Decomposing Value Added Growth Over Sectors into Explanatory Factors

Figure 2: Sector 2 Level of TFP, Input Mix, Value Added Efficiency and Technology

Cost Constrained Value Added Function for a Sector

- A sector produces M net outputs, $y \sum [y_1,...,y_M]$, using N primary inputs $x \sum [x_1,...,x_N] \varnothing 0_N$.

- If $y_m > 0$, then the sector produces the mth net output during period t while if $y_m < 0$, then the sector uses the mth net output as an intermediate input.

- Strictly positive vector of net output prices $p \sum [p_1,...,p_M] >> 0_M$ and strictly positive vector of input prices $w \sum [w_1,...,w_N] >> 0_N$

- *Period t production possibilities set* for the sector S^t

Cost Constrained Value Added Function for a Sector

Period t cost constrained value added function:

$$R^t(p,w,x) \sum \max_{y,z} \{p \lceil y : (y,z) \subset S^t; w \lceil z \, \Omega \, w \lceil x\}$$

$R^t(p,w,x)$ is well defined even if there are increasing returns to scale in production; i.e., the constraint $w \lceil z \, \Omega \, w \lceil x$ leads to a finite value for $R^t(p,w,x)$.

If (y^*,z^*) solves this constrained maximization problem, then sectoral value added $p \lceil y$ is maximized subject to the constraints that (y,z) is a feasible production vector and primary input expenditure $w \lceil z$ is equal to or less than "observed" primary input expenditure $w \lceil x$.

Cost Constrained Value Added Function for a Sector

Observed value added, $p^t \int y^t$, may not equal the optimal value added.

Value added efficiency of the sector during period t:

$$e^t \sum p^t \int y^t / R^t(p^t, w^t, x^t) \Omega 1$$

The cost constrained valued added function has some interesting properties. If S^t is a cone, so that production is subject to constant returns to scale, can show that

Cost Constrained Value Added Function for a Sector

A brief digression to prove this result

Value added function:

$M^t(p,x) \equiv \max_y \{p \cdot y : (y,x) \in S^t\}$.

The cost constrained value added function $R^t(p,w,x)$ has the following representation:

$$R^t(p,w,x) \equiv \max_{y,z} \{p \cdot y : (y,z) \in S^t; w \cdot z \leq w \cdot x;\}$$
$$= \max_z \{M^t(p,z) : w \cdot z \leq w \cdot x; z \geq 0_N\}.$$

- Holding p constant, we can define the period t *"utility" function* $f^t(z) \equiv M^t(p,z)$.

Cost Constrained Value Added Function for a Sector

Then we have the following “utility” maximization problem:

$R^t(p,w,x) = \max_z \{f^t(z) : w\lceil z \Omega w\lceil x; z \varnothing 0_N\}$

where $w\lceil x$ is the consumer’s “income”.

For u in the range of $M^t(p,z)$ over the set of nonnegative z vectors and for $w >> 0_N$, we can define the *cost function* $C^t(u,w)$ that corresponds to $f^t(z)$ as follows:

$C^t(u,w) \sum \min_z \{w\lceil z : f^t(z) \varnothing u; z \varnothing 0_N\} = \min_z \{w\lceil z : M^t(p,z) \varnothing u; z \varnothing 0_N\}$.

Cost Constrained Value Added Function for a Sector

If $M^t(p,z)$ increases as all components of z increase, then $C^t(u,w)$ will be increasing in u and we can solve the following maximization problem for a unique u^*:

$$\max_u \{u: C^t(u,w) \Omega w\lceil x\}$$

$$R^t(p,w,x) = u^* \text{ with } C^t(u^*,w) = w\lceil x.$$

The above formulae simplify considerably if S^t is a cone, so that production is subject to constant returns to scale:

- $M^t(p,z)$ is linearly homogeneous in z and hence, so is $f^t(z) \sum M^t(p,z)$.

Decomposing Value Added Growth for a Sector into Explanatory Factors

Change in value added efficiency

$$\kappa^t \sum e^t/e^{t41} = [p^t \int y^t/R^t(p^t,w^t,x^t)]/[p^{t41} \int y^{t41}/R^{t41}(p^{t41},w^{t41},x^{t41})]$$

If $\kappa^t > 1$, then value added efficiency has *improved* going from period t41 to t whereas it has *fallen* if $\kappa^t < 1$.

