

## **MMAN4951**

FYgYUfW\"H\Yg]g"5

HYfa 'C bY''##'&\$&%

## 7ci fgY Cj Yfj ]Yk

## GHJZ:7cbHJWh'8YHJ]`g

#### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUhjcb	D\cbY
HfUWJY 6UfVYf	h"VUfVYf4 i bgk "YXi "Ui	D`YUgY`WcbHJWhVm	>%+ž'(\$%5	
	_	Ya U]`cf'HYUa g		

#### GWkcc~7cbhJWh~abZcfa Uhlcb

#### @cWUhlcb

IBGK AYVXUb]WU`UbXAUbiZUWiflb['9b[]bYYflb[

5]bgk cfh\ 'Vi ]`X]b[ '>%+ž'@Yj Y` '%

5Vcj Y 7 c ZZYY cb 7 Ua di g

#### < ci fg

-.\$\$-).\$\$da ž'A cbXUm-: f]XUm

ł7`cgYX'cb'di V']W\c']XUmgž'GW\cc`'gW\YXi `YX'Yj Ybhg'UbX'l b]j Yfg]hmG\i hXck b

#### K YV

GWXcc`cZA YWXUb]WU`UbX'A Ubi ZUWi f]b[ '9b[ ]bYYf]b[

9b[]bYYf]b[]Gh XYbhGi ddcfhGYfi ]Wa

9b[]bYYf]b['\delta Xi glf]U\'HfU]b]b[

<u>I\_BGK\_'Ghi Xm5VfcUX'UbX'9I VXUb[Y</u>'flcf']bVci bX'ghi XYbhgŁ

#### I BGK : i hi fY Ghi XYbhg

#### D\cbY

fŽ\*%&Ł-',),)\$\$ - Bi WYig Gh XYbh<i V

fŽ\*%%&L'-',) '+\*\*%'-'9b[]bYYf]b['\&Xi glf]U``HfU]b]b[

fŽ\*%%&E-',) '' %+- '-' BGK 'Gh Xm5VfcUX'UbX'I BGK '91 WXUb[Y'fZcf']bVci bX'gh XYbhgŁ

fŽ\*%"&Ł"-',) '(\$-+"-"GW\cc`"CZ]WYłł

#### 9a U]`

<u>9b[]bYYf]b[ 'Gh XYbhGi ddcfhGYfj ]W</u>yg'-'W ffYbhgh XYbhYbei ]f]Yg

• Y"[""Ybfc`a Ybhž'dfc[fYgg]cbž'WUg\"fYei Yghgž'Wci fgY']ggi Yg'cf'dfc[fUa!fY`UhYX'ei Yf]Yg

<u>9b[]bYYf]b[ '±xi glf]U`'HfU]b]b[</u> '-'±xi glf]U`'lfU]b]b[ 'ei Ygl]cbg

<u>I BGK "Gh Xm5VfcUX"</u>—"gh XmUVfcUX"gh XYbhYbei ]f]Yg"fZcf"]bVci bX"gh XYbhgŁ

<u>I\_BGK\_'9| W.Ub[Y</u>'-'gh XYbhY| W.Ub[Y'Ybei]f]Yg'flcf']bVci bX'gh XYbhgŁ

I BGK : i h fY'Gh XYbhg'-'dchYbhJU'gh XYbhYbei ]f]Yg

• Y"["UXa]gg]cbgžZYYgždfc[fUagžWfYX]hlfUbgZYf

<u>GWXcc``CZZJWY</u>'--'GWXcc``[YbYfU``cZZJWY'UXa]b]ghfUh]cb'Ybei]f]Yg

• B6. h\Y fY`Yj UbhhYUa g``]ghYX UVcj Y a i ghVY WcbHUWYX -

gh XYbhYbei ]f]Yg

 $\label{thm:condition} $$HY'WcifgY']g'HUi[\hrUg'Ub']bX]j']XiU''fYgYUfWX'dfc'YWz'hc'XYjY`cd'U''YjY''cZfYgYUfWX'g_]`g'UbXUihcbcam''$ 

## 5ggYgga Ybh

## 5ggYgga YbhHUg\_g

5ggYgga Ybh'hJg_	K Y][ \h	8i Y`8UHY	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX
±bHYf]a 'F Ydcfh'⁄ 'Dfc^YWh D`Ub'	%) ၊	: f]XUmiK_YY_`%\$	%ž'&ž' ž'(ž')

## 5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% +bhYf]a FYdcfh/ Dfc^YWhD`Ub

8YHJ]`g.

