

MANF9400

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ł7`cgYX'cb'di V']W\c']XUmgž'GW\cc`'gW\YXi `YX'Yj Ybhg'UbX'l b]j Yfg]hmG\i hXck b

K YV

GW.cc`cZA YW.Ub]WU`UbX'A Ubi ZUWi f]b['9b[]bYYf]b[

9b[]bYYf]b[]Gh XYbhGi ddcfhGYfi]Wa

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<u>I_BGK_'Ghi Xm5VfcUX'UbX'9I VXUb[Y</u>'flcf']bVci bX'ghi XYbhgŁ

I BGK : i hi fY Ghi XYbhg

D\cbY

fŽ*%&Ł-',),)\$\$ - Bi WYig Gh XYbh<i V

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fŽ*%%&E-',) '' %+- '-' BGK 'Gh Xm5VfcUX'UbX'I BGK '91 WXUb[Y'fZcf']bVci bX'gh XYbhgŁ

fŽ*%"&L'-',) '(\$-+'-'GW\cc`'CZ]WY}

 $\label{thm:linear_continuity} $$ \frac{1}{D}YUgY'bchY'h\Uhih\Y'GW\&cc`'CZ]WY'k]``bch_bck'k\Yb\#Zmcif'WcifgY'WcbjYbcf']g'cb'WUadig'cf'UjUJ'UV'Y$$

7ci fgY 8YHJ] g

7 fYX]hDc]bhg * *

Gi a a UfmcZh\Y 7ci fgY

H\Y'di fdcgY'cZh\]g'Wci fgY']g'hc'dfcj]XY'Ub'i bXYfghUbX]b['cZh\Y'h\Ycf]Yg'UbX'df]bW]d`Yg'cZa cXYfb a UbU[Ya Ybh'UbX'YbWci fU[Y'h\Y'Wci fgY'dUfh]W]dUbhg'hc'\Uj Y'Ub'UddfYW]Uh]cb'cZh\YgY'df]bW]d`Yg']b fY`Uh]cb'hc'h\Y]f'ck b'Yl dYf]YbWYg'UbX'gY`YWhYX'a UbU[Yf]U`'WUgY'ghi X]Yg"'=ggi Yg']bj c'j]b['Yh\]Wgž gi ghU]bUV]`]hmž]bbcj Uh]cb'UbX'W\Ub[Yž'[`cVU`]nUh]cb'UbX'k cf_ZcfWY'X]j Yfg]hmk]``'VY'Yl hYbg]j Y`mXYU`hk]h\]b'h\Y'Wci fgY'hc'YbUV'Y'h\Yghi XYbhg'hc'i bXYfgHUbX'h\YgY'df]bW]d`Yg"

7 ci fgY 5]a g

H\Y`U]a g`cZh\Y`Wci fgY`UfY`hc`i bXYfgHUbX`h\Y`VUg]Wdf]bW]d`Yg`cZa UbU[Ya YbhUbX`h\Y`Zci f`a U^cf \\ b\W]cbg`cZa UbU[Yfg`!`]"Y"`d`Ubb]b[z`cf[Ub]n]b[z`YUX]b[`UbX`Wcbffc``]b[`!`UbX`\ck `a UbU[Yfg`bYYX`hc cdYfUhY"`Ghi XYbfg`k]```VY`fYei]fYX`hc`h\]b_`\Wf]h]\W\``mUbX`gffUhY[]\W\``mUvci ha UbU[Ya Ybhh\Ycf]Yg`UbX]ggi Ygz'k \]\W\`k]```YbUV`Y`h\Ya `hc`XYj Y`cd`h\Y]f`XY\W]g]cb!a U_]b[`UbX`UbU`mh]\W\``g_]``g"`H\Ymk]```VY]bj c`j YX`]b`Udd`]\\JhY`Ufd]Yg`q`UbX`\WJqY`qhi X]Yg`k \]\W\`k]```Uqq]qhh\Ya `hc`XYj Y`cd`[fUXi UhY`Uhff]Vi hYg"

7 ci fqY @YUfb]b['Ci hWca Yq

5 ZhYfgi WWggZ ``mWcad`Yh]b['h\]g'WcifgYžmcig\ci`X'VY'UV`Y'hc.

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%"I bXYfgHJbX'h\Y'h\Ycf]Yg'UbX'df]bW]d`Yg'cZa cXYfb'a UbU[Ya Ybh UbX'Udd`mh\Y'WcbW'dhg'hc'h\Y'a UbU[Ya Ybh'cZcf[Ub]gUh]cbg']b df]j UhY'UbX'di V`]WgYWcf	D9%"%žD9%" žD9%"(žD9%"*
&"I bXYfgfUbX`\ck a UbU[Yfg`WUb`YZZYWf]j Y`mid`Ub`]b`hcXUmfig XmbUa]WYbj]fcba Ybhž`VY`ZJa]`]Uf`k]h\`h\Y`XYg][b`cZcf[Ub]gUh]cb ghfi Wfi fY`UbX`XYgWf]VY`\ck f % †	WhifY' bU[

