

# **MMAN3200**

@bYUf'GmghYa g'UbX'7cbhfc`

HYfa 'C bY''##'&\$&%

# 7ci fgY Cj Yfj ]Yk

### GHJZZ7cbHJWh8YHJ]`g

#### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]m	@cWJhjcb	D\cbY
>cgY'; i ]j Ubh	^*[i]jUbh4ibgk"YXi"Ui	0 0	6i ]`X]b[ `>%+ž F cca `) %\$8	-',)')*-'

#### @YWhi fYfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D\cbY
NcfUb'Ji`cj]W	, , , ,	, 0	6i]`X]b[`>%+ž Fcca'' %%8	-',)`*&*%

### GWXcc`7cbHJWh =bZcfa Uh]cb

### @cWUhjcb

I BGK 'A YWXUb]WU``UbX'A Ubi ZUWfi f]b[ '9b[ ]bYYf]b[

5]bgk cfh\ 'Vi ]`X]b[ '>%+ž'@Yj Y`'%

5Vcj Y 7 c ZZYY cb 7 Ua di g

#### <ci fg

-.\$\$-).\$\$da ž'A cbXUm-: f]XUm

ł7`cgYX'cb'di V`]W\c`]XUmgž'GW\cc``gW\YXi `YX'Yj Ybhg'UbX'l b]j Yfg]hm'G\i hXck b

#### K YV

GWXcc`cZA YWXUb]WU`UbX'A Ubi ZUWi flb[ '9b[ ]bYYflb[

9b[]bYYf]b[ Gh XYbhGi ddcfhGYfj ]Wg

9b[]bYYf]b['=bXi ghf]U`'HfU]b]b[

I BGK Ghi Xm5VfcUX UbX 91 W. Ub[Y flcf]bVci bX ghi XYbhgŁ

LBGK: i hi fY'Ghi XYbhq

#### D\cbY

fŽ\*%%L'-',);,)\$\$"—"Bi WYi g"Gh XYbh'<i V

fŽ\*%'&Ł'-',) '+\*\*%'-'9b[]bYYf]b['=bXi glf]U`'HfU]b]b[

fŽ\*%%&E-',) " %+- - I BGK "Gh Xm5VfcUX"UbX"I BGK "91 WXUb[Y fTCf"]bVci bX gh XYbhgk

fŽ\*%"&Ł"-',) "(\$-+"-"GW\cc`"CZZ]WYłł

 $\label{thm:linear_continuity} $$ \frac{1}{D}YUgY'bchY'h\Uhih\Y'GW\&cc`'C\Z_JWY'k\]``bch_bck'k\Yb\#Zmci\ f'Wci\ fgY'Wcbj\Ybcf']g'cb'WUa\ di\ g'cf'Uj\UJ'YUV'Y}$ 

#### 9a U]`

9b[]bYYf]b['Gh XYbhGi ddcfhGYfj ]Wg - W ffYbhgh XYbhYbei ]f]Yq

• Y"[""Ybfc`a Ybhž'dfc[fYgg]cbž'WUg\"fYei Yghgž'Wci fgY']ggi Yg'cf'dfc[fUa!fY`UhYX'ei Yf]Yg

<u>9b[]bYYf]b['±bXi glf]U`'HfU]b]b[</u>'--'±bXi glf]U`'lfU]b]b['ei Ygl]cbg

LBGK Gh Xm5VfcUX - gh XmUVfcUX gh XYbhYbei ]f]Yg fZcf ]bVci bX gh XYbhgŁ

<u>I BGK '91 WXUb[Y</u>'-ghi XYbhYl WXUb[Y'Ybei]f]Yg'flcf']bVci bX'ghi XYbhgŁ

I BGK : i h fy Gh XYbhg - dch bh U gh XYbh Ybei ]f] Yg

• Y"["UXa]gg]cbgžZYYgždfc[fUagžWfYX]hlfUbgZYf

<u>GW.cc``CZZWY</u>'--'GW.cc``[YbYfU``cZZ]WY'UXa]b]glfUljcb'Ybei]f]Yg

• B6. ThY fY Yj UbhthYUa g``]ghYX UVcj Y a i ghVY WcbtUWYX Zcf U```gh XYbhYbei ]f]Yg

