

MMAN9001

A $UghYf^*cZ9b[]bYYf]b[~GW]YbWW^*Dfc^YWh5$

HYfa 'C bY''##'&\$&%

7ci fgY⁻Cj Yfj]Yk

GHJZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWUh]cb	D\cbY
Fcb [.] 7\Ub	f"W. Ub4 i bgk "YXi "Ui	#i]g fyWca a ybxyx mci 'Ya U] 'h\y Wci fgy Wcbj ybcf h a U_Y 'U'gdyWJZJW Uddc]bha ybh]Zmci byyx hc X]gW gg Ub]a dcffUbh]ggi yg"	5]bgk cfh\ 6i]`X]b[-',)%)')

GWXcc`7cbHJWhi±bZcfa Uh]cb

@cWUh]cb

I BGK 'A YVX.Ub]\\\U`\UbX\'A Ubi \\\U\\\forall f]\b[\'9b[]\bYYf]\b[

5]bgk cft\\ 'Vi]`X]b['>%+ž'@Yj Y`'%

5Vcj Y 7 c ZZYY cb 7 Ua di g

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-.\$\$-).\$\$da ž'A cbXUm-: f]XUm

K YV

GWXcc``cZA YWXUb]WU``UbX`A Ubi ZUWi f]b['9b[]bYYf]b[

 $f\tilde{Z}^{*}\%\&E^{-}\text{'}\text{'},\text{)}\text{'}(\$-+\text{'}\text{-}\text{'}GVWcc``C}Z\!J\!WHH$

7ci fgY 8YHJ] g

7 fYX]h'Dc]bhg'*

Gi a a UfmcZh\Y'7ci fgY

D; '7 ci fgY'K cf_'Dfc YWf]g'i gi U``m'Wca d`YhYX']b'hk c'WcbgYW h]j Y'hYfa g'Xi f]b['h\Y``UghUWUXYa]WmYUf'' Gh XYbhg'k]```k cf_']b'hYUa g'hc'WcbXi WfU'fYgYUfW.'VUgYX'dfc YWf''H\Y'dfc YWfWcbHJ]bg'U'`Uf[Y'Ua ci bhcZ cf][]bU'`fYgYUfW.'UbX#cf'bcj Y``XYg][b'k cf_'cf'UbU'mg]g''' #i]g'bchh\Y'fYgdcbg]V]`]mmcZh\Y'gi dYfj]gcfhc'hY``h\Y'gh XYbhhYUa k\Uhhc'Xcž'bcf'g\ci 'X']hVY'Uggi a YX'h\Uhh\Y'gi dYfj]gcf']g'Ub'YI dYfh]b'U```UfYUg'cZ Yb[]bYYf]b[''''H\YmUfY'h\YfY'hc'cZYf'[i]XUbW''UbX'UXj]Wfz'Ug'UfY'`UVcfUhcfmghUZz'k cf_g\cd'ghUZz'UbX ch\Yfg']b'h\Y'gW\cc`'h\Uha Um\Uj Y'YI dYfh]gY']b'h\Y'UfYU'cZmci f'dfcYWf'H\Y'gi WWggZ ``YI YW h]cb'cZh\Y dfcYWf]g'gc'Y`mh\Y'fYgdcbg]V]`]hmcZh\Y'ghi XYbh'

Dfc YWi5]g hc 'VY 'Hu_Yb]b hk Y 'Zbu'` 'UWUXYa]WmY Uf 'Zcf' hk Y 'Wca d 'Yhjcb' c Z U`` fYei]fYa Ybhg 'Zcf' hk Y 'Uk UfX' c Z hk Y 'XY[fYY" 'Hk]g 'Wci fgY z hc [Yhk Yf k]hk 'A A 5B - \$\$& 'Dfc YWi6 z k \]Wk ']g hc 'VY 'Hu_Yb ']b hk Y 'Zc ``ck]b['hY fa z fYei]fYg YUWk 'ghi XYbhhc 'XYa cbghfuh' 'a Ubu[Yf]u' z hYwk b]Wb'` Ubx' dfc Zygg]cbu'` 'g_]``g ']b 'd `Ubb]b['UbX YI YW h]b['Ub' Uddfcj YX 'Yb[]bYYf]b['dfc YWik]hk]b 'U gh]di `UhYX 'h]a Y ``]a]h' '9 UWk 'ghi XYbhi]g '[i]XYX 'VmU gi dYfj]gcfz 'Vi hgi WW ggZ ``md`Ubb]b[z YI YW h]b['Ubx fYdcfh]b['cb'hk Y 'dfc YWiUfY 'hk Y 'gc `Y 'fYgdcbg]V]`]hmcZ hk Y ghi XYbhihYUa "'