Decomposing Value Added Growth for a Sector into Explanatory Factors

Family of *input quantity indexes*:

$$\eta(x^{t41},x^{t},w) \sum w\lceil x^{t}/w\lceil x^{t41}.$$

$\eta_L{}^t \sum w^{t41}\lceil x^t/w^{t41}\lceil x^{t41}$;
$\eta_P{}^t \sum w^t\lceil x^t/w^t\lceil x^{t41}$.

Preferred overall measure of input quantity growth:

$\eta^t \sum [\eta_L{}^t \eta_P{}^t]^{1/2}$.

Decomposing Value Added Growth for a Sector into Explanatory Factors

Family *of (global) returns to scale measures:*

$$\iota(x^{t41},x^{t},p,w,s) \sum [R^{s}(p,w,x^{t})/R^{s}(p,w,x^{t41})]/[w\int x^{t}/w\int x^{t41}].$$

$$\iota_{L}{}^{t} \sum \iota(x^{t41},x^{t},p^{t41},w^{t41},t41) \sum R^{t41}(p^{t41},w^{t41},x^{t})/R^{t41}(p^{t41},w^{t41},x^{t41})]/[w^{t41}\int x^{t}/w^{t41}\int x^{t41}];$$

$$\iota_{P}{}^{t} \sum \iota(x^{t41},x^{t},p^{t},w^{t},t) \sum [R^{t}(p^{t},w^{t},x^{t})/R^{t}(p^{t},w^{t},x^{t41})]/[w^{t}\int x^{t}/w^{t}\int x^{t41}].$$

$$\iota^{t} \sum [\iota_{L}{}^{t}\, \iota_{P}{}^{t}]^{1/2}$$

Decomposing Value Added Growth for a Sector into Explanatory Factors

Six explanatory growth factors:

1. Change in cost constrained value added efficiency: $\kappa^t \sum e^t/e^{t41}$
2. Change in output prices: $\zeta(p^{t41},p^t,w,x,s)$
3. Change in input quantities: $\eta(x^{t41},x^t,w)$
4. Change in input prices: $\nu(w^{t41},w^t,p,x,s)$
5. Changes due to technical progress: $\vartheta(t41,t,p,w,x)$

A Nonparametric Approximation to the Cost Constrained Value Added Function

National Value Added Growth Decompositions: The Sectoral Weighted Average Approach

National Value Added Growth Decompositions: The Sectoral Weighted Average Approach

Use some approximations (drawing on Schlömilch's inequality) to write:

$$\ln v^{t}/v^{t41} - \mathrm{O}_{k=1}^{K} (1/2)(s^{kt} + s^{k,t41})\ln(v^{kt}/v^{k,t41})$$

$$= \mathrm{O}_{k=1}^{K} (1/2)(s^{kt} + s^{k,t41})\ln(\zeta^{kt}\,\eta^{kt}\,\nu^{kt}\,\iota^{kt}\,\kappa^{kt}\,\vartheta^{kt})$$

$$= \ln \zeta^{t¢} + \ln \eta^{t¢} + \ln \nu^{t¢} + \ln \iota^{t¢} + \ln \kappa^{t¢} + \ln \vartheta^{t¢}$$

National Total Factor Productivity Growth:

$$\mathrm{TFPG}^{t} \sum [v^{t}/v^{t41}]/\zeta^{t¢}\,\eta^{t¢} - \nu^{t¢}\,\iota^{t¢}\,\kappa^{t¢}\,\vartheta^{t¢}$$

National Value Added Growth Decompositions: The Sectoral Weighted Average Approach

- Assume that the technology of each sector can be represented by a translog value added function with the restrictions on technical progress that are described in Diewert and Morrison (1986) and Kohli (1990).
- These papers also assumed constant returns to scale and competitive profit maximizing behavior. Under these assumptions:

$$v^{kt}/v^{k,t41} = \zeta^{kt}$$

National Value Added Growth Decompositions: The Sectoral Weighted Average Approach