# 7ci fgY 8YW] g

7 fYX]hDc]bhg\*\*

Gi a a UfmcZh\Y 7ci fgY

@YUfb]b[ 'Ci HWca Y	95 GW[Y %7ca dYHYbW]Yg
UbX`Wca d`YI ž`Xca U]bg	

### HYUWX]b[ GhfUhY[]Yg

@YWn fYg`]b'h\Y`Wci fgY`UfY`XYg][bYX`hc`dfcj]XY`h\Y`VUg]Wh\Ycfm`VY\]bX`h\Y`WcbWYdhg`hUi [\h': cf`a cgh WUggYgž``YWn fY`bchYg`g`]XYg`k]```VY`Uj U]`UV`Y`cb`]bY`UbX`VYZcfY\UbX"`Gh XYbhg`UfY`YbWci fU[YX`hc`Ug\_ei Ygh]cbg`Xi f]b[`h\Y`WUggYg"

#i]g'j Yfm']a dcftUbh'Zcf'th.]fX'mYUf'ghi XYbhg'tc'VY'UV'Y'hc'i gY'a i 't]d'Y'gci fW'g": cf'th.UhfYUgcbž'UdUfh'Zca th.Y'hYI hVcc\_'UbX'dfcj ]XYX''YWh fY'bchYgž'gYj YfU''fYWca a YbXYX'hYI hg'UfY'']ghYX'''NCi 'UfY'k Y'Wca Y'tc Wcbgi 'thmci f''YWh fYfg'cb'th.]g"

8Ya cbglfUhjcbgʻUfYʻXYg][ bYXʻZcfʻdfUWhjWU``Udd`]\WUhjcbgʻcZh\Yʻh\YcfYhjWU``WcbW'dhgʻ]bHfcXi WYXʻ]b``YWh fYg" 5 Wca dfY\Ybg]j YʻgYhcZhi hcf]U``dfcV`Ya gʻk]```VYʻdfcj ]XYX`VYZcfY\UbX"`Hk cʻhmdYgʻcZXYa cbglfUhjcbgʻk]```VYʻcf[ Ub]gYXž'ghUbXUfX`UbX`]bHYfUWhj Y"

±) "ghubxufx" XYa cbghfuhjcbgž jh jg h Y XYa cbghfuhcf k \c gyhg h Y duwr ubx k cf\_g cb gY YwhYl Ua d Yg H Y h a yg c Zh cgY wugg yg uf Y Zci b X jb mci f h a Yhuv yg "

±ɔʻʻ]bhYfUWlji Y"`XYa cbghfUhjcbgži]hi]gʻghi XYbhgʻk \cʻk cf\_jbX]ji ]Xi U``mcfʻ]bʻga U```[ fci dgžʻUbXʻh\YfYZcfYʻ]hj]g i dʻhcʻh\Ya 'hcʻgY`YWh\YʻYI Ua d`YgʻUbX`X]WUhYʻh\YʻdUWY"'H\YʻXYa cbghfUhcfgʻUbXʻh\Yʻ`YWhi fYf`k ]``VYʻcb \UbXʻhcʻdfcj ]XY`[ i ]XUbWY"'H\YgY`XYa cbghfUhjcbgʻk ]```fi bʻ]bʻh\Yʻa ]XX`Yʻ() 'a ]bi hYgʻcZYUW\``YWhi fYʻVʻcW\_ž cbʻK YXbYgXUmg"

 $: ]bU``m\check{z}'h\Y``UV'k\ cf\_']g']a\ dcffUbh']b'[]j\ ]b[\ 'mci\ 'h\Y''dfUWf]WU``UgdYWfg'cZgca\ Y'cZh\Y'WcbWfdhg'`YUfbh']b'WUggYg''$ 

