux.8 708 Tm [12 159.96 708 Tm ( )Tj 0.001 Tc 0.004 Tw 0.255 0 Td (= [)Tj /C2_0 1

The national counterpart to the sectoral indexes defined by (78) is defined as follows:

(80) $W_L^t \equiv R^t(p^{t-1},w^{t-1},x^t)/R^{t-1}(p^{t-1},w^{t-1},x^{t-1})$ 9R15(p>Tj/T1 25d <5C2_.002 TwTw 1.012 -0 10>T8 -4 Tw 2.01 311sp85..00 $t = 2,\ldots,T$

$= \sum_{k=1}^{K} R^{kt}(p^{k,t-1},w^{k,t-1},x^{kt}) / \sum_{i=1}^{K} R^{i,t-1}(p^{i,t-1},w^{i,t-1},x^{it})$

$= \sum$

(84) $G_L^{kt}\ \{[R^{k,t\ \ 1}(p^{k,t\ \ 1},w^{k,t\ \ 1},x^{kt})/R$

(95) $v^t/v^{t\ 1} = D_L^t\ E_P^t\ J_{PLL}^t\ G_P^t\ H^t\ W_P^t$,

(96) $v^t/v^{t\ 1} = D^t\ E^t\ J^t\ G^t\ H^t\ W^t$.

We conclude this section by deriving a useful approximation to the exact decomposition defined by (96).

Recall the exact decompositions for D_L^t and D_P^t defined by (70) and (71). Approximate the theoretical sector k, period t 1 shares $V^{k,t\ 1}$ in (70) by the observed sector k, period t $^{k,t\ 1}$

^{kt}. Then approximate the resulting weighted arithmetic mean $6_{k=1}^{K}\ 6^{k,t\ 1}\ D_L^{kt}$

7. TFP Growth for the U.S. Corporate Nonfinancial Sector, 1960-2014

The US Bureau of Economic Analysis (BEA), in conjunction with the Bureau of Labor Statistics (BLS) and the Board of Governors of the Federal Reserve, have developed a new set of production accounts (the Integrated Macroeconomic Accounts or IMA) for two major private sectors of the US economy: the Corporate Nonfinancial Sector and the Noncorporate Nonfinancial Sector. The Balance Sheet Accounts in the IMA cover the years 1960-2014 but do not provide a decomposition of output, input and asset values into price and quantity components. Diewert and Fox (2016a) provided such a decomposition and we will use their data in this study.

In this section, we will use their output and input data for the U.S. Corporate Nonfinancial Sector (which we denote as Sector 1) for the 55 years 1960-2014. The year t output y^{1t} is real value added[41] and the corresponding year t value added deflator is denoted as p^{1t}. The ten inputs used by this sector are labour and the services of nine types of asset.[42] The output and input data are listed in Appendix A of Diewert and Fox (2016b). The year t input vector for this sector is $x^{1t} \equiv [x_1^{1t},x_2^{1t},...,x_{10}^{1t}]$ where x_1^{1t} is year t labour input measured in billions of 1960 dollars and $x_2^{1t},...,x_{10}^{1t}$ are capital service inputs measured in billions of 1960 capital stock dollars. The corresponding year t input price vector for Sector 1 is $w^{1t} \equiv [w_1^{1t},w_2^{1t},...,w_{10}^{1t}]$ for t = 1960,...,2014.

