

MMAN9001

A $UghYf^*cZ9b[]bYYf]b[~GW]YbWW^*Dfc^YWh5$

HYfa 'H\fYY''##'&\$&\$

7ci fgY⁻Cj Yfj]Yk

GHJZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWUh]cb	D\cbY
Fcb [.] 7\Ub	f"W. Ub4 i bgk "YXi "Ui	#i]g fyWca a ybxyx mci 'Ya U] 'h\y Wci fgy Wcbj ybcf h a U_Y 'U'gdyWJZJW Uddc]bha ybh]Zmci byyx hc X]gW gg Ub]a dcffUbh]ggi yg"	5]bgk cfh\ 6i]`X]b[-',)%)')

GWXcc`7cbHJWhi±bZcfa Uh]cb

@cWUh]cb

I BGK 'A YVX.Ub]\\\U`\UbX\'A Ubi \\\U\\\forall f]\b[\'9b[]\bYYf]\b[

5]bgk cft\\ 'Vi]`X]b['>%+ž'@Yj Y`'%

5Vcj Y 7 c ZZYY cb 7 Ua di g

<ci fg

-.\$\$-).\$\$da ž'A cbXUm-: f]XUm

K YV

GWXcc``cZA YWXUb]WU``UbX`A Ubi ZUWi f]b['9b[]bYYf]b[

 $f\tilde{Z}^{*}\%\&E^{-}\text{'}\text{'},\text{)}\text{'}(\$-+\text{'}\text{-}\text{'}GVWcc``C}Z\!J\!WHH$

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

D; '7 ci fgY'K cf_'Dfc^YWi]g'i gi U``mWta d`YhYX']b'ik c'WtbgYW h]j Y'hYfa g'Xi f]b['h\Y``UghUWUXYa]WmYUf'' Ghi XYbhg'k]```k cf_']b'hYUa g'hc'WtbXi Wi'U'fYgYUfW.'VUgYX'dfc^YWi'H\Y'dfc^YWi'WtbHJ]bg'U'`Uf[Y'Ua ci bhcZ cf][]bU'`fYgYUfW.'UbX#cf'bcj Y``XYg][b'k cf_'cf'UbU'mg]g''' \delta']g'bchh\Y'fYgdcbg]V]`]micZh\Y'gi dYfj]gcf'hc'hY'``h\Y'ghi XYbh'hYUa 'k \Uh'hc'Xcž'bcf'g\ci `X']h'VY'Uggi a YX'h\Uh'h\Y'gi dYfj]gcf']g'Ub'YI dYfh]b'U```UfYUg'cZ Yb[]bYYf]b["''H\YmUfY'h\YfY'hc'cZYf'[i]XUbW''UbX'UXj]W''zUg'UfY'``UVcfUhcfmigHJZz'k cf_g\cd'ghJZz'UbX

H\YfY`]gʻbcʻZcfa U``hYUW\]b[`Vi hih\Y`ghi XYbhg``YUfb`Zfca `Vch\`]bhYfbU``UbX`YI hYfbU``gci fWYg"``H\Y gi dYfj]gcfž'ch\Yf`UWUXYa]WgʻUbX``UVcfUhcfm#k cf_g\cd`ghUZZUfY`h\Y`]bhYfbU``gci fWYgžik \YfYUgʻh\Y`@]VfUfmž]bhYfbYhUbX`]bXi ghfma YbhcfgʻUfY`h\Y`YI hYfbU``gci fWYg"`H\YfY`UfY`bc`gYh`YWh fYgʻZcf`h\]g`Wci fgYž'Vi hU bi a VYf`cZk cf_g\cdg`k]```VY`dfcj]XYX`hc`Ugg]ghighi XYbhgʻhc`Wca d`YhY`h\Y]f`h\Yg]g`hc`U`\][\`ghUbXUfX"`5`` k cf_g\cdg`k]```VY`fYWcfXYX`UbX`a UXY`Uj U]`UV`Y`hc`qhi XYbhgʻcb`]bY"

5ggYgga Ybh

5ggYgga YbhHUg_g

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
Dfc[fYgg'9jU'i Uh]cb'k]h\ A Ybhcfg'	%) ।	K YY_`m	%ž'&ž' ž'(ž')
8Y`]j YfUV`Yg`hc`A ccX`Y ; fci d`: cfi a `	%) ।	K YY_`m	%ž'&ž' ž'(ž')
Dfc[fYgg'F Ydcfh'	+\$1	&' #%%#&\$&\$`%&.\$\$`5A	%ž [*] .&ž [*] ' ž [*] (ž [*])

5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% Dfc[fYgg 9j Ui Uh]cb k]h\ A Ybhcfg

GhUfhiXUhY. K YY_`m

@Yb[h\." \$ a]bi hYg

8YHJ]`g.