5XX]h]cbu``a UhhYfg. 'GYj YfU``bYWYggUfma Uh\Ya Uh]WU``WcbWYdhg`UfY`fY[ UfXYX`Ug`Uggi a YX`\_bck `YX[ Y`Zcf

# 5ggYgga Ybh

## 5ggYgga Ybh'HUg\_g

5ggYgga Ybh'tUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[ Ci hWca Ygʻ5ggYggYX
HYghg <sup>-</sup>	( &ı		%ž'&
: ]bU``91 Ua '	(\$1		%ž <sup>*</sup> &
@UV F Ydcfhi	% ।	9bX'cZhYfa 'fk YY_'%\$Ł	%ž <sup>*</sup> &

### 5ggYgga Ybh8YHJ]`g

5ggYgga Ybh% HYghg

8 YHJ]`g.

H\YfY`k]``VY`lk c`Wca dcbYblgž'HYgh%f]b`K YY\_'' Ł'k cfh\`'&i `UbX'HYgh&'f]b`K YY\_'+Ł'k cfh\`' \$i

5XX]h]cbU`XYhU]`g.

H\Y'hYghg'k]``fib'j]U'AccX`Y"

Gi Va ]gg]cb bchYg. cb ]bY hYghg

5ggYgga Ybh& : ]bU`9l Ua

8 YW] g.

: ]bU`YI Ua ]bUhcb ZcW g]b[ 'cb h\Y a UhYf]U`Wcj YfYX`]b h\Y gYWcbX`\U`ZcZh\Y`hYfa "

Gi Va ]gg]cb bchYg. H\Y YI Ua k ]``VY cb`]bYžj ]U A ccX`Y"

5ggYgga Ybh' @UV F Ydcfh

8YW]`q.

5dd`mjb[ 'ghUhY 'Yghja Uhjcbž'UbX 'Wcblfc` 'UddfcUWX Ygž'k cf\_jb[ 'k ]h\ 'U'[ ]j Yb 'gmghYa "

H\Y`dfc`YWfg`gdYWJZJWUhjcbg`k]```VY`fY`YUgYX`]b`kYY\_`\*žhc`VY`gc`j YX`Xi f]b[`gi VgYei YbhkYY\_g"Gi Va]gg]cb`cZfYgi`hg`k]``VY`]b`kYY\_`%\$"

5df]`		D8 ž D=UbX D=8 'Wcblfc `Yfgž XY Zb]hjcb 'UbX 'UbU`mg]g j ]U'F cch'@cW g'UbX '6cXY" F cVi glbYgg" '7 `cgYX `ccd DYfZcfa UbWY 'U[ U]bgha cXY` i bW fhU]bhjYg 'UbX dYfh fVUhjcbg" .  @YWh fY '& fK YXŁ GhUhY 'gdUWY 'fYdfYgYbhUhjcb" 'DfcWfgg'a cXY` cZ@H= gmghYa g" 'G=6C 'UbX 'A =A C 'gmghYa g" 'Bcb` ]bYUf WUgYg" '@]bYUf]nUhjcb 'cZbcb` `]bYUf dfcWfgg'a cXY` g" .
K YY_', .') '5df]`'!'- '5df]`	@YWhifY	HfUbgZYf'Z bWfjcb'fl cf'@H=G+GC'UbX'A +A C'WJgYgŁ" 7 cblfc``UV'Y'WJbcb]WJ``Zcfa "'G]a ]'Uf]hm lfUbgZcfa Uhjcb"'Gc`i hjcb'cZglUhY'gdUWY'Yei Uhjcb" A Ulf]I 'YI dcbYbhjU'"
K_YY_''\%\'5df]`'!'\%* 5df]`	@YWhifY	GtUV]`]lmiUbU`mg]gž`Y][Ybj U`i Yg"'GtUlY'ZYYXVUW_" Dc`Y'd`UWa Ybh''7cblfc``UV]`]lm'
K_YY_`%\$.`%_`5df]``!`&' 5df]`	@YWhifY	GHUHY 'YgHja UHjcb" 'CVgYfj Yfg" 'CVgYfj UV]`]hm' ≠ad`YaYbHjb[ 'gHUHY 'ZYYXVUW_'j ]U 'YgHjaUHYX'gHUHYg"