Н%

Dfc YWnfYdcfnf[i]XY UbX a Uf\_]b[ fi Vf]WUj U] UV Y cb h\Y Wci fgY A G HYUa g g]hYŁ"

H&.

Dfc YWni dXUhY fYdcfhif[ i ]XY UbX a Uf\_]b[ fi Vf]WUj U] UVY cb h\Y Wci fgY A G HYUa g g]hY L" H'.

: ]bU``dfc^YWifYdcfhif[i]XY`UbX`a Uf\_]b[`fi Vf]WUj U]`UV`Y`cb`h\Y`Wci fgY`A G`HYUa g`g]hYt'' DfYgYbhUh]cb`f]bZcfa Uh]cb`k]``VY`Uj U]`UV`Y`cb`h\Y`Wci fgY`A G`HYUa g`g]hYt''

## F Ygci fWyg

FYWca a YbXYX FYgci fW/g

BchUj U]`UV`Y

7 ci fgY '9j U'i Uhcb 'UbX '8 Yj Y`cda Ybh

@UVcfUhcfmK cf\_g\cd \pdzcfa Uhlcb

 $7\,C\,J = 8\,1\% B\,C\,H9. \ 'H\ 'GVX\ cc```UVcfUlrcfm'ZJWJ']hJYg'UfY'cdYb'hc'fYgYUfW. \ 'h\ Yg]g'gh' XYbhg/\ck Yj Yfž'UWW'gg ]g``]a ]hYX"'Hc'Wca d`mk ]h\ '[cj Yfba Ybhfg'gcWJU`X]ghUbWJb['fYei ]fYa Ybhgž'Y"['z'('gei UfY'a YhYfg'dYf dYfgcbž'YUW\ fcca `\Ug'U'a Ul ]a i a `WUdUWJhm`]a ]hž'k \]WX'k ]```VY'a cb]hcfYX'Vmh\Y'gi dYfj ]g]b[ dfcZYgg]cbU``ghUZ''D`YUgY'WXYW_k ]h\ 'h\ Y``UVcfUlrcfmcZJWYf']b'mci f'UfYU'VYZcfY'UWW'gg]b['UbX'cVgYfj Y'h\ Y`]a ]hYbX'U```ch\Yf'gUZYhmdfYWUi hJcbg''$ 

•

Mci a i ghgYY\_[i ]XUbWY'UbX'Uddfcj U``Zfca 'mci f'H\Yg]g'gi dYfj ]gcf'df]cf'hc'Ubm`UVcfUhcfmUWfj ]h]Yg"

.

.

= mci f'dfc YWh']bj c'j Yg''UVcfUhcfmik cf\_= WcbHUWhih\Y'c=Z]WYf!]b!W\Uf[Y'fC=7 E'cZh\Y''UVcfUhcfmi]b'k\]W\'mci k]'''VY'k cf\_]b['Ug'gccb'Ug'dcgg]V'Y'hc'X]gWigg'mci f'fYei]fYa Ybhg"'H\Ymk]''']ggi Y'mci 'k]h\'U'@UVcfUhcfm 5 WWgg'5ddfcj U''fl@55E'Zcfa 'k\]W\'mci 'a i gh'Wca d'YhY'UbX'fYh fb'hc'h\Y'C=7"

.

: cf'a cfY']bZcfa Uhjcb'UVci h'UVWYgg]b['h\Y'GW\cc''g''UVcfUhcf]Ygž'd'YUgY'j ]g]hh\Y'GW\cc'\
k YVg]hY.`\hdg.#k k k "Yb[]bYYf]b["i bgk "YXi "Ui #a YV\Ub]\W'!Yb[]bYYf]b[#Ygci f\Wg#UV!U\W\gg!\ck!hc!\
Zcfa q

## K cf g\cd

.