7 ci fgY 5]a g

H\Y'dfc YWidfcj]XYg'Ub'cddcffi b]miZcf'h\Y'ghi XYbhihc'Vf]b['lhc[Yh\Yf'Yb[]]bYYf]b['df]bW]d'Yg'YUfbYX'cj Yf h\Y]f'dfYj]ci g'mYUfg'cZghi XmUbX'Udd'mh\YgY'df]bW]d'Yg'hc']bbcj Uhjj Y'mgc'j Y'dfcV'Ya g'gi W\'Ug'h\YXY XYj Y'cda YbhicZU'gdYW]ZWXYg][bž'dfcWygg'UbX#cf'h\Y']bj Ygh][Uhjcb'cZU'\mdch\Yg]g''H\Yg]g'dfc YWg'a i gh VY'Wca d'Yl ž'cdYb!YbXYX'dfcV'Ya g'h\UhU'`ck 'fcca 'Zcf'ghi XYbhiWfYUhjj]lmž'UbX'h\Y'UWei]g]hjcbž'UbU'mg]g UbX']bhYfdfYhUhjcb'cZfYgi 'hg''H\YfY'a i ghVY'a i 'h]d'Y'dcgg]V'Y'gc'i hjcbg'cf'WcbWi g]cbg'Uhih\Y'ci hgYhUbX gi ZZW]YbhWca d'Yl]lmhc'fYei]fY'U'XY[fYY'cZdfc YWfd'Ubb]b['Zfca 'h\Y'ghi XYbh''H\Y'h\Yg]g'fYei]fYg'h\Y ghi XYbhhc 'Zcfa i 'UhY'dfcV'Ya g']b'Yb[]bYYf]b['hYfa gž'a UbU[Y'Ub'Yb[]bYYf]b['dfc YWfUbX'Z]bX'gc'i hjcbg'Vm Udd'm]b['Yb[]bYYf]b['a Yh\cXg''Ghi XYbhg'U'gc'XYj Y'cd'h\Y]f'UV]']mhc'k cf_']b'U'fYgYUfW\'UbX'XYj Y'cda Ybh Ybj]fcba Ybh'

7ci fgY @YUfb]b[Ci hWca Yg

 $52 HY f^* gi WW gg Z \ ``mW ca d`Yh] b ['h] g^* W ci fg Y z^* m ci 'g \ ci `X `V Y `UV `Y `h c. \\$

@YUfb]b['Ci HWca Y	95 GHJ[Y %7ca dYHYbW]Yg
%":7cbXi Whi]bXYdYbXYbhfYgYUfW\"UbX"Udd`mYgHUV`]g\YX"h\Ycf]Yghc"UXXfYgg"Ub"Yb[]bYYf]b['dfcV`Ya 'h\Uh'XcYg'bch'\Uj Y'U'k Y``!XYZ]bYX'gc`i h]cb	D9&"% D9&" žD9&"(žD9' "'
&":5bU`mgY`Wf]hJWJ``nžfYZYWicb`UbX`gmbh\Yg]gY`Wcad`YI]bZcfaUhJcbždfcV`YagžWcbWYdhg`UbX`h\Ycf]Yg"	D9&"%ŽD9&" ŽD9&"(
' "'=bHYfdfYhUbX'HfUbga]hi_bck `YX[Yž'g_]``g'UbX']XYUg'hc'gdYW]U`]gh UbX'bcb!gdYW]U`]ghUi X]YbWYg"	D9&"(ž'D9' "&ž'D9' "(
("'8Ya cbghfUhY'a UbU[Yf]U`'g_]``g'UbX']bX]j]Xi U`'fYgdcbg]V]`]hmhc Wca d`YhY'U'dfcYWik]h\]b'`]a]hYX'hJa Y'UbX'fYgci fWYg"	D9' "(ž˙D9' ") ž˙D9' "*

HYUWX]b[GhfUhY[]Yg

H\YfY`]gʻbcʻZcfa U``HYUW\]b[`Vi hh\Y`ghi XYbhg``YUfbʻZca `Vch\`]bHYfbU``UbX`YI HYfbU``gci fWYg"``H\Y gi dYfj]gcfž'ch\Yf`UWJXYa]WgʻUbX``UVcfUhcfm#k cf_g\cd`gHJZUfY`h\Y`]bHYfbU``gci fWYgž'k \YfYUgʻh\Y`@]VfUfmž]bHYfbYhUbX`]bXi ghfma YbhcfgʻUfY`h\Y`YI HYfbU``gci fWYg"`H\YfY`UfY`bc`gYh`YWhi fYgʻZcf`h\]gʻWci fgYž'Vi hU bi a VYf`cZk cf_g\cdg`k]```VY`dfcj]XYX`hc`Ugg]ghighi XYbhgʻhc`Wca d`YhY`h\Y]f`h\Yg]gʻhc`U`\][\`gHUbXUfX"`5`` k cf_g\cdg`k]```VY`fYWcfXYX`UbX`a UXY`Uj U]`UV`Y`hc`ghi XYbhgʻcb`]bY"