Our year t technology set for Sector 1, S^{1t}, is defined as the free disposal cone spanned by the observed output and input vectors for the sector up to and including the year t observation. For convenience, we label the years 1960-2014 as years 1-55 in definitions (100)-(104) below. Thus S^{1t} is defined as follows:

$$(100)\quad S^{1t} \equiv \{(y,x): y \le \textstyle\sum_{s=1}^{t} y^{1s}\lambda_s,\ x \ge \sum_{s=1}^{t} x^{1s}\lambda_s,\ \lambda_1 \ge 0, ..., \lambda_s \ge 0\}; \qquad t = 1,...,55.$$

We adapt definition (41) of section 4 to the present situation and define the Sector 1 year t

Using the cost constrained value added functions defined by (101), we can readily calculate the Sector 1 counterparts to the year t generic value added growth decompositions (32)-(33) that we derived in section 3 above. Using our present notation for the Sector 1 prices and quantities, these decompositions can be written as follows for t = 2,...,55:[43]

(102) $v^{1t}/v^{1,t\ 1} = p^{1t}y^{1t}/p^{1,t\ 1}y^{1,t\ 1} = H^{1t} D_P^{1t} E_L^{1t} J_{LPP}^{1t} W_L^{1t}$;
(103) $v^{1t}/v^{1,t\ 1} = H^{1t} D_L^{1t} E_P^{1t} J_{PLL}^{1t} W_P^{1t}$;
(104) $v^{1t}/v^{1,t\ 1} = H^{1t} D^{1t} E^{1t} J^{1t} W^{1t}$.

As in section 3, we define year t Total Factor Productivity Growth for Sector 1 as value added growth divided by output price growth D^{1t} times input quantity growth E^{1t}:

(105) $TFPG^{1t} \{ [v^{1t}/v^{1.t\ 1}]/[D^{1t} E^{1t}] = H^{1t} J^{1t} W^{1t}$; t = 1961,...,2014.

Since we have only a single value added output, $D^{1t} \{ p^{1t}/p^{1,t\ 1}$ can be interpreted as a Fisher output price index and $[v^{1t}/v^{1.t\ 1}]/D^{1t}$ can be interpreted as a Fisher output quantity index going from year t 1 to year t. E^{1t} is the Fisher input quantity index going from year t 1 to year t. Thus $TFPG^{1t}$ is equal to a conventional Fisher productivity growth index in this one output case.

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It can be verified that the TFP growth decomposition defined by (105) holds; i.e., for each year t, nonparametric TFP growth $TFPG^t$ equals the product of value added efficiency growth H^{1t} times the year t input mix growth factor J^{1t} times the year t technical progress measure W^{1t}. It can be seen that the input mix factors are all very close to one. It can also be seen when value added efficiency in year t, e^{1t}, is less than one, then the year t technical progress measure W^{1t} always equals one so that there is no t4.9 T66i(ci) pioduow4(a)-1(s)-1(

Table 2: U.S. Corporate Nonfinancial Value Added Year t Levels $v^{1t}/v^{1.1960}$, Output Price Levels A^{1t}, Input Quantity Levels B^{1t}, TFP Levels TFP^{1t}, Input Mix Levels C^{1t}, Value Added Efficiency Levels E^{1t} and Technical Progress Levels T^{1t} where all Levels are Relative to 1960

Year t	$v^{1t}/v^{1,1960}$	A^{1t}	B^{1t}	TFP^{1t}	C^{1t}	E^{1t}	T^{1t}
1960	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1961	1.02696	1.00305	1.00469	1.01906	1.00000	1.00000	1.01906
1962	1.12113	1.00954	1.03945	1.06839	1.00000	1.00000	1.06840
1963	1.19615	1.01454	1.06476	1.10730	1.00000	1.00000	1.10730
1964	1.29190	1.02388	1.09407	1.15329	1.00000	1.00000	1.15330
1965	1.42514	1.04188	1.14350	1.19620	1.00000	1.00000	1.19622
1966	1.57518	1.07230	1.20194	1.22217	1.00000	1.00000	1.22220
1967	1.65648	1.09623	1.23486	1.22368	1.00000	1.00000	1.22370
1968	1.81864	1.13024	1.27771	1.25934	1.00000	1.00000	1.25937
1969	1.97106	1.17785	1.32956	1.25864	0.99994	0.99950	1.25937
1970	2.02656	1.22161	1.32822	1.24899	0.99888	0.99287	1.25937
1971	2.18245	1.26574	1.33802	1.28866	0.99894	1.00000	1.29004
1972	2.43141	1.31076	1.39306	1.33157	0.99894	1.00000	1.33300
1973	2.73032	1.38762	1.45963	1.34803	0.99894	1.00000	1.34947
1974	2.95307	1.52395	1.47611	1.31275	0.99860	0.97416	1.34947