# FYgci fW/g

### DfYgWf]VYX F Ygci fWYg

@YWn fy bchyg Ufy dfcj ]XYX Vmh\Y `YWn fyfg"

91 Ua d'Y'WcXY'cZg]a i `Uhjcbg']b'A UhUV'UbX'G]a i `]b\_"

### FYWca a YbXYX FYgci fWYg

8\UbU`U\_g\a ]ž?"`~A cXY`]b[ žUbU`mg]g`UbX`Wcblfc``cZXmbUa ]WgmghYa g~f&bX`YXbEVmK ]``]Ua `>"`DU`a `=\mathbf{z} \c\b`K ]`YmV `Gcbgž'\mathbf{z}`BYk `Mcf\_z`&\$\$(\mathbf{z}'\mathbf{k}) \( +\%\mathbf{k} +\' +\mathbf{k} -\"~f&\mathbf{k}\mathbf{k}\"\"

>c\bgcbzïA "ž'>"K ]`\_]Yž'UbX'F "'? UhYV]"'"7 cblfc``9b[]bYYf]b[-Ub'\=blfcXi Wcfm7 ci fgY""1&\$\$&!"

B]gYžBcfa Ub·G"~7cblfc~gmghYa g·Yb[]bYYf]b[~"">c\b·K]YmV~Gcbgž&\$&\$"

: fUb\_`]bž; YbY`: "ž>"`8Uj ]X`Dck Y``ž`UbX`5VVUg`9a Ua ]!BUY]b]"`": YYXVUW\_`Wcblfc``cZXmbUa ]WgmghYa g"" @cbXcb.`DYUfgcbž'&\$%) "

 $fA cghcZhYgYVcc\_gUfYUjU]UVY]bhXYY]VfUfmŁ$ 

### 7ci fgY'9j U'i Uh]cb'UbX'8Yj Y'cda Ybh

%: \subsection \text{%\text{\fight} \text{\fight} \te

&£'5\Wi U``m`Wcj Yf]b[ 'U'fY`Yj Ubhhcd]\W'GhUhY`Ygh]a Uh]cb'#cVgYfj Yfgž'Zcf'VY]b[ 'UV`Y'hc']a d`Ya YbhghUhY ZYYXVUW\_']b''U'fYU']gh]\Wk Um''K Y'k]``'U'gc'j Yf]Zm]hz'k cf\_]b[ z'h\fci [ \ 'fYU']gh]\Wg]a i `Uh]cbg']b A UhUV#G]a i `]b\_"

'EFYd`UWXX'c`X'ZJg\]cb`\UbX'a UXY'6cXY'd`chg'Vmi g]b['dfcdYf'UbU`mg]g'j ]U'Wca di hYf'gcZhk UfY

(£'9| dYf]a YbHJ`k cf\_']g'XYg][ bYX'hc'VY']bX]i [Xi U`ž'UbX'k \]W. WUb'VY'gc`i YX'Zfca '\ca Y'cf'Uhi b]"

@UVcfUhcfmK cf\_g\cd \DZcfa Uhlcb

UggYgga YbhidYfZcfa UbWYžmci 'UfY'Y`][]V`Y`hc'Udd`m'Zcf'GdYWJU``7cbg]XYfUhJcb'df]cf'hc'gi Va]htjb['UbUggYgga Ybhicf'g]htjb['Ub'Yl Ua"

