

# **ENGG9744**

Bi WYUf'GUZYhmž'GYW f]hm'UbX'GUZY[i UfXg

HYfa 'Hk c''##'&\$&%

## 7ci fgY Cj Yfj ]Yk

## GHJZ:7cbHJWh8YHJ]`g

#### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]m	@cWUhjcb	D\cbY
9Xk UfX <sup>-</sup> C VVUfX	Y"cVVUfX4 i bgk "YXi "Ui	D`YUgY'Ya U]`'a Y'hc	5]bgk cfh	-',)+*&)
		UffUb[Y'U'a YYh]b[	6i ]`X]b[	

#### GWkcc~7cbhJWh~abZcfa Uhlcb

#### @cWUhlcb

IBGK AYVXUb]WU`UbXAUbiZUWiflb['9b[]bYYflb[

5]bgk cfh\ 'Vi ]`X]b[ '>%+ž'@Yj Y` '%

5Vcj Y 7 c ZZYY cb 7 Ua di g

#### < ci fg

-.\$\$-).\$\$da ž'A cbXUm-: f]XUm

ł7`cgYX'cb'di V']W\c']XUmgž'GW\cc`'gW\YXi `YX'Yj Ybhg'UbX'l b]j Yfg]hmG\i hXck b

#### K YV

GWXcc`cZA YWXUb]WU`UbX'A Ubi ZUWi f]b[ '9b[ ]bYYf]b[

9b[]bYYf]b[]Gh XYbhGi ddcfhGYfi ]Wa

9b[]bYYf]b['\delta Xi glf]U\'HfU]b]b[

<u>I\_BGK\_'Ghi Xm5VfcUX'UbX'9I VXUb[Y</u>'flcf']bVci bX'ghi XYbhgŁ

#### I BGK : i hi fY Ghi XYbhg

#### D\cbY

fŽ\*%&Ł-',),)\$\$-BiWYigGhXYbh<iV

fŽ\*%%&L'-',) '+\*\*%'-'9b[]bYYf]b['\&Xi glf]U``HfU]b]b[

fŽ\*%%&E-',) '' %+- '-' BGK 'Gh Xm5VfcUX'UbX'I BGK '91 WXUb[Y'fZcf']bVci bX'gh XYbhgŁ

fŽ\*%"&L'-',) '(\$-+'-'GW\cc`'CZ]WY}

## 9a U]`

9b[]bYYf]b[ 'Gh XYbhGi ddcfhGYfj ]Wg - W ffYbhgh XYbhYbei ]f]Yg

• Y"[""Ybfc`a Ybhž'dfc[fYgg]cbž'WUg\"fYei Yghgž'Wci fgY']ggi Yg'cf'dfc[fUa!fY`UhYX'ei Yf]Yg

9b[]bYYf]b[ \*\DXi ghf]U\cdot'HfU]b]b[ \( --\Dxi\) ghf]U\cdot'hfU]b]b[ \( \text{iei Ygh]cbg} \)

<u>I BGK 'Gh Xm5VfcUX</u>'--gh XmUVfcUX'gh XYbhYbei ]f]Yg'fZcf']bVci bX'gh XYbhgŁ

 $\underline{\mathsf{LBGK}}. \underline{\mathsf{SIWWUb}}[\underline{\mathsf{Y}}. - \underline{\mathsf{gh}}. \mathsf{XYbhYI}. \mathsf{WWUb}[\underline{\mathsf{Y}}. \mathsf{Y}] \mathsf{bei}. ] f] \mathsf{Yg}. \underline{\mathsf{ffcf}}. \mathsf{bVci}. \mathsf{bX}. \underline{\mathsf{gh}}. \mathsf{XYbhg}. \underline{\mathsf{bM}}. \underline{\mathsf{VM}}. \underline{\mathsf{bM}}. \underline{\mathsf{M}}. \underline{\mathsf{bM}}. \underline{\mathsf{M}}. \underline{$ 