Ghi XYbhihYua 'Webxi Wiu'zJVW!hc!ZJVW'cf'cb`]bY']bhYfj]Yk 'fdfc[fYgg'WXYW_Łk]h\'h\Y'Wei fgY'WecfX]bUhcf UbX'hi hcfg'hc'X]gW gg'h\Y]f'dfcYWigWedYž'dfcV'Ya 'ghUhYa YbhUbX'a Yh\cXc`c[mhc'gc'j Y'h\Y'dfcV'Ya " Ghi XYbhihYua g'[Yh]bghUbh'ZYYXVUW_'UZhYf'h\Y'Yj U'i Uh]cb'gYgg]cb

5ggYgga Ybh& 8Y`]j YfUV`Yg hc A ccX`Y; fci d: cfi a

GHUfhXUhY. K YY `m

8YHJ]`q.

HYUa 'gi Va]hg'Ub'U'[fci d'XcW a Ybhhc'A ccX'Y 'hc'dfcj]XY'k YY_`mi dXUhY'cb'h\Y]f'dfc[fYgg"

5ggYgga Ybh' . 'Dfc[fYgg F Ydcfh

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g.

 $Gh\ XYbhgi\ Va\]hg`Ub`]bX]j\]Xi\ U``fYdcfh!\ ci\ h]b]b[\ `_YmZ]bX]b[\ g`cZDfc\,\Upsilon Wh5\ `UbX`dfcXi\ WF`U`XYhU]`YX`d`Ub`ZcfDfc\,\Upsilon Wh6\ `]b`h\Y`bYI\ hhYfa\ "$

FYgci fWfg

DfYgWf]VYX F Ygci fWYg

Bch'Uj U]`UV`Y

FYWca a YbXYX FYgci fWYg

7 cbh/bhcb h\Y 'Wci fgY 'A ccX`Y 'dU[Y 'k]```VY 'i dXUh/X`cZh/b`k]h\ 'h]dgžX]gW gg]cbg 'UbX`fYgci fW/gžgc 'mci UfY`ghfcb[`mUXj]gYX`hc`a U_Y`gi fY`mci 'W\YW_'Zcf'U```i dXUh/y"

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

± 'h\]g'Wci fgYžfYWYbh']a dfcj Ya Ybhg'fYgi `h]b['Zfca 'ghi XYbhZYYXVUW_']bWi XY'\Uj]b['ga U``Yf'hYUa 'g]nYg hc']a dfcj Y'ghi XYbhg!a Ybhcf'Yb[U[Ya Ybh'

@UVcfUhcfmK cf_g\cd \DZcfa Uh]cb

B#5

UggYgga YbhidYfZcfa UbWYžmci 'UfY'Y`][]V`Y`hc'Udd`m'Zcf'GdYWJU``7cbg]XYfUhJcb'df]cf'hc'gi Va]hhJb['UbUggYgga Ybhicf'g]hhJb['Ub'Yl Ua"

: cf`XYhJ]`g`cZUdd`m]b['Zcf`GdYW]U``7 cbg]XYfUh]cb`UbX`WcbX]h]cbg`Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhUfm UggYgga Ybhž'd`YUgY`gYY`h\Y`]bZcfa Uh]cb`cb`I BGK 'g`<u>GdYWJU``7 cbg]XYfUh]cb`dU[Y</u>"

D`YUgY'bchY'h\Uhghi XYbhg'k]``bch'VY'fYei]fYX'hc'dfcj]XY'Ubm'XcW'a YbhUfm'Yj]XYbWY'hc gi ddcfh'UVgYbWg'Zfca 'Ubm'WUggYg'a]ggYX'**VYWUi gY'cZ'7CJ=8!%** 'di V`]W\YU'h\ 'a YUgi fYg'gi W\'Ug]gc`Uh]cb"'| BGK 'k]``bch'VY']bg]gh]b['cb'a YX]WU`'VW'fh]ZJWUh'Yg'Zfca 'UbmcbY'XYYa YX'hc'VY'U'dcg]h]j Y'WUgYZ cf'k\Yb'h\Ym\Uj Y'fYWcj YfYX"'Gi W\'WYfh]ZJWUh'Yg'UfY'X]ZZJW 'hhc'cVhU]b'UbX'di h'Ub'i bbYWYggUfmghfU]b'cb ghi XYbhg'UbX'a YX]WU`'ghUZZ''

 $5 \ dd^{\wdf} \ duff \ duff$

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[`U`W`hifY`cZ`YUfb]b[`]bZcfaYX`VmUWUXYa]W]bHY[f]hm'`5``IBGK ``ghiXYbhg`\UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'`D`U[]Uf]gaibXYfa]bYg`UWUXYa]W]bHY[f]hmUbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVW{\WSeaXafZWbES`VbSee[YfZW_aX\Sekagdai`ž$

D`U[]Uf]ga `]g`U`hmdY`cZ]bhY``YWi U``h\YZh'`±iWUb`HU_Y`a UbmZcfa gžZfca `XY`]VYfUhY`W\YUh]b[`hc`UWV]XYbHU``m Wcdm]b[`Zfca `U`gci fWYk]h\ci hUW_bck `YX[Ya Ybh'`I BGK `\Ug`dfcXi WYX`U`k YVg]hY`k]h\`U`k YU`h\`cZ fYgci fWg`hc`gi ddcfhighi XYbhg`hc`i bXYfgHUbX`UbX`Uj c]X`d`U[]Uf]ga žij]g]h`ghi XYbh'i bgk "YXi "Ui #d`U[]Uf]ga "H\Y`@YUfb]b[`7 YbhfY`Ugg]ghg`ghi XYbhg`k]h\`i bXYfgHUbX]b[`UWUXYa]W]bhY[f]hmUbX`\ck 'bchhc'd`U[]Uf]gY" H\YmU`gc`\c`X`k cf_g\cdg`UbX`WUb`\Y`d`ghi XYbhg`cbY!cb!cbY"

Mci 'UfY'U'gc'fYa]bXYX'h\Uh'WUfYZ\ ``h]a Y'a UbU[Ya Ybh']g'Ub']a dcfhUbhdUfhcZgh XmUbX'cbY'cZh\Y]XYbh]Z]YX'WUi gYg'cZd'U[]Uf]ga ']g'dccf'h]a Y'a UbU[Ya Ybh'Gh XYbhg'g\ci `X'U``ck 'gi ZZ]W]Ybh'h]a Y'Zcf

5 WUXYa]W±bZcfa Uh]cb

7 fYX]hdc]bhg

7 ci fgY'WfYX]hi]g'WU'W'`UhYX']b'I b]hg!C Zi 7 fYX]hifl C 7 L"'H\Y'bcfa U``k cf_`cUX'YI dYWfUh]cb'Zcf'cbY'I C 7]g'UddfcI]a UhY`m'&) '\ci fg'dYf'hYfa "'H\]g']bWi XYg'WUgg'WcbhUWh'\ci fgž'df]j UhY'ghi Xmž'ch\Yf'`YUfb]b[UWfj]]h]Ygž'dfYdUfUh]cb'UbX'h]a Y'gdYbh'cb'U``'UggYggUV`Y'k cf_"

A cgh'Wci fgYk cf_'Wci fgYg'Uhl BGK 'UfY'*'I C 7 'UbX']bj c`j Y'Ub'Ygh]a UhYX'%) \$`\ci fg'hc'Wca d`YhY'''9UW\ Wci fgY']bWi XYg'U'dfYgWf]VYX'bi a VYf'cZ\ci fg'dYf'k YY_'fl\#\ Ł'cZgW\YXi `YX'ZJ\WY!hc!ZJ\W''UbX#cf'cb`]bY WcbhJ\W''5bmUXX]h]cbU`'hja Y'VYmcbX'h\Y'dfYg\fy]VYX'WcbhJ\Wi\ci fg'g\ci `X'VY'gdYbh]b'a U_]b['gi fY'h\Uhmci ii bXYfghJbX'h\Y''YWfi fY'a UhYf]U'ž'Wca d'Yh]b['h\Y'gYhUgg][ba Ybhgž'Z fh\Yf'fYUX]b['ž'UbX'fYj]g]b['Zcf UbmYl Ua]bUh]cbg"

Cb!WUa di g WUgg UHYbXUbW

Di V`]WX]gHJbW]b[`WcbX]h]cbg'a i gh'VY`Zc``ck YX`Zcf'U```H' `ZJ\WY!hc!ZJ\WY`WUggYg":Hc`Ybgi fY`h\]gž'cb`m ghi XYbhg`Ybfc``YX`]b`h\cgY`WUggYg'k]```VY`U``ck YX`]b`h\Y`fcca "`7`Ugg`fcghYfg`k]```VY`UhtUW\YX`hc WcffYgdcbX]b[`fcca g`UbX`W]fW``UhYX`Ua cb[``UV`XYa cbghfUhcfg"'Bc'cj Yf!Ybfc`a Ybh]g`U``ck YX`]b`ZJ\WY!hc! ZJ\WY`WUgg"'Ghi XYbhg`Ybfc``YX`]b`cb`]bY`WUggYg`WJb`gk Ud`h\Y]f`Ybfc`a YbhZca 'cb`]bY`hc'ch\Yf`UXX]h]cbU`ž' Vi h``]a]hYXž'bi a VYf`cZcb!WJa di g`WUggYg`VmGi bXUmž'K YY_`%"D`YUgY`fYZYf`hc`mci f`Wci fgYfg`A]\WcgcZh HYUa g`UbX`A ccX`Y`g]hYg`Zcf'a cfY`]bZcfa Uh]cb`UVci h`WUgg`UhhYbXUbWY`Zcf']b!dYfgcb`UbX`cb`]bY`WUgg gY\W]cbg#J\W]j]h]Yg"

 $\label{thmuh} $$ Mci f'Yu'h' `UbX'h'Y'YU'h' `cZh'cgY']b'mci f'WUgg']g'Wf]hJWU'`m]a dcfhUbh''Mci 'a i ghghUmUh'\ca Y']Zmci 'UfY g]W_'cf'\Uj Y'VYYb'UXj]gYX'hc'gY'Z]gc`UhY'VmBGK_'\YU'h' `cf'[cj Yfba Ybh'Ui h'\cf]hJYg'''7 i ffYbh'U'Yfhg'UbX'U `]gh'cZ\chgdchg'WUb'VY'Zci bX'$

a dcfhUbh'@]b_g