Mci 'a i ghgYY\_'[i]XUbW''UbX'Uddfcj U`'Zfca 'mci f'H\Yg]g'gi dYfj]gcf'df]cf'hc'fYei Ygh]b['Ubmik cf\_g\cd UWn]j]h]Yg"

.

5```ghi XYbhUWhj ]hjYg`fYei ]f]b[ 'a Ubi ZJWhi fY`]b`h\Y`A YW\Ub]WJ``UbX`A Ubi ZJWhi f]b[ '9b[ ]bYYf]b[ 'fA A 9Ł k cf\_g\cd'g\ci `X`VY`X]gW ggYX`k ]h\ 'h\Y`k cf\_g\cd'dYfgcbbY``Uhh\Y`]bWYdh]cb'cZh\Y`k cf\_"'H\Y`k cf\_g\cd dYfgcbbY``a i gh\Uj Y`h\Y`cddcfhi b]mhc`UXj ]gY`UbX`]bZi YbWY`h\Y`XYg][ b`hc`\Y`d`a ]b]a ]gY`UggYa V`mž a Ubi ZJWhi fY`cf`Z bWh]cbU``dfcV`Ya g"

•

UggYgga YbhidYfZcfa UbWYžmci 'UfY'Y`][]V`Y`hc'Udd`m'Zcf'GdYWJU``7cbg]XYfUhJcb'df]cf'hc'gi Va]htjb['UbUggYgga Ybhicf'g]htjb['Ub'Yl Ua"

: cf`XYhJ]`g`cZUdd`m]b[ 'Zcf`GdYW]U``7cbg]XYfUh]cb`UbX`WcbX]h]cbg`Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhUfm UggYgga Ybhž'd`YUgY`gYY`h\Y`]bZcfa Uh]cb`cb`I BGK 'g`<u>GdYW]U``7cbg]XYfUh]cb`dU[Y</u>"

D`YUgY'bchY'h\Uhghi XYbhg'k]``bch'VY'fYei]fYX'hc'dfcj]XY'Ubm'XcW'a YbhUfm'Yj]XYbWY'hc gi ddcfh'UVgYbWg'Zfca 'Ubm'WUggYg'a]ggYX'**VYWUi gY'cZ'7CJ=8!%** 'di V`]W\YU'h\ 'a YUgi fYg'gi W\'Ug]gc`Uh]cb"'| BGK 'k]``bch'VY']bg]gh]b[ 'cb'a YX]WU`'VW'fh]ZJWUh'Yg'Zfca 'UbmcbY'XYYa YX'hc'VY'U'dcg]h]j Y'WUgYZ cf'k\Yb'h\Ym\Uj Y'fYWcj YfYX"'Gi W\'WYfh]ZJWUh'Yg'UfY'X]ZZJW 'hhc'cVhU]b'UbX'di h'Ub'i bbYWYggUfmghfU]b'cb ghi XYbhg'UbX'a YX]WU`'ghUZZ''

 $5\,dd^{3}WUhlcbg^{3}Zcf^{3}gdYW_{U}^{3}W^{3}bg^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}xYfUhlcb^{$ 

## 5 WUXYa ]W<cbYglmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[``U`W``hifY`cZ`YUfb]b[`]bZcfaYX`VmUWUXYa]W]bHY[f]hm'`5``IBGK `ghiXYbhg`UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'D`U[]Uf]gaibXYfa]bYg`UWUXYa]W]bHY[f]hmUbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVW{`WSege[YfZW_aXSekagdai`ž$ 

Mci 'UfY'U'gc'fYa ]bXYX'h\UhWUfYZ ``h]a Y'a UbU[ Ya Ybh]g'Ub']a dcfhUbhdUfhcZgh XmUbX'cbY'cZh\Y ]XYbh]Z]YX'WUi gYg'cZd`U[ ]Uf]ga ']g'dccf'h]a Y'a UbU[ Ya Ybh'Gh XYbhg'g\ci `X'U``ck 'gi Z]W]Ybh'h]a Y'Zcf fYgYUfW\žXfUZ\]b[ 'UbX'h\Y'dfcdYf'fYZYfYbW]b[ 'cZgci fW/g']b'dfYdUf]b[ 'U```UggYgga Ybh'hUg\_g''