5ggYgga Ybh

5ggYgga YbhHUg_g

5ggYgga Ybh'hUg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
Dfc[fYgg*9jU*iUh]cb*k]h\ AYbhcfg*	%) ।	Bch5dd`]WUV`Y	%ž'&ž' ž'(
Dfc YVViDfYgYblUljcb	%) ।	Bch5dd`]WUV`Y	%ž'&ž'' ž'(
Dfc[fYgg'FYdcfh	+\$1	K YY_`%\$´: f]XUm	%ž'&ž'' ž'(

5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% Dfc[fYgg 9j Ui Uh]cbk]h\AYbhcfg

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g.

Gh XYbhhYUa g'k]``VY`[fUXYX`k YY_`mVmh\Y]f'a Ybhcfg`VUgYX`cb'h\Y]f'k YY_`mUWUXYa]Wdfc[fYgg"

5ggYgga Ybh& Dfc YWhDfYgYbhUh]cb

GhUfhXUhY. Bch5dd`\\\VY

8 YW] g.

 $\label{eq:continuous} Gh XYbhihYUa g'k]``VY'[fUXYX'k YY_`m'Vmh\Y]f'a Ybhcfg'VUgYX'cb'h\Y]f'cf[Ub]gUh]cb'g_]``g'UbX'ghm`Y'cZ dfYgYbhUh]cb''$

5ggYgga Ybh' . Dfc[fYgg F Ydcfh

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.')!%\$'dU[Yg'Ž'Zfcbh'/ YbX'a UffYf

8YW]`q.

 $Gh XYbhgi Va]hg `Ub `]bX]j]Xi U``fYdcfh! `ci h]b]b[`_YmZ]bX]b[g `cZDfc `YWh5 `Ub X'dfc Xi WY `U` XYhU]`YX'd` Ub `Zcf Dfc `YWh6 `]b `h\ Y `bYI hhYfa "$

FYgci fW/g

 $\label{thm:condition} $$ \ensuremath{\mathsf{U}} = \ensuremath{\mathsf{U}} =$

I BGK 'bck '\Ug'U': <u>|hhc'G|h#Gi Va |hfi 'Y</u>ž'k \]W\ 'a YUbg't\\Uh']Zmci 'UhhYa dh'Ub'YI Ua 'cf'gi Va]h'U'd]YW\'cZ UggYgga Ybhž'mci 'UfY'XYWUf]b['mci fgY`ZZ]hYbci [\'hc'Xc'gc'UbX'WUbbch`UhYf'Udd`m'Zcf'GdYW]U` 7 cbg]XYfUh]cb"

: cf`XYhJ]`g`cZUdd`m]b['Zcf`GdYVJU``7 cbg]XYfUh]cb`UbX`WcbX]h]cbg'Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhUfm UggYgga Ybhž'd`YUgY`gYY`h\Y`]bZcfa Uh]cb`cb`l BGK 'g`<u>GdYVJU``7 cbg]XYfUh]cb`dU[Y</u>"

D`YUgY'bchY'h\Uhigh XYbhg'k]``bchVY'fYei]fYX'hc'dfcj]XY'UbmXcW a YbhUfmYj]XYbW'hc gi ddcfhUVgYbWg'Zfca 'UbmWUggYg'a]ggYX'

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[``U`W``hifY`cZ`YUfb]b[`]bZcfaYX`VmUWUXYa]W]bHY[f]hm'`5``IBGK `ghiXYbhg`\UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'D`U[]Uf]gaibXYfa]bYg`UWUXYa]W]bHY[f]hmUbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVW{`WSege[YfZW_aXSekagdai`ž$

Mci 'UfY'U'gc'fYa]bXYX'h\UhWUfYZ ``h]a Y'a UbU[Ya Ybh]g'Ub']a dcfhUbhdUfhcZgh XmUbX'cbY'cZh\Y]XYbh]Z]YX'WUi gYg'cZd`U[]Uf]ga ']g'dccf'h]a Y'a UbU[Ya Ybh'Gh XYbhg'g\ci `X'U``ck 'gi Z]W]Ybh'h]a Y'Zcf fYgYUfW\žXfUZ\]b['UbX'h\Y'dfcdYf'fYZYfYbW]b['cZgci fW/g']b'dfYdUf]b['U```UggYgga Ybh'hUg_g''