I BGK 'bck '\Ug'U': <u>|hhc'G|h#Gi Va |hfi `Y</u>ž'k \]W\ 'a YUbg't\\Uh']Zmci 'UthYa dh'Ub'Yl Ua 'cf'gi Va ]h'U'd]YW'cZ UggYgga Ybhž'mci 'UfY'XYWUf]b[ 'mci fgY`ZZ]h'Ybci [\'hc'Xc'gc'UbX'WUbbch`UhYf'Udd`mZcf'GdYW]U` 7 cbg]XYfUh]cb"

: cf`XYhJ]`g`cZUdd`m]b[ 'Zcf`GdYW]U``7cbg]XYfUh]cb`UbX`WcbX]h]cbg`Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhUfm UggYgga Ybhž'd`YUgY`gYY`h\Y`]bZcfa Uh]cb`cb`I BGK 'g`<u>GdYW]U``7cbg]XYfUh]cb`dU[Y</u>"

D`YUgY'bchY'h\Uhghi XYbhg'k]``bch'VY'fYei]fYX'hc'dfcj]XY'Ubm'XcW'a YbhUfm'Yj]XYbWY'hc gi ddcfh'UVgYbWg'Zfca 'Ubm'WUggYg'a]ggYX'**VYWUi gY'cZ'7CJ=8!%** 'di V`]W\YU'h\ 'a YUgi fYg'gi W\'Ug]gc`Uh]cb"'| BGK 'k]``bch'VY']bg]gh]b[ 'cb'a YX]WU`'VW'fh]ZJWUh'Yg'Zfca 'UbmcbY'XYYa YX'hc'VY'U'dcg]h]j Y'WUgYZ cf'k\Yb'h\Ym\Uj Y'fYWcj YfYX"'Gi W\'WYfh]ZJWUh'Yg'UfY'X]ZZJW 'hhc'cVhU]b'UbX'di h'Ub'i bbYWYggUfmghfU]b'cb ghi XYbhg'UbX'a YX]WU`'ghUZZ''

 $5\,dd^{3}WUhlcbg^{3}Zcf^{3}gdYW_{U}^{3}W^{3}bg^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}xYfUhlcb^{$ 

# 5 WUXYa ]W<cbYglmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[``U`W``hifY`cZ`YUfb]b[`]bZcfaYX`VmUWUXYa]W]bHY[f]hm'`5``IBGK `ghiXYbhg`UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'D`U[]Uf]gaibXYfa]bYg`UWUXYa]W]bHY[f]hmUbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVW{`WSege[YfZW_aXSekagdai`ž$ 

Mci 'UfY'U'gc'fYa ]bXYX'h\UhWUfYZ ``Hja Y'a UbU[ Ya Ybh']g'Ub']a dcffUbhdUfhcZgh XmUbX'cbY'cZh\Y ]XYbhJZYX'WUi gYg'cZd`U[ ]Uf]ga ']g'dccf'hja Y'a UbU[ Ya Ybh''Gh XYbhg'g\ci 'X'U``ck 'gi ZZWYbh'Hja Y'Zcf fYgYUfW\žXfUZ\]b[ 'UbX'h\Y'dfcdYf'fYZYfYbW]b[ 'cZgci fWYg']b'dfYdUf]b[ 'U```UggYgga Ybh'HUg\_g''

₹d`U[]Uf]ga i]gʻZci bXi]bimci fik cf\_ik \Ybimci 'UfYi]bʻZ[fghmYUfžimci fi`YWi fYfik]``cZYfimci 'Ugg]ghUbWYihc
]a dfcj Yimci fi'UWUXYa]Wg\_]``g'''H\Yma UmUg\_imci ihci`cc\_'Uhigca Yicb`]bYifYgci fWygžiUhhYbXih\Yi@YUfb]b[
7 YbhfYžcfigca Yh]a YgifYgi Va]himci fik cf\_ik]h\h\YidfcV`Ya iZ[l YX'''< ck Yj Yfia cfYigYf]ci gi]bghUbWygi]biZ[fgh
mYUfžigi W\ugighYU`]b[iUbch\Yfighi XYbhgik cf\_icfidUm]b[igca YcbYihciXcimci fik cf\_žia UmVYi]bj Ygh][UhYX
i bXYfih\YiGhi XYbhA]gWcbXi WhDfcWYXi fYg''