National Value Added Growth Decompositions: The National Cost Constrained Value Added Function Approach

Sector k share of national best practice value added in period t:

$\omega^{kt} \sum R^{kt}(p^{kt},w^{kt},x^{kt})/R^{t}(p^{t},w^{t},x^{t})$

National efficiency Level:

$e^{t} \sum v^{t}/R^{t}(p^{t},w^{t},x^{t})$

$= O_{k=1}^{K} \omega^{kt} e^{kt}$

National efficiency change:kt

National Value Added Growth Decompositions: The National Cost

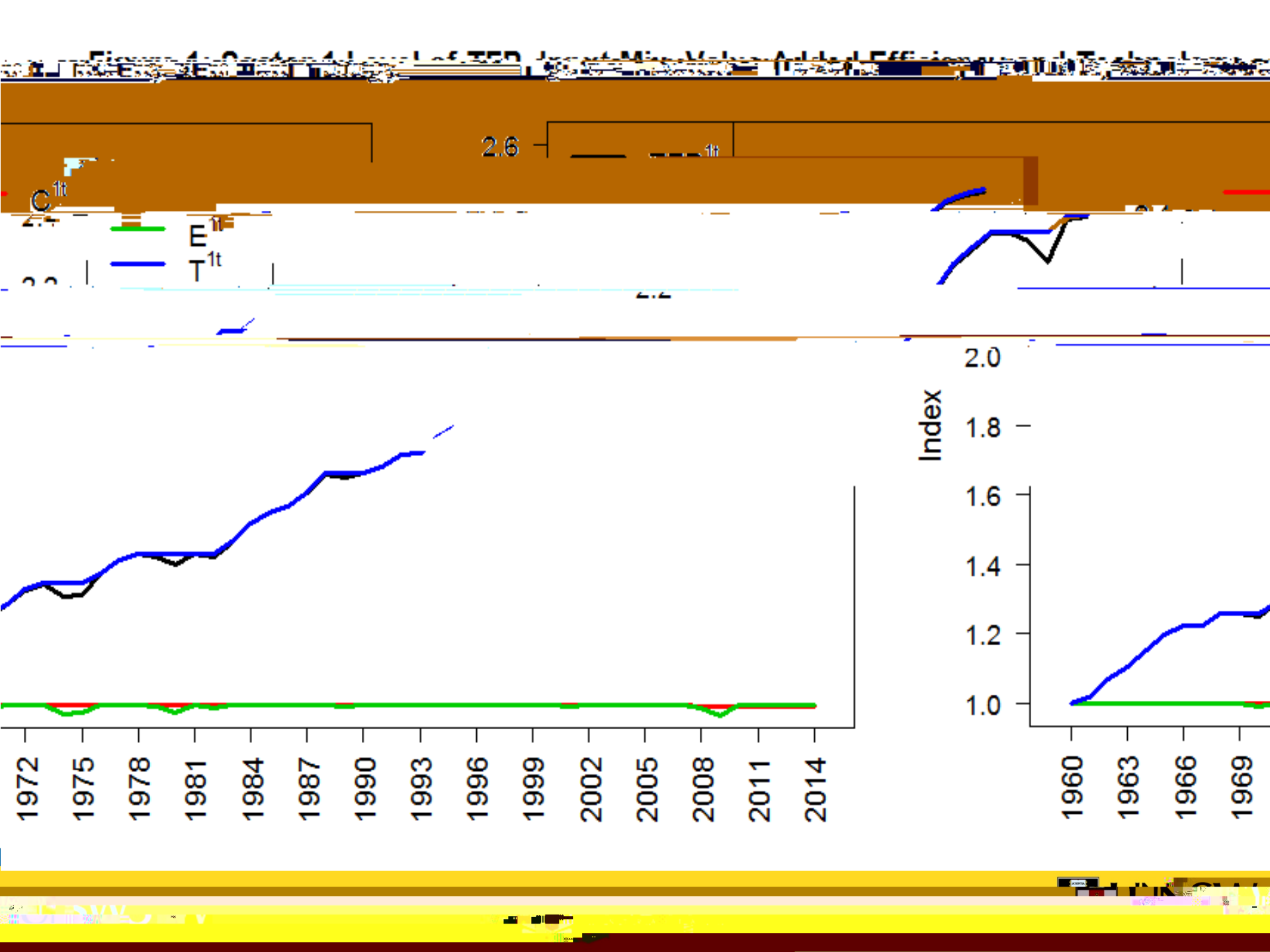
TFP Growth for the U.S. Corporate Nonfinancial Sector, 1960-2014