I BGK : i h fY Gh XYbhg

7ci	fgY	`8YW]`g	
7 fYX	(]hˈDo	c]b <b>h</b> g <sup>·</sup> * ·	

Gi a a UfmcZ'h\Y'7ci fgY

Gli XYbligik]```YUfbiZica idfcZYgg]cbU`gʻUhih\Yʻ5 i glifU`]UbʻB i WYUfʻGW]YbWYʻUbXʻHYW\bc`c[mCf[Ub]gUh]cbfbBGHCE'UbXʻZica ih\Yʻ5 i glifU`]UbʻGUZY[i UfXgʻUbXʻBcb!dfc`]ZYfUh]cbʻCZZJWYʻfbGBCE'Vch\`\ck igd`YZc`]ZYfUh]cbD

H\]g`Wci fgY`U`gc`dfcj ]XYg`gYj YfU``Wca dYHYbWmUfYUg`cZh\Y`=595`=bHYfbUh]cbU``Bi WYUf`A UbU[Ya Ybh 5WJXYa mf\B A 5\E``YUfb]b[`ci \Wca Yg`Zcf`a UgHYfg'``Yj Y``Wci fgY`]b`Bi WYUf`HYW\bc`c[ mA UbU[ Ya Ybh

-BA 5 7 ca dYhYbWm5fYUł	-BA 5 7ca dYhYbWm
	@Yj Y`
%"& ±bH/fbUh]cbU`bi WYUf`cf[Ub]nUh]cbg	%
%" Buhjcbu`bi WYufhYW\bc`c[mdc`]Whžd`Ubb]b['UbX'dc`]hJWg	%
%"( Bi WYUf ghub Xuf Xg	%
%") Bi WYUf``Uk	%
%", 'Bi WYUf``]WYbg]b[ ž``]WYbg]b[ 'VUg]g'UbX'fY[i `UlcfmdfcWYggYg	&
%"- "Bi WYUf'gYW f]lm	&
%"%\$"Bi WYUf'gUZY[i UfXg	&
%"%%"HfUbgdcfhcZbi WYUf"[ccXg"UbX a UhYf]U`g	%
&"%Bi WYUfdck Yfd`UbhUbX`ch\YfZUWj]lmXYg][b'df]bWjd`Yg	%
&"&BiWYUfdckYfd`Ubh#ZUWj`]micdYfUhjcbU`gmghYag	%
&"* Bi WYUfgUZYhmdf]bWjd`Yg`UbX`UbU`mg]g	1
&"+"FUX]c`c[]WU``gUZYhm'UbX'dfchYWnJcb	1
' "% 'Bi WYUf']bW]XYbha UbU[Ya Ybhž'Ya Yf[YbWnd`Ubb]b[ 'UbX	%
fYgdcbgY	
("' '7 ca a i b]\WUh]cb 'g\rf\UhY[]\Yg'\Zcf``\YUX\Yfg']\b'bi \W\Y\Uf	%

 $fcgVc]g\ddot{z}>"XY\ddot{z}: "`5XUWX]\ddot{z}`UbX`<"`<]fcgY"`&$%+"`"$hYfbUhJcbU``Bi WYUf`A UbU[Ya Ybh'5WUXYa miA UghYf'g Dfc[fUa g`]b`Bi WYUf`HYWXbc`c[miA UbU[Ya Ybh'3":<math>=$ 595"

#### HYUWX]b[ GhfUhY[]Yg

: i ``micb`]bY"'H\]g'Wci fgY']g'I BGK 'X][ ]hU`'i d`]ZhWcbhYbhž'k \]W\']g'hUi [\hicb`]bY'h\fci [\'h\Y'cb']bY'`YUfb]b[ d`UhZcfa "