Note that the final level of TFP in 2014, 2.46873, is slightly less than the level of technology in 2014, which was 2.47559. This small difference is explained by the fact that the cumulative input mix level, 0.99725, is slightly less than 1 in 2014. We plot TFP^{1t}, C^{1t}, E^{1t} and T^{1t} in Figure 1.

It can be seen that there was a substantial decline in value added efficiency over the years 2006-2009 and in fact, TFP has grown at a slower than average rate since 20068 0 Td 57y0 Td(e)y in 20
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Table 4:

Table 5: U.S. National Nonfinancial Value Added Growth $V^t/V^{t\;1}$, Output Price Growth $D^{t x}$, Input Quantity Growth $E^{t x}$, TFP Growth $TFPG^{t x}$, Input Mix Growth Factors $J^{t x}$, Value Added Efficiency Growth

From Table 6, it can be seen that the final level of approximate national TFP relative to 1960 was 2.33486 which is somewhat below the final level of technology, which was 2.35219. This difference is explained by the below unity final level of the input mix explanatory factor, C^{2014}

As was noted in the previous section, the year t sector k explanatory growth factors, D^{kt}, E^{kt}, J^{kt}, H^{kt} and W^{kt}, are listed in Tables 1 and 3 above. Now apply definitions (69)-(92) in order to define the year t national growth factors, D^t, E^t, J^t, G^t, H^t and W^t. The year t exact decomposition of national value added growth is given by (96), which we repeat here as equations (113):

$$(113)\quad v^t/v^{t\ 1} = D^t E^t J^t G^t H^t W^t ; \qquad t = 1961,...,2014.$$

Define the national year t TFP growth factor as $TFPG^t \{ [v^t/v^{t\ 1}]/D^t E^t$. This definition and equations (113) imply that we have the following national exact TFP growth decomposition into explanatory factors:

$$(114)\quad TFPG^t \{ [v^t/v^{t\ 1}]/D^t E^t = J^t G^t H^t W^t ; \qquad t = 1961,...,2014.$$

The growth decomposition components that appear in (114) are listed in Table 7.

The arithmetic means of the growth rates over the 54 years 1961-2014 are listed in the last row of Table 7.[52] It can be seen that the average rate of national TFP growth over these years was 1.59 percent per year, just below the average rate of technical progress, which was 1.60 percent per year. The results in Table 7 are very similar to the results listed in Table 5 above. Again, technical progress growth was not present for 8 years: 1974, 1975, 1979, 1980, 1982, 1989, 2007 and 2009. For those years, the rate of growth of value added efficiency was below unity and this translated into negative rates of TFP growth for those years. The national exact input mix and returns to scale growth factors, the J^t and G^t, are all very close to unity. The exact equation (114) was not quite exact due to rounding errors: the absolute value of the difference between $TFPG^t$ and $J^t G^t H^t W^t$ was always less than 0.00003 and the mean difference was -0.0000027.

To conclude this section, apply definitions (37)-(40) to national value added in order to obtain the following exact levels decomposition for national Total Factor Productivity in year t relative to the year 1960, TFP^{t*}:

$$(115)\quad TFP^t = [v^t/v^{1960}]/[A^t B^t] = C^t D^t E^t T^t ; \qquad t = 1960,...,2014.$$

Table 8 lists the various levels that appear in (115) along with the variable $Check^t$, which is defined as TFP^t minus $C^t D^t E^t T^t$. It can be seen that $Check^t$ is quite small although the rounding errors build up as time goes on.

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[52] The mean for V^{1t} is for all 55 observations.

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