₹d`U[]Uf]ga i]gʻZci bXi]bimci fik cf_ik \Ybimci i'UfYi]b iZ[fghmYUfžimci fiYVWi fYfik]``cZYfimci i'Ugg]gHUbWYihc
]a dfcj Yimci fiUWUXYa]Wg_]``g'''H\Yma UmUg_imci ihci`cc_'Uhigca Yicb`]bYifYgci fWygžiUhhYbXih\Yi@YUfb]b[
7 YbhfYžicfigca Yh]a YgifYgi Va]himci fik cf_ik]h\h\YidfcV`Ya iZ[l YX'''< ck Yj Yfia cfYigYf]ci gi]bghUbWygi]biZ[fgh
mYUfžigi W\ugighYU`]b[iUbch\Yfigh XYbhgik cf_icfidUm]b[igca YcbYihciXcimci fik cf_žia UmVYi]bj Ygh][UhYX
i bXYfih\YiGh XYbhA]gWcbXi WhDfcWYXi fYg''

FYdYUHYX'd`U[]Uf]ga 'fYj Yb']b'Z|fghmYUfłź'd`U[]Uf]ga 'UZHYf'Z|fghmYUfźcf'gYf]ci g']bghUbWYgža Um'U`gc'VY]bj Ygh][UhYX'i bXYf'h\Y'Gh XYbhA]gWcbXi WhDfcWYXi fYg"'H\Y'dYbU'h]Yg'i bXYf'h\Y'dfcWYXi fYg'WUb']bWi XY U'fYXi Whcb']b'a Uf_gž'ZU]`]b['U'Wci fgY'cf'Zcf'h\Y'a cghgYf]ci g'a UhHYfg'fi]_Y'd`U[]Uf]ga ']b'Ub'\cbci fg h\Yg]gŁYj Yb'gi gdYbg]cb'Zfca 'h\Y'i b]j Yfg]mm''H\Y'Gh XYbhA]gWcbXi WhDfcWYXi fYg'UfY'Uj U]`UV'Y'\YfY.

kkk"[g"i bgk"YXi"Ui #dc]Wn#XcWa Ybhg#ghi XYbha]gWcbXi WhdfcWYXi fYg"dXZ

5 WUXYa]W = b Zcfa Uh cb

7 fYX]hdc]bhg

7 ci fgY'WfYX]h']g'WU'W'`UhYX']b'I b]hg!C ½7 fYX]hfl C 7 L'''H\Y'bcfa U``k cf_`cUX'YI dYWfUhJcb'Zcf'cbY'I C 7]g'UddfcI]a UhY`m'&) '\ci fg'dYf'hYfa "'H\]g']bWi XYg'WUgg'WcbhUWh'\ci fgz'df]j UhY'ghi Xmz'ch\Yf'`YUfb]b[UWfj]]h]Ygz'dfYdUfUhJcb'UbX'hJa Y'gdYbh'cb'U``'UggYggUV`Y'k cf_"

Cb!WUa di g WUgg UHYbXUbW

Di V`]WX]ghJbV}b['WcbX]hJcbg'a i ghVY'Zc``ck YX'Zcf'U```ZJWY!hc!ZJWY'WUggYg"'Hc'Ybgi fY'h\]gž'cb`migh XYbhg

a dcfhUbh'@]b_g

- A ccX`Y
- <u>@UV:5\W\/aa</u>
- < YU`h\ 'UbX'GUZY m
- <u>7 ca di h|b[: UW|^]h|Yq</u>
- Gh XYbhF Ygci fWg
- 7 ci faY 'C i hlbYa
- 9b[]bYYf]b['Gh XYbhGi ddcfhGYfj]\Wg:7YblfY
- A U YfadUW
- I BGK 'H]a YHUV'Y
- I BGK '< UbXVcc
- I BGK 'A YVX Ub]WU`'UbX'A Ubi ZUWi f]b['9b[]bYYf]b[
- <u>9ei]hUV`Y`@YUfb]b['GYfj]WYg</u>

a U[Y'7fYX]h

GmbYf[]Yg^]b^GcibX^&\$%*

7F = 7CG

7F = 7CG Dfcj]XYf 7cXY. \$\$\$-,;