## 5XX]h]cbU`7ci fgY ±bZcfa Uh]cb

 $H\[g']g'U'dcgh[fUXi\ UhY'Wci\ fgY'Wcbj\ YbYX'Vmh\Y'GW\[cc^*cZA\ YW\[Ub]WU''UbX'A\ Ubi\ ZUWi\ f]b[''9b[]bYYf]b[''=h']g'dUfhicZh\Y'A\ 9b[GW]'Bi\ WYUf'9b[]bYYf]b[''gdYW]U']nUh]cb''UbX'WUb'VY''hU_Yb''Ug''Ub'Y'YWh]j\Y'Vm'' fX'cf (h\ 'mYUf'ghi\ XYbhg'Zca ''ch\Yf'gW\[cc''g'UbX'ZUW'''h]Yg''cb''h\Y''Uddfcj\U''cZ\[ca\Y'gW\[cc'''UbX'h\Y''9B; ; -+(($ 

Gh XYbhg'k cf\_']b'[ fci dg'hc'dfcXi WY'U'XYg][ b'VUg]g'h\fYUh'UggYgga Ybh'Zcf'U'bi WYUf'ZUW]']m''H\Y Ugg][ ba Ybh']g'[ fUXYX'Vm'U'fi Vf]W'': cf'`UhY'gi Va ]gg]cbgž'&\$i 'cZa Uf\_g'UfY'XYXi WYX'dYf'XUmž'fYUW\]b[ nYfc'a Uf\_g'cb':\&h\'>i `mž'&\$&\"'A Uf\_g'UfY'fYh fbYX'Vm'\(h\'>i `m'

Gi Va ]gg]cb bch/g. Gi Va ]hh/X h\fci [\ Hi fb]h]bž]b a ccX`Y"

 $\label{eq:higher} \mbox{Hi fb]h]b"gYh]b"[."H\]g"Ugg][ba Ybh]g"gi Va ]hhYX"h\fci [\ \ 'Hi fb]h]b"UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b"UbX"gh XYbhg"Xc"bchgYY Yh Xybhg"Xc Yh Xybhg Yh$ 

5ggYgga Ybh' . 9a Yf[ YbWmGWYbUf]c

GHJfh'XUhY. "%(#\$\*#&\$&%"\$-.\$\$"5A

## 5 hhyb XUb WY F Yei ]fYa Ybhg

H\Y`Wci fgY`]g`WcbXi WYX`Z``mcb`]bY"`H\Y`WcbhYbh]g`YI hYbg]j Y`UbX`gca Yh]a Yg`WkU``Yb[]b[`a YUb]b[`h\Uh UhhYbXUbWY`Uhk YY\_`mhi hcf]U`g`k]h\`h\Y`Wci fgY`Wcbj YbYf`k\]`Y`cdh]cbU``]g`ghfcb[`mYbWci fU[YX"``Ghi XYbhg k]```bYYX`hc`a U\_Y`UffUb[Ya Ybhg`hc`WcbXi Wh[fci d`k cf\_`hc[Yh\Yf"`Gca Y`Ugg][ba Ybhgžgi Wk`Ug`h\Y`gUZYhm WUgY`dfYgYbhUh]cb`UfY`gWXYXi `YX`]b`UXj UbWY`UbX`UhhYbXUbWY`]g`a UbXUhcfmhc`cVhU]b`h\Y`[fUXYg"

## 7ci fgY GWX YXi `Y

#### J ]Yk 'WUgg'ha YHJV'Y

## H]a YHJV`Y

8Uh/	Hmd Y	7 cbh/bh
C 'K YY'&) 'A Um'!'&,		
A Um		
K YY_"%"' %"A Umi!"(">i bY	Hcd]W	Bi WYUf`GUZY[i UfXg
K YY_'&.'+'>i bY'!'%%	Hcd]W	A UhYf]U``5VWci bh]b[
>i bY		
_ ` `	Hcd]W	FUX]Uh]cb'GUZYhm
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>i bY		
K YY_")."&, ">i bY"!"&">i `m	Hcd]W	Bi WYUf'GYW f]hm
K YY_'*.')	Hcd]W	Bi WYUfgUZYmiZ bXUa YbtU`gʻ
K_YY_"+."%&">i_`m!"%*">i_`m	Hcd]W	H\Y`gUZYlm`WUgY`
K_YY_', .'%-'>i `m!'&' '>i `m	Hcd]W	GUZYmi5ggYgga Ybh'''
K YY_''&*'>i `m!'' \$'>i `m	Hcd]W	GUZYmi5bU`mg]g