₹d`U[]Uf]ga i]gʻZci bXi]bimci fik cf\_ik \Ybimci i'UfYi]b iZ[fghmYUfžimci fiYVWi fYfik]``cZYfimci i'Ugg]gHUbWYihc
]a dfcj Yimci fiUWUXYa]Wg\_]``g'''H\Yma UmUg\_imci ihci`cc\_'Uhigca Yicb`]bYifYgci fWygžiUhhYbXih\Yi@YUfb]b[
7 YbhfYžicfigca Yh]a YgifYgi Va]himci fik cf\_ik]h\h\YidfcV`Ya iZ[l YX'''< ck Yj Yfia cfYigYf]ci gi]bghUbWygi]biZ[fgh
mYUfžigi W\ugighYU`]b[iUbch\Yfigh XYbhgik cf\_icfidUm]b[igca YcbYihciXcimci fik cf\_žia UmVYi]bj Ygh][UhYX
i bXYfih\YiGh XYbhA]gWcbXi WhDfcWYXi fYg''

FYdYUHYX'd`U[]Uf]ga 'fYj Yb']b'Z|fghmYUfłź'd`U[]Uf]ga 'UZHYf'Z|fghmYUfźcf'gYf]ci g']bghUbWYgža Um'U`gc'VY ]bj Ygh][UhYX'i bXYf'h\Y'Gh XYbhA ]gWcbXi WhDfcWYXi fYg"'H\Y'dYbU'h]Yg'i bXYf'h\Y'dfcWYXi fYg'WUb']bWi XY U'fYXi Whcb']b'a Uf\_gž'ZU]`]b['U'Wci fgY'cf'Zcf'h\Y'a cghgYf]ci g'a UhHYfg'fi]\_Y'd`U[]Uf]ga ']b'Ub'\cbci fg h\Yg]gŁYj Yb'gi gdYbg]cb'Zfca 'h\Y'i b]j Yfg]mm''H\Y'Gh XYbhA ]gWcbXi WhDfcWYXi fYg'UfY'Uj U]`UV'Y'\YfY.

kkk"[g"i bgk"YXi"Ui #dc]Wn#XcWa Ybhg#ghi XYbha ]gWcbXi WhdfcWYXi fYg"dXZ

## 5 WUXYa ]W = b Zcfa Uh cb

## 7 fYX]hdc]bhg

7 ci fgY'WfYX]h']g'WU'W'`UhYX']b'I b]hg!C ½7 fYX]hfl C 7 L'''H\Y'bcfa U``k cf\_`cUX'YI dYWfUhJcb'Zcf'cbY'I C 7 ]g'UddfcI ]a UhY`m'&) '\ci fg'dYf'hYfa "'H\]g']bWi XYg'WUgg'WcbhUWh'\ci fgz'df]j UhY'ghi Xmz'ch\Yf'`YUfb]b[ UWfj] ]h]Ygz'dfYdUfUhJcb'UbX'hJa Y'gdYbh'cb'U``'UggYggUV`Y'k cf\_"

### Cb!WUa di g WUgg UHYbXUbW

Di V`]WX]ghJbV}b[ 'WcbX]hJcbg'a i ghVY'Zc``ck YX'Zcf'U```ZJWY!hc!ZJWY'WUggYg"'Hc'Ybgi fY'h\]gž'cb`migh XYbhg

## a dcfhUbh'@]b\_g

- A ccX`Y
- <u>@UV:5\W\/aa</u>
- < YU`h\ 'UbX'GUZY m
- <u>7 ca di h|b[ : UW|^]h|Yq</u>
- Gh XYbhF Ygci fWg
- 7 ci fgY 'Ci h]bYg
- 9b[]bYYf]b['Gh XYbhGi ddcfhGYfj]\Wg:7YblfY
- A U YfadUW
- I BGK 'H]a YHUV'Y
- I BGK '< UbXVcc
- LBGK 'A YWXUb]WU`'UbX'A Ubi ZUWi f]b[ '9b[ ]bYYf]b[
- <u>9ei ]hUV`Y`@YUfb]b[ 'GYfj ]WYg</u>