5W_bck `YX[Ya YbhcZ7ci bffm

 $K Y : UW\underline{b} ck `YX[Y : h, Y : 6YXY[U : dY cd `Y : k \ c : UfY : h, Y : hfUX]h\underline{b} cb U : W ghcX]Ubg : cZh, Y ``UbXg : cb : k \]WX : I B G K ? Ybg]b[hcb : WUa di g :]g ``cWUhYX''$

5ddYbX]I . '9b[]bYYfg'5i ghfU`]U'f95Ł'DfcZYgg]cbU` '9b[]bYYf'7ca dYhYbWm GHJbXUfX

Dfc[fUa =bhybXYX=@YUfb]b['Ci hWca Yg	
?bck `YX[Y`UbX'g_]```VUgY	
D9%"%7ca dfY\Ybg]j Yž'h\YcfmVUgYX'i bXYfghUbX]b['cZh\Y'i bXYfd]bb]b['bUhi fU`'UbX d\mg]\WJ`'g\WJYb\Wg'UbX'h\Y'Yb[]bYYf]b['Z bXUa YbhJ`g'Udd`]\WJV`Y'hc'h\Y'Yb[]bYYf]b['X]g\WJd`]bY	
D9%"&"7cbWYdh U``i bXYfgHJbX]b['cZh\Y'a Uh\Ya Uh]Wgžbi a Yf]WJ``UbU`mg]gžgHJh]gh]Wgž'UbX Wca di hYf`UbX`]bZcfa Uh]cb'gW]YbWYg'k\]W\'i bXYfd]b'h\Y'Yb[]bYYf]b['X]gW]d`]bY	
D9%" '=b!XYdh\'i bXYfghUbX]b['cZgdYW]U`]ghVcX]Yg'cZ_bck `YX[Y'k]h\]b'h\Y'Yb[]bYYf]b[X]gW]d`]bY	
D9%'('8]gWfba YbhcZ_bck `YX[Y 'XYj Y`cda YbhUbX'fYgYUfWX 'X]fYWf]cbg'k]h\]b'h\Y Yb[]bYYf]b['X]gW]d`]bY	
D9%) '? bck 'YX[Y'cZYb[]bYYf]b['XYg][b'dfUWjWY'UbX'WcbhYl hi U`'ZJWcfg']a dUWjb['h\Y Yb[]bYYf]b['X]gWjd`]bY	
D9%"* I bXYfgHJbX]b['cZh\Y'gWcdYž'df]bW]d`Ygž'bcfa gž'UWVci bHJV]`]h]Yg'UbX'Vci bXg'cZ gi gHJ]bUV`Y'Yb[]bYYf]b['dfUWfJWY']b'h\Y'gdYW]ZJWX]gW]d`]bY	
9b[]bYYf]b['Udd`]WUh]cb'UV]`]hm	
D9&"%5dd`]WUh]cb`cZYgHUV`]g\YX`Yb[]bYYf]b[`a Yh\cXg`hc`Wca d`Yl `Yb[]bYYf]b[`dfcV`Ya gc`j]b[
D9&"&": `i YbhiUdd`]WUh]cb'cZYb[]bYYf]b['hYVXb]ei Ygžhcc`g'UbX'fYgci fWYg	
D9&" '5dd`]WUh]cb`cZgmghYa Uh]WYb[]bYYf]b[`gmbh\Yg]g`UbX`XYg][b`dfcWYggYg	
D9&"('5dd`]WUh]cb'cZgmghYa Uh]WUddfcUWXYg'hc'h\Y'WcbXi WhUbX'a UbU[Ya YbhcZYb[]bYYf]b[dfcYWng	
DfcZYgg]cbU`'UbX'dYfgcbU`'Ultf]Vi hYg	
D9' "%'9h\]WU`'WcbXi Wh'UbX'dfcZYgg]cbU`'UWWci bHJV]`]hm	
D9' "&'9ZYWAj Y'cfU`'UbX'k f]MYb'Wcaaib]WUAjcb']b'dfcZYgg]cbU`'UbX'`Um'XcaU]bg	
D9' "' '7 fYUhjj Yž']bbcj Uhjj Y'UbX'dfc!UWhj Y'XYa YUbci f	
D9' "('DfcZ/gg]cbU`i gY'UbX'a UbU[Ya YbhcZ]bZcfa Uh]cb	
D9' ") 'C fXYf'mia UbU[Ya YbhcZgY`Zi'UbX`dfcZygg]cbU``WcbXi Wh	
D9' "* '9ZZYWNjj Y'NYUa 'a Ya VYfg\]d'UbX'NYUa 'YUXYfg\]d	