## FYgci fWyg

#### DfYgWf]VYX F Ygci fWYg

H\Y a cgh]a dcffUbhfYgci fWg UfY dfcj ]XYX Ug dXZXcW a Ybhg Ug dUfhcZh\Y Wci fgY"

#### FYWca a YbXYX FYgci fWYg

%"Bi WYUf'GUZY[i UfXgžGYW f]mmUbX'Bcbdfc`]ZYfUf]cb. '5W,]Yj ]b[ 'GYW f]mmk ]h\ 'HYV\\bc`c[ mUbX'Dc`]Wn 5i h\cf. '>Ua Yg'8cm`Y

€6B'-+,!\$+)\$\*, \*+'\$

MYUf'Di V`]g\YX'&\$\$,

Di V`]g\Yf'< Y]bYa Ubb!6i HYfk cfh\

&"`H\fYY`A ]`Y`=g`UbX.`5`Bi WYUf`7f]g]g`]b`< ]ghcf]WU``DYfgdYWf]j Y 5i h\cf.`>"`GUa i Y``K U`\_Yf =6B`-+,!\$) &\$&(\*,'\* MYUf`Di V`]g\YX`&\$\$\*

' "'5V`UnY'fH\Y'GlrcfmcZh\Y'< YfcYg'UbX'J ]Wf]a g'cZ7\YfbcVmŁ 5i h\cf. 'D]Yfg'DUi ``F YUX €6B'-+, !\$\*+- (\$, % & Di V`]g\Yf'F UbXca' < ci gY

("K YVg]hYg. DUfh]W`Ufmi=595 XcW a Ybhgʻfdfcj ]XYX`]bʻh\Y`Wci fgY`a UhYf]U`gŁ" '5`gc. K B5ž 5BGžB9₹K ⊕G

### 7 ci fgY '9j U'i Uh]cb 'UbX '8 Yj Y'cda Ybh

: YYXVUW\_cb^h\Y`Wci fgY^]g^[Uh\YfYX'dYf]cX]WU``mi g]b[`j Uf]ci gʻa YUbgž]bWi X]b[`h\ I BGK A m9I dYf]YbWI`dfcWIggž]bZcfa U``X]gWi gg]cb^]b^h\Y`Wci fgYžUbX'h\Y`GW\cc^fgj^Gh XYbh#GhUZ'a YYh]b[g"'Mci fZYYXVUW\_']g^hU\_Yb^gYf]ci g`mžUbX`Wcbh]bi U``]a dfcj Ya Ybhg^UfY`a UXY`hc^h\Y`Wci fgY`VUgYXž]b^dUfhžcb^gi W\ZYYXVUW\_"

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

5ggYgga Ybhgi Va ]gg]cb UbX a Uf\_]b[ Wf]hYf]U

G\ci`X'h\Y'WcifgY'\UjY'Ubmbcb!Y`YWfcb]WUggYgga Ybhgi Va ]gg]cbz'h\YgY'g\ci`X'\UjY'U'ghUbXUfX GW\cc``WcjYf'g\YYh'

5```gi Va ]gg]cbg`UfY`YI dYWYX`lc`VY`bYUh`UbX`WYUf`mgYhci H'`Mci f`fYgi `\g`UfY`h\Y`d]bbUWY`cZU```mci f`\UfX k cf\_'UbX`g\ci `X`VY`lfYUhYX`k ]h\`Xi Y`fYgdYWf`DfYgYbh]b[ `fYgi `\g`WYUf`m[ ]j Yg`h\Y`a Uf\_Yf`h\Y`VYghW\UbW\ cZi bXYfgHJbX]b[ `mci f`a Yh\cX/Yj Yb`]Zh\Y`bi a Yf]WJ``fYgi `\g`UfY`]bWcffYWf