Υ.

fYgi `hg"; fci d`X]gWi gg]cb`cZWUgY`ghi X]Yg`UbX`dfYgYbHUh]cb`k]```U``ck `Wca a i b]WUh]cb`UbX`]bhYfUWh]cb`cZ]XYUg`UbX`U``ck `h\Y`ghi XYbhg`hc`VY`UWh]j Y`]b`h\Y``YUfb]b[`dfcWigg`UbX`Wca dfY\YbX`\ck `h\Y`df]bW]d`Yg`cZ a UbU[Ya YbhWUb`VY`Udd`]YX`]b`gc`j]b[`cf[Ub]nUh]cbU``dfcV`Ya g"

C bY 'DfY!@YWi fY 'cb`]bY 'UggYgga YbhfB ch; fUXYXŁ]g'dfcj]XYX`]b'k YY_'cbY'hc 'UggYgg'h\Y'[YbYfU`_bck `YX[Y'cZh\Y'ghi XYbhg'Uvci hh\Y'hcd]W'H\Y'di fdcgY']g'hc'dfYdUfY'h\Y'ghi XYbhg'Zcf'h\Y'Wci fgY'UbX dfcj]XY'a cfY'fYgci fWrg']ZbYYXYX"'C bY 'cb`]bY'a i `h]d`Y!W\c]W'hYghff fUXYX'A]X!gYgg]cb'hr'ghz'k]``'VY'\Y'X]b'h\Y'Z]fgh\U'ZcZh\Y'gYgg]cb'hc'dfUWfJW'h\Y'cj YfU``'UddfYWJUh]cb'cZh\Y'[YbYfU`'WcbW'dhg']bj c'j YX']b'h\Y dfYj]ci g'Wci fgY'a UhYf]U'"

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5ggYgga YbhHUg_g

5ggYgga Ybh'hUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
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7 UgY Gh Xm5gg][ba Ybhg	%\$ı	Bch5dd`]WUV`Y	%ž'&ž'' ž'(
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A]X!GYgg]cb'hYgh	&) ı	%(#\$+#&\$&%`\$*.\$\$`DA	% <u>*</u> *&

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5ggYgga Ybh%:]bU`9l Ua]bUh]cb

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.'&'\ci fg

8YHJ]`g.

H\Y`Z]bU``YI Ua]bUh]cb`k]```]bj c`j Y`a i `h]d`Y!W\c]WY`ei Ygh]cbg`Ug`k Y```Ug`XYgWf]dh]j Y`ei Ygh]cbg"
5```Wci fgY`WcbhYbh'UfY`]bWi XYX"'

5XX]h]cbU`XYhJ]`g.

:]bU`YI Ua]bUh]cbg'Zcf'h\Y'Wci fgY']g'\Y`X'Xi f]b['h\Y'I b]j Yfg]lmiYI Ua]bUh]cb'dYf]cX'UbX'H67"

8 Y U X`]b Y 'Zcf' U V gc`i h Y 'ZU]`. 'B #5

A Uf_g'UfY'fYhi fbYX.'K]h\]b'hk c'k YY_g'UZhYf'gi Va]gg]cb'

Bi a VYf'cZUggYgga Ybhg']bWi XYX']b'h\]g'UggYgga Ybh'cbY'Z|bU``Yl Ua '

Gi Va]gg]cb bchYg. J]U A ccXY

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh& 7UgY Gh Xm5gg][ba Ybhg

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. %) \$\$ k cfXg

8 YW] g.

7 UgY 'gh Xm'Ugg][ba Ybhg'k]```]bj c`j Y'k cf_]b[']b'[fci dg'cZ(!) 'Ugg][bYX'cb'WUgYgž'gi Va]hh]b['U'g\cfh fYdcfh'

5XX]h]cbU`XYhJ]`g.

Bia VYfcZUggYgga Ybhg.cbY WUgY ghi Xmi

 $HY'Z_{fgh}WUgY'ghXm'Ug'U'kcfh'cZ''')$ ı UbX'hY'fYgh'UjY'U'kcfh'cZ'''+i

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8 YUX`]bY 'Zcf'UVgc`i hY 'ZU]`. 'cbY 'k YY_'UZhYf'gi Va]gg]cb 'XYUX`]bY 'fk YY_'' ž'(ž) ž'*ž-ž'%\$£'

K\Yb'A Uf_g'UfY'fYh fbYX.'k]h\]b'cbY'kYY_'UZhYf'gi Va]gg]cb'

.

Gi Va]gg]cb bchYg. J]U A ccXY

 $\label{thm:continuous} \begin{tabular}{ll} Hi fb]h]b"gYh]b[."H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\Thi fb]h]b"UbX"gh XYbhg"WUb"gYY"Hi fb]h]b"g]a]"Uf]hm fYdcfhg" \end{tabular}$

5ggYgga Ybh' .; fci d 5gg][