F YdYUHYX'd`U[]Uf]ga `fYj Yb`]b`ZjfghmYUfłžd`U[]Uf]ga `UZHYf`ZjfghmYUfžcf`gYf]ci g`]bgHUbWYgža Um'U`gc`VY ]bj Ygh][UHYX`i bXYf`h\Y`Ghi XYbhA ]gWcbXi WhDfcWYXi fYg"`H\Y`dYbU`h]Yg`i bXYf`h\Y`dfcWYXi fYg`WUb`]bWi XY U`fYXi Whcb`]b`a Uf\_gž`ZU]`]b[`U`Wci fgY`cf`Zcf`h\Y`a cghgYf]ci g`a UHHYfg`fi]\_Y`d`U[]Uf]ga `]b`Ub`\cbci fg h\Yg]gŁYj Yb`qi qdYbq]cb`Zfca `h\Y`i b]j Yfq]mm'`H\Y`Ghi XYbhA ]qWcbXi WhDfcWYXi fYg`UfY`Uj U]`UV`Y`\YfY.

kkk"[g"i bgk"YXi"Ui #dc]Wn#XcWa Ybhg#ghi XYbha ]gWcbXi WhdfcWYXi fYg"dXZ

### 5 WUXYa ]W = b Zcfa Uh cb

### 7 fYX]hdc]bhg

7 ci fgY'WfYX]h']g'WU'W'`UhYX']b'I b]hg!C ½7 fYX]hfl C 7 L'''H\Y'bcfa U``k cf\_`cUX'YI dYWfUhJcb'Zcf'cbY'I C 7 ]g'UddfcI ]a UhY`m'&) '\ci fg'dYf'hYfa "'H\]g']bWi XYg'WUgg'WcbhUWh'\ci fgz'df]j UhY'ghi Xmz'ch\Yf'`YUfb]b[ UWfj] ]h]Ygz'dfYdUfUhJcb'UbX'hJa Y'gdYbh'cb'U``'UggYggUV`Y'k cf\_"

### Cb!WUa di g WUgg UHYbXUbW

Di V`]WX]ghJbV}b[ 'WcbX]hJcbg'a i ghVY'Zc``ck YX'Zcf'U```ZJWY!hc!ZJWY'WUggYg"'Hc'Ybgi fY'h\]gž'cb`migh XYbhg

### a dcfhUbh'@]b\_g

- A ccX`Y
- <u>@UV:5\W\/aa</u>
- < YU`h\ 'UbX'GUZY m
- <u>7 ca di h|b[ : UW|^]h|Yq</u>
- Gh XYbhF Ygci fWg
- 7 ci faY 'C i hlbYa
- 9b[]bYYf]b['Gh XYbhGi ddcfhGYfj]\Wg:7YblfY
- A U YfadUW
- I BGK 'H]a YHUV'Y
- I BGK '< UbXVcc
- I BGK 'A YVX Ub]WU`'UbX'A Ubi ZUWi f]b[ '9b[ ]bYYf]b[
- <u>9ei ]hUV`Y`@YUfb]b[ 'GYfj ]WYg</u>

### a U[Y'7fYX]h

GmbYf[]Yg^]b^GcibX^&\$%\*

#### 7F = 7CG

7F = 7CG Dfcj ]XYf 7cXY. \$\$\$-,;

### 5W\_bck `YX[ Ya YbhcZ7ci blfm

 $K Y : UW\underline{b} ck `YX[Y : h, Y : 6YXY[U : dY cd `Y : k \ c : UfY : h, Y : hfUX]h\underline{b} cb U : W ghcX]Ubg : cZh, Y ``UbXg : cb : k \ ]WX : I B G K ? Ybg]b[hcb : WUa di g : ]g ``cWUhYX''$ 

	<u>i</u>			