 $\label{thm:condition} $$ \ensuremath{\mathsf{U}} = \ensuremath{\mathsf{U}} =$ 

I BGK 'bck '\Ug'U': <u>|hhc'G|h#Gi Va |hfi 'Y</u>ž'k \]W\ 'a YUbg't\\Uh']Zmci 'UhhYa dh'Ub'YI Ua 'cf'gi Va ]h'U'd]YW\'cZ UggYgga Ybhž'mci 'UfY'XYWUf]b[ 'mci fgY`ZZ]hYbci [\'hc'Xc'gc'UbX'WUbbch`UhYf'Udd`m'Zcf'GdYW]U` 7 cbg]XYfUh]cb"

: cf`XYhJ]`g`cZUdd`m]b[ 'Zcf`GdYVJU``7 cbg]XYfUh]cb`UbX`WcbX]h]cbg'Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhUfm UggYgga Ybhž'd`YUgY`gYY`h\Y`]bZcfa Uh]cb`cb`l BGK 'g`<u>GdYVJU``7 cbg]XYfUh]cb`dU[Y</u>"

D`YUgY'bchY'h\UhighiX\Ybhhgik\]bchVY'fYei]fYX'hc'dfcj]XY'UbmXcWa YbhUfmYj]XYbWY'hc giddcfhUVgYbWYg'Zfca'UbmWUggYg'a]ggYX'VYWUigY'cZ'7CJ=8!% 'diV`]W\YU'h\'a YUgifYg'giW\'Ug

## 5 WUXYa ]W<cbYglmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa] ha Ybhhc`ZcghYf]b[``U`W`hi fY`cZ`YUfb]b[`]bZcfa YX`VmUWUXYa]W]bHY[f]hm'`5``IBGK ``ghi XYbhg`\Uj Y`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'UbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[e VWX]`WWSege[`YfZWiadVead[VWSeaXafZWbeS`VbSee[`YfZW_aXSekagdai`ž$ 

D`U[]Uf]ga i]g`U`ImdY`cZ]bhY``YWi U``I\YZI'`±iWUb`HU\_Y`a UbmZcfa gžZfca 'XY`]VYfUhY`W\YUIJb[`Ic`UWJXYbHJ``m Wcdm]b[`Zfca 'U`gci fWYk]h\ci hUW\_bck `YX[Ya Ybh'I BGK '\Ug`dfcXi WYX`U`k YVg]hY`k]h\ci hU'k YU`I\`cZ fYgci fWYg`Ic`gi ddcfhighi XYbhg`Ic`i bXYfgHJbX`UbX`Uj c]X`d`U[]Uf]ga žj]g]h`ghi XYbh'i bgk "YXi "Ui #J`U[]Uf]ga "H\Y`@YUfb]b[`7 YbhY`Ugq]ghg`ghi XYbhg`k]h\i bXYfgHJbX]b[`UWJXYa]W]bHY[f]mYUbX`\ck 'bchIc'd`U[]Uf]gY"

## 5WUXYa]W±bZcfaUhjcb

## 7 fYX]hdc]bhg

 $7\,ci\,fgY\,'WfYX]hi]g\,'WU\,'W'\,'Uh'X\,']b\,'I\,\,b]hg!\,C\,Z\,7\,fYX]hfl\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,C\,(A)$ 

# 5ddYbX]l. '9b[]bYYfg'5i glfU`]U'f95Ł'DfcZYgg]cbU`'9b[]bYYf'7ca dYhYbWm GHJbXUfX

Dfc[fUa '±bHYbXYX'@YUfb]b['CiHWca Yg				