Use the (BEA, BLS, Fed Reserve) Integrated Macroeconomic Accounts to construct a data set for two major sectors of the U.S. economy in Diewert and Fox (2016) :

“Alternative User Costs, Rates of Return and TFP Growth Rates for the US Nonfinancial Corporate and Noncorporate Business Sectors: 1960-2014”

Sector 1: US Corporate Nonfinancial Sector

Sector 2: US Noncorporate Nonfinancial Sector


2.6
C 1t
E 1t
T 1t
Index
2.0
1.8
1.6
1.4
1.2
1.0
1972
1975
1978
1981
1984
1987
1990
1993
1996
1999
2002
2005
2008
2011
2014
1960
1963
1966
1969

TFP Growth for the U.S. Corporate Nonfinancial Sector, 1960-2014

- There was a substantial decline in value added efficiency over the years 2006-2009
- TFP has grown at a slower than average rate since 2006. The level of TFP also fell in the 1974, 1979, 1982, 1989 and 2001 recessions when efficiency growth dipped below one.
- On the whole, TFP growth in the U.S. Corporate Nonfinancial Sector has been satisfactory.

Figure 2: Sector 2 Level of TFP, Input Mix, Value Added Efficiency and Technology

Figure 3: Approximate National Level of TFP, Input Mix, Value Added Efficiency and Technology