## a U[Y'7fYX]h

GmbYf[]Yg^]b^GcibX^&\$%\*

#### 7F = 7CG

7F =7 C G 'Dfcj ]XYf'7cXY. '\$\$\$-,;

### 5W\_bck `YX[ Ya YbhcZ7ci blfm

 $K Y : UW\underline{b}ck `YX[Y : h, Y : 6YXY[U : dYcd `Y : k \c : UfY : h, Y : hfUX]h\underline{b}cb U ``W ghcX]Ubg : cZh, Y ``UbXg : cb : k \] WX : I B G K ? Ybg]b[hcb : WUa di g : ]g ``c WUhYX''$ 

# 5ddYbX]I. '9b[]bYYfg'5i ghfU`]U'f95Ł'DfcZYgg]cbU`'9b[]bYYf'7ca dYhYbWmGHJbXUfX

Dfc[fUa '±bhYbXYX'@YUfb]b['CihWca Yg	
?bck `YX[Y'UbX'g_]```VUgY	
D9%%7ca dfY\Ybg]j Yž'h\YcfmVUgYX'i bXYfgHJbX]b[ 'cZh\Y'i bXYfd]bb]b[ 'bUhi fU`'UbX d\mg]WJ`'gWJYbWYg'UbX'h\Y'Yb[ ]bYYf]b[ 'Z bXUa YbHJ`g'Udd`]WJV`Y'hc'h\Y'Yb[ ]bYYf]b[ 'X]gWJd`]bY	
D9%%.7cbWrdhi U`i bXYfghUbX]b[ cZh\Ya Uh\Ya Uh\Ya Uh]Wgžbi a Yf]WU`UbU`mg]gžghUh]gh]WgžUbX Wca di hYf'UbX`]bZcfa Uh]cb'gW]YbWrg'k\]W\i bXYfd]b'h\Y'Yb[]bYYf]b[ 'X]gW]d`]bY	
D9%" '±0!XYdh\'i bXYfgh\bX]b[ 'cZgdYW]U`]ghVcX]Yg'cZ_bck `YX[ Y`k ]h\]b'h\Y'Yb[ ]bYYf]b[ X]gW]d`]bY	
D9%"( '8]gWYfba YbhcZ_bck 'YX[ Y'XYj Y`cda YbhUbX'fYgYUfW\'X]fYWIjcbg'k ]h\]b'h\Y Yb[ ]bYYf]b[ 'X]gWJd`]bY	
D9%") "? bck "YX[Y"cZYb[]bYYf]b[ "XYg][b"dfUWf]VY" "UbX" WcbhYl hi U" "ZJWcfg"]a dUWf]b[ "h\YYb[]bYYf]b[ "X]gW]d"]bY	
D9%"* I bXYfgHJbX]b[ 'cZh\Y'gWcdYž'df]bWjd`Ygž'bcfa gž'UWVci bHJV]`]hjYg'UbX'Vci bXg'cZ gi gHJ]bUV`Y'Yb[ ]bYYf]b[ 'dfUWjWY']b'h\Y'gdYWjZJWX]gWjd`]bY	
9b[]bYYf]b['Udd`]WUh]cb'UV]`]hm	
D9&"%5dd`]WUhjcb`cZYghUV`]g\YX`Yb[]bYYf]b[`a Yh\cXg`hc`Wca d`Yl `Yb[]bYYf]b[`dfcV`Ya gc`j ]b[	
D9&"&": `i Ybh'Udd`]WUh]cb'cZYb[]bYYf]b['hYVXb]ei Ygž'hcc`g'UbX'fYgci fWfg	
D9&"' '5dd`]\WInjcb'cZgmghYa Unj\WYb[]bYYf]b['gmbh\Yg]g'UbX'XYg][b'dfc\WggYg	