

# **MMAN9451**

A UghYfg Dfc YWh5

HYfa "H\fYY" ## "&\$&\$

# 7ci fgY Cj Yfj ]Yk

### GHJZ:7cbHJWh8YHJ]`g

#### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D\cbY
HfUWJY 6UfVYf	U U	D`YUgY`WcbHJWfiVm Ya U]`'cf'HYUa g	>%+ <sup>·</sup> ( \$%5	-',)(\$,%

### GWkcc~7cbhJWh~abZcfa Uhlcb

#### @cWUhlcb

IBGK 'AYVXUb]WU`UbX'AUbiZUWif]b['9b[]bYYf]b[

5]bgk cfh\ 'Vi ]`X]b[ '>%+ž'@Yj Y` '%

5Vcj Y '7 c ZZYY 'cb '7 Ua di g

### <ci fg

-.\$\$-).\$\$da ž'A cbXUm-: f]XUm

ł7`cgYX'cb'di V']W\c']XUmgž'GW\cc`'gW\YXi `YX'Yj Ybhg'UbX'l b]j Yfg]hmG\i hXck b

#### K YV

GWXcc`cZA YWXUb]WU`UbX'A Ubi ZUWi f]b[ '9b[ ]bYYf]b[

9b[]bYYf]b[]Gh XYbhGi ddcfhGYfi ]Wa

9b[]bYYf]b['\delta Xi glf]U\'HfU]b]b[

<u>I\_BGK\_'Ghi Xm5VfcUX'UbX'9I VXUb[Y</u>'flcf']bVci bX'ghi XYbhgŁ

#### I BGK : i hi fY Ghi XYbhg

#### D\cbY

fŽ\*%&Ł-',),)\$\$ - Bi WYig Gh XYbh<i V

fŽ\*%%&L'-',) '+\*\*%'-'9b[]bYYf]b['\&Xi glf]U``HfU]b]b[

fŽ\*%%&E-',) '' %+- '-' BGK 'Gh Xm5VfcUX'UbX'I BGK '91 WXUb[Y'fZcf']bVci bX'gh XYbhgŁ

fŽ\*%"&L'-',) '(\$-+'-'GW\cc`'CZ]WY}

 $\label{thm:condition} $$ \frac{1}{D}YUgY'bchY'h\Uhh\Y'GW\&cc'CZ]WY'k]''bch_bck'k\Yb\#Zmcif'WcifgY'WcbjYbcf']g'cb'WUadig'cf'UjU]'UV'Y$$ 

### 9a U]`

<u>9b[]bYYf]b[ 'Gh XYbhGi ddcfhGYfj ]W</u>yg'-'W ffYbhgh XYbhYbei ]f]Yg

• Y"[""Ybfc`a Ybhž'dfc[fYgg]cbž'WUg\"fYei Yghgž'Wci fgY']ggi Yg'cf'dfc[fUa!fY`UhYX'ei Yf]Yg

<u>9b[]bYYf]b[ '±xi glf]U`'HfU]b]b[</u> '-'±xi glf]U`'lfU]b]b[ 'ei Ygl]cbg

<u>I BGK "Gh Xm5VfcUX"</u>—"gh XmUVfcUX"gh XYbhYbei ]f]Yg"fZcf"]bVci bX"gh XYbhgŁ

<u>I\_BGK\_'9| W.Ub[Y</u>'-'gh XYbhY| W.Ub[Y'Ybei]f]Yg'flcf']bVci bX'gh XYbhgŁ

I BGK : i h fY'Gh XYbhg'-'dchYbhJU'gh XYbhYbei ]f]Yg

• Y"["UXa]gg]cbgžZYYgždfc[fUagžWfYX]hlfUbgZYf

<u>GWXcc``CZZJWY</u>'--'GWXcc``[YbYfU``cZZJWY'UXa]b]ghfUh]cb'Ybei]f]Yg

• B6. h\Y fY`Yj UbhhYUa g``]ghYX UVcj Y a i ghVY WcbHUWYX -

gh XYbhYbei ]f]Yg

# 7ci fgY 8YW] g

### 7 fYX]h'Dc]bhg'('

### Gi a a UfmcZh\Y 7ci fgY

H\Y'dfc YWidfcj ]XYg'Ub'cddcfii b]lmZcfmci 'lc'Vf]b[ 'lc[Yh\YfYb[]bYYf]b[ 'df]bW]d`Yg 'Yg'YvfbYX'cj Yf mci f'dfYj ]ci g'mYUfg'cZghi XmUbX'Udd`mh\YgY'df]bW]d`Yg'hc']bbcj Uhjj Y`mgc'j Y'dfcV`Ya g'gi W\'Ug'h\Y XYj Y`cda YbhcZU'gdYW]ZJWXYg][ bž'dfcWygg'UbX#cfh\Y']bj Ygh][ Uhjcb'cZU'\mdch\Yg]g" A UghYf'dfc YWg'UfY Wca d`Yl ž'cdYb!YbXYX'dfcV`Ya g'h\UhU``ck 'fcca 'Zcf'mci f'WfYUhjj ]lmž'UbX'h\Y'UWei ]g]hjcbž'UbU`mg]g'UbX ]bhYfdfYhUhjcb'cZfYgi 'lg"'H\YfY'UfY'a i 'hjd`Y'dcgg]V'Y'gc`i hjcbg'cf'WcbWi g]cbg'Uhh\Y'ci hgYhUbX'gi ZZJW]Ybh Wca d`Yl ]lmhc fYei ]fY'cZdfc YWid`Ubb]b[ "'H\Y'dfc YWifYei ]fYg'mci 'lc'Zcfa i `UhY'dfcV`Ya g']b gW]Ybh]ZJWcf'Yb[]bYYf]b[ 'hYfa gž'a UbU[ Y'Ub'hYW\b]WU'dfc YWifUbX'Z]bX'gc`i hjcbg'VmUdd`mjb[ 'gW]Ybh]ZJWUbX Yb[ ]bYYf]b[ 'a Yh\cXg"'Mci 'k ]``'U`gc'XYj Y`cd'h\Y'UV]`]mhc'k cf\_']b'U'fYgYUfW\'UbX'XYj Y`cda Ybh Ybj ]fcba Ybh''Mci 'a i gh]XYbh]ZmU'gi dYfj ]gcf'UbX'dfc YWidf]cf'rc'Ybfc``]b[ ']b'h\]g'Wci fgY"'H\]g']g'h\Y'Z]fgh Wci fgY''cZh\Y'' 'Wci fgY'dfc'YWighfi Wi fY"

### 7 ci fgY 5]a g

H\Y'h\Yg]g'dfcj ]XYg'Ub'cddcfhi b]hmZcf'h\Y'ghi XYbhhc'Vf]b[ 'hc[Yh\Yf'Yb[ ]bYYf]b[ 'df]bW]d'Yg'YUfbYX'cj Yf h\Y]f'dfYj ]ci g'mYUfg'cZghi XmUbX'Udd'mh\YgY'df]bW]d'Yg'hc']bbcj Uhji Y'mgc'j Y'dfcV'Ya g'gi W\'Ug'h\Y XYj Y'cda YbhcZU'gdYW]ZWXYg][ bž'dfcW\'gg'UbX\\chi\Y']bj Ygh][ Uhjcb'cZU'\mdch\Yg]g''H\Yg]g'dfc'YWg'a i gh VY'Wca d'Yl ž'cdYb!YbXYX'dfcV'Ya g'h\UhU'`ck 'fcca 'Zcf'ghi XYbhWYUhjj ]hmž'UbX'h\Y'UWei ]g]h]cbž'UbU'mg]g UbX']bhYfdfYhUhjcb'cZfYgi 'hg''H\YfY'a i ghVY'a i 'h]d'Y'dcgg]V'Y'gc'i h]cbg'cf'WcbWi g]cbg'Uhih\Y'ci hgYhUbX gi ZZ[W]YbhWca d'Yl ]hmhc'fYei ]fY'U'XY[ fYY'cZdfc'YWhd'Ubb]b[ 'Zfca 'h\Y'ghi XYbh''H\Y'h\Yg]g'fYei ]fYg'h\Y ghi XYbhhc 'Zcfa i 'UhY'dfcV'Ya g']b'Yb[ ]bYYf]b[ 'hYfa gž'a UbU[ Y'Ub'Yb[ ]bYYf]b[ 'dfc'YWhUbX'Z]bX'gc'i h]cbg'Vm Udd'm]b[ 'Yb[ ]bYYf]b[ 'a Yh\cXg''Ghi XYbhg'U'gc'XYj Y'cd'h\Y]f'UV]']mhc'k cf\_']b'U'fYgYUfW\'UbX'XYj Y'cda Ybh Ybj ]fcba Ybh'

# 7ci fgY @YUfb]b[ 'Ci hWca Yg

- %" 8 Yj Y`cd`U`XYg][ b`cf`U`dfcW'gg`cf`]bj Ygh][ UhY`U`\mdch\Yg]g`Zc``ck ]b[`]bXi ghfm'UbX`dfcZYgg]cbU` Yb[]bYYf]b[`ghUbXUfXg"`
- &" 7 f]h]WU``mifYZYWiicb 'U'gdYW]U`]ghiVcXmicZ\_bck `YX[Y'fY`UhYX'hc'h\Y]f'h\Yg]g'hcd]W'
- ' " 5dd`mgWYbfjZWUbX'Yb[]bYYf]b[ 'a Yh\cXg'hc'gc`j Y'Ub'Yb[]bYYf]b[ 'dfcV'Ya "'
- ("5bU`mgY`XUHU`cV'YWhj Y`mi g]b[ei UbhjHuhj Y`UbXa Uh\Ya UhjWU`a Yh\cXg"
- ) "8 Ya cbglfUlY cfU`UbX k f]ltYb Wca a i b]WUljcb ]b dfcZ/gg]cbU`UbX `UmXca U]bg"

## HYUW ]b[ GhfUhY[]Yg

H\Y`Wci fgY`]g`HJi [\h'Ug`Ub`]bX]j ]Xi U``fYgYUfW\`dfc`YWžhc`XYj Y`cd`U``Yj Y``cZfYgYUfW\`g\_]``g`UbX Ui hcbca m'

5```ghi XYbhUWhj ]hjYg`fYei ]f]b[ 'a Ubi ZJWhi fY`]b`h\Y`A YW\Ub]WJ``UbX`A Ubi ZJWhi f]b[ '9b[ ]bYYf]b[ 'fA A 9Ł k cf\_g\cd'g\ci `X`VY`X]gW ggYX`k ]h\ 'h\Y`k cf\_g\cd'dYfgcbbY``Uhh\Y`]bWYdh]cb'cZh\Y`k cf\_"'H\Y`k cf\_g\cd dYfgcbbY``a i gh\Uj Y`h\Y`cddcfhi b]mm'c`UXj ]gY`UbX`]bZi YbWY`h\Y`XYg][ b`hc`\Y`d`a ]b]a ]gY`UggYa V`mž a Ubi ZJWhi fY`cf`Z bWh]cbU``dfcV`Ya g"

•

H\Y'k cf\_g\cd']g'i gi U``m']b'j Yfm'\][\'XYa UbX"'=\mathref{Trei}]fY'h\Y'k cf\_g\cd'hc'a Ubi \mathref{Z}\whiter fy' Yei ]da Ybh YggYbh]U``hc'mci f'h\Yg]g\tink\Yb'a U\_Y'gi fY'h\Uhmci 'X]g\W gg'mci f'fYei ]fYa Ybhg'Ug'YUf'mUg'dcgg]V`Y'k ]h\'h\Y K cf\_g\cd#@UVcfUhcfmA UbU[Yf''Mci 'g\ci `X'dfcj ]XY'Yb[]bYYf]b['XfUk]b[g'k\]\W\'UfY'\namedfy \mathref{Z}\fgh'Uddfcj YX'Vm h\Y'`UVcfUhcfmC=\mathref{Tree}''Mci 'g\ci 'X'a U\_Y'Yj YfmY\mathref{Z}\fgh'hc'a ]b]a ]gY'h\Y'K cf\_g\cd''cUX'Vma cX]\mathref{Z}\fgh'b['Yi]gh'b['Yei]da YbhfUh\Yf'h\Ub'Vi]\X]b['\mathref{Z}\fca 'bYk \tink\Ym\_YYd]b['mci f'XYg][bg'g]a d'Y

UggYgga YbhidYfZcfa UbWYžmci 'UfY'Y`][]V`Y`hc'Udd`m'Zcf'GdYWJU`'7cbg]XYfUhJcb'df]cf'hc'gi Va]hjb['UbUggYgga Ybhicf'g]hjb['Ub'Yl Ua"

I BGK 'bck '\Ug'U': <u>|hhc'G|h#Gi Va |hfi `Y</u>ž'k \]W\ 'a YUbg't\\Uh']Zmci 'UthYa dh'Ub'Yl Ua 'cf'gi Va ]h'U'd]YW'cZ UggYgga Ybhž'mci 'UfY'XYWUf]b[ 'mci fgY`ZZ]h'Ybci [\'hc'Xc'gc'UbX'WUbbch`UhYf'Udd`mZcf'GdYW]U` 7 cbg]XYfUh]cb"

: cf`XYhJ]`g`cZUdd`m]b[ 'Zcf`GdYW]U``7cbg]XYfUh]cb`UbX`WcbX]h]cbg`Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhUfm UggYgga Ybhž'd`YUgY`gYY`h\Y`]bZcfa Uh]cb`cb`I BGK 'g`<u>GdYW]U``7cbg]XYfUh]cb`dU[Y</u>"

D`YUgY'bchY'h\Uhghi XYbhg'k]``bch'VY'fYei]fYX'hc'dfcj]XY'Ubm'XcW'a YbhUfm'Yj]XYbWY'hc gi ddcfh'UVgYbWg'Zfca 'Ubm'WUggYg'a]ggYX'**VYWUi gY'cZ'7CJ=8!%** 'di V`]W\YU'h\ 'a YUgi fYg'gi W\'Ug]gc`Uh]cb"'| BGK 'k]``bch'VY']bg]gh]b[ 'cb'a YX]WU`'VW'fh]ZJWUh'Yg'Zfca 'UbmcbY'XYYa YX'hc'VY'U'dcg]h]j Y'WUgYZ cf'k\Yb'h\Ym\Uj Y'fYWcj YfYX"'Gi W\'WYfh]ZJWUh'Yg'UfY'X]ZZJW 'hhc'cVhU]b'UbX'di h'Ub'i bbYWYggUfmghfU]b'cb ghi XYbhg'UbX'a YX]WU`'ghUZZ''

 $5 \ dd^{\wdf} \ duff \ duff$ 

# 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa] ha Ybhhc`ZcghYf]b[``U`W`hi fY`cZ`YUfb]b[`]bZcfa YX`VmUWUXYa]W]bHY[f]hm'`5``IBGK ``ghi XYbhg`\Uj Y`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'UbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[e VWX]`WWSege[`YfZWiadVead[VWSeaXafZWbeS`VbSee[`YfZW_aXSekagdai`ž$ 

D`U[]Uf]ga `]g`U'hmdY'cZ]bhY``YWi U``h\YZh''±i\Wb'hU\_Y'a UbmZcfa gžZfca `XY`]VYfUhY'\W.YUh]b[ 'hc'U\W]XYbhU``m Wcdm]b[ 'Zfca 'U'gci f\W'k ]h\ci h'U\w\_bck `YX[ Ya Ybh''I BGK '\Ug'dfcXi \W'X'U'k YVg]hY'k ]h\'U'k YU`h\'cZ fYgci f\W'g'hc'gi ddcfh'ghi XYbhg'hc'i bXYfghUbX'UbX'Uj c]X'd`U[ ]Uf]ga ž'j ]g]h'ghi XYbh'i bgk "YXi "Ui #d`U[ ]Uf]ga

# 5 WUXYa ]W±bZcfa Uh]cb

### 7 fYX]hdc]bhg

7 ci fgY'WfYX]hi]g'WU'W'`UhYX']b'I b]hg!C Zi 7 fYX]hifl C 7 L"'H\Y'bcfa U``k cf\_`cUX'YI dYWfUh]cb'Zcf'cbY'I C 7 ]g'UddfcI ]a UhY`m'&) '\ci fg'dYf'hYfa "'H\]g']bWi XYg'WUgg'WcbhUWh'\ci fgž'df]j UhY'ghi Xmž'ch\Yf'`YUfb]b[ UWfj] ]h]Ygž'dfYdUfUh]cb'UbX'h]a Y'gdYbh'cb'U``'UggYggUV`Y'k cf\_"

A cgh'Wci fgYk cf\_'Wci fgYg'Uhl BGK 'UfY'\*'I C 7 'UbX']bj c`j Y'Ub'Ygh]a UhYX'%) \$`\ci fg'hc'Wca d`YhY'''9UW\ Wci fgY']bWi XYg'U'dfYgWf]VYX'bi a VYf'cZ\ci fg'dYf'k YY\_'fl\#\ Ł'cZgW\YXi `YX'ZJ\WY!hc!ZJ\W''UbX#cf'cb`]bY WcbhJ\W''5bmUXX]h]cbU`'hja Y'VYmcbX'h\Y'dfYg\fy]VYX'WcbhJ\Wi\ci fg'g\ci `X'VY'gdYbh]b'a U\_]b['gi fY'h\Uhmci ii bXYfghJbX'h\Y''YWfi fY'a UhYf]U'ž'Wca d'Yh]b['h\Y'gYhUgg][ ba Ybhgž'Z fh\Yf'fYUX]b['ž'UbX'fYj ]g]b['Zcf UbmYl Ua ]bUh]cbg"

### Cb!WUa di g WUgg UHYbXUbW

Di V`]WX]gHJbW]b[ `WcbX]h]cbg'a i gh'VY`Zc``ck YX`Zcf'U```H' `ZJ\WY!hc!ZJ\WY`WUggYg":Hc`Ybgi fY`h\]gž'cb`m ghi XYbhg`Ybfc``YX`]b`h\cgY`WUggYg'k ]```VY`U``ck YX`]b`h\Y`fcca "`7`Ugg`fcghYfg`k ]```VY`Uh'U\XYX`hc WcffYgdcbX]b[ `fcca g`UbX`W]f\W``Uh'X`Ua cb[ ``UV`XYa cbghfUhcfg"'Bc'cj Yf!Ybfc`a Ybh]g`U``ck YX`]b`ZJ\WY!hc! ZJ\WY`WUgg"'Ghi XYbhg`Ybfc``YX`]b`cb`]bY`WUggYg`WJb`gk Ud`h\Y]f`Ybfc`a Ybh\Zca 'cb`]bY`hc'ch\Yf`UXX]h]cbU`ž' Vi h``]a ]hYXž'bi a VYf`cZcb!\WJa di g`WUggYg`VmGi bXUmž'K YY\_'%"D`YUgY`fYZ\f`hc`mci f`\dci fgYfg`A]\WfcgcZh HYUa g`UbX`A ccX`Y`g]hYg`Zcf'a cfY`]b\Zcfa Uh]cb`UVci h`WUgg`UhhYbXUb\W`Zcf`]b!dYfgcb`UbX`cb`]bY`\WUgg gY\M]cbg#J\Mj] ]h]Yg"

 $\label{thmuh} $$ Mci f'Yu'h' `UbX'h'Y'YU'h' `cZh'cgY']b'mci f'WUgg']g'Wf]hJWU'`m]a dcfhUbh''Mci 'a i ghghUmUh'\ca Y']Zmci 'UfY g]W_'cf'\Uj Y'VYYb'UXj ]gYX'hc'gY'Z]gc`UhY'VmBGK_'\YU'h' `cf'[cj Yfba Ybh'Ui h'\cf]hJYg'''7 i ffYbh'U'Yfhg'UbX'U `]gh'cZ\chgdchg'WUb'VY'Zci bX'$ 

a dcfhUbh'@]b\_g