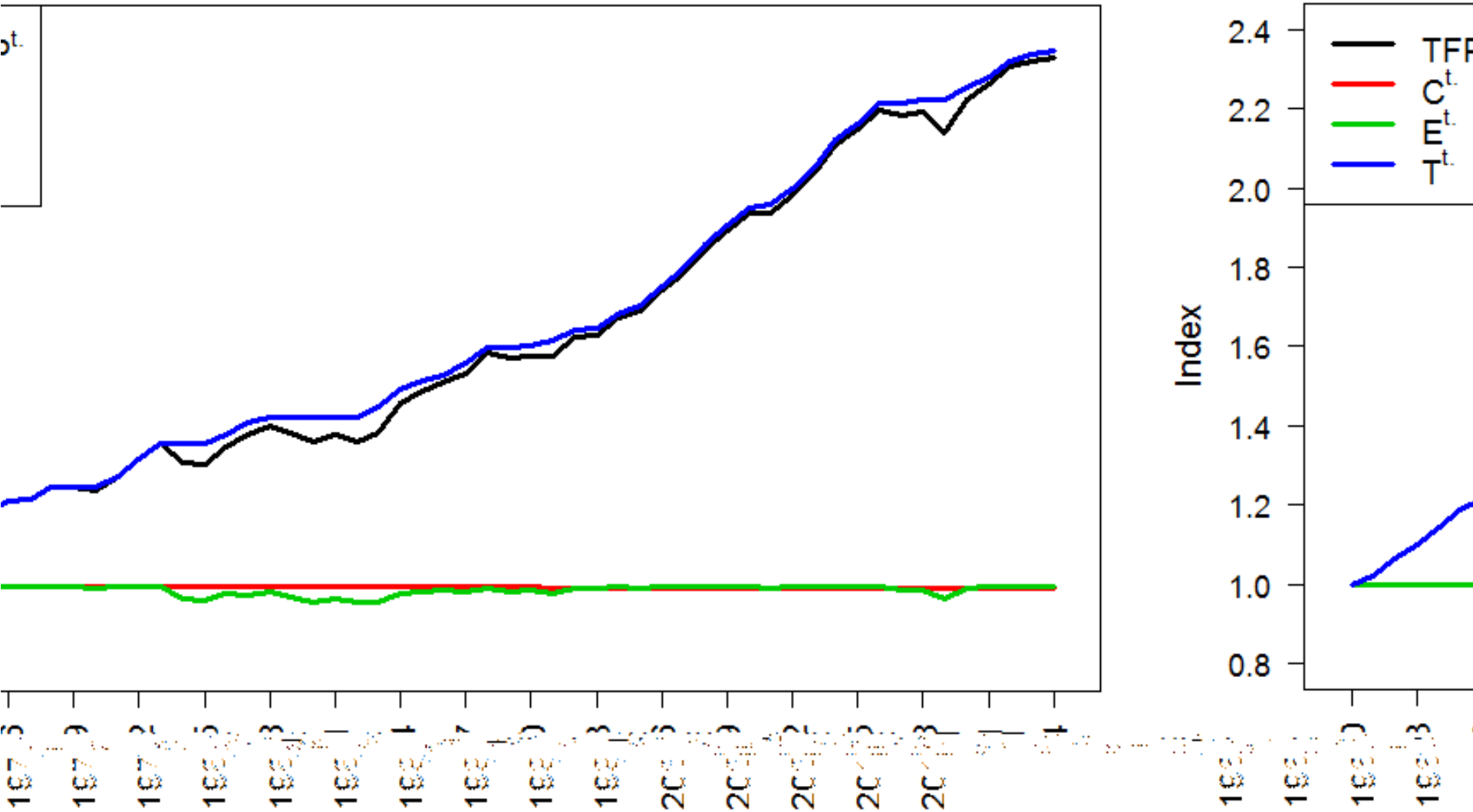
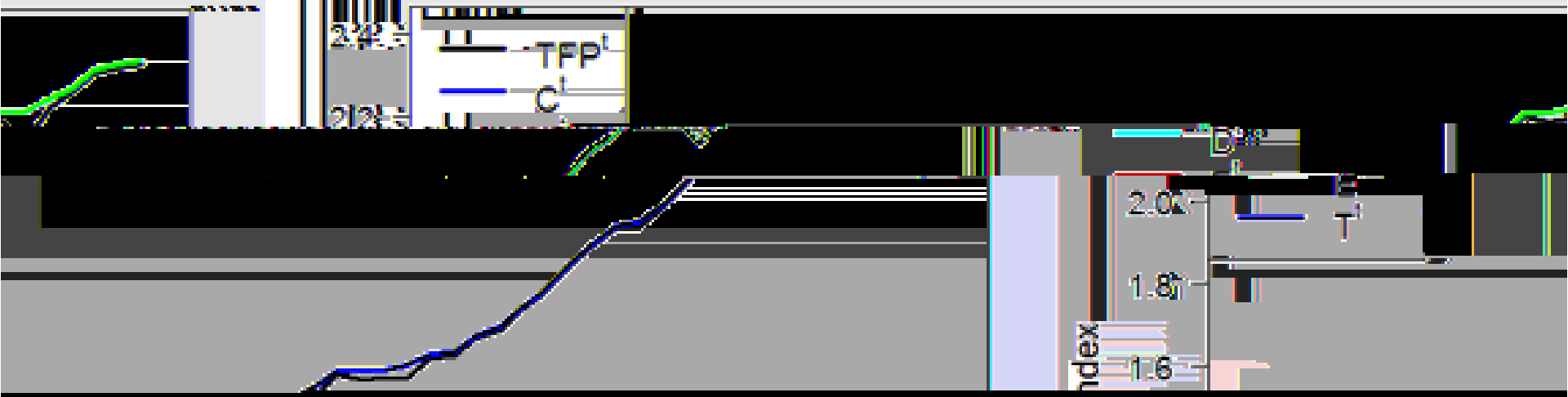


Figure 4: National Level of TFP, Input Mix, Returns to Scale,
and Productivity Growth



Summary

- Derived decompositions of nominal value added growth (and TFP growth) for a single sector into explanatory factors.
- We also used two alternative approaches to relating the sectoral decompositions to a national growth decomposition:
  - a weighted average sectoral approach and
  - a national value added function approach.
- A main advantage of our new approach is that our new nonparametric measure of technical progress never indicates technical regress.
- During recessions, value added efficiency drops below unity and depresses TFP growth.

Summary

- For our U.S. data set, TFP growth is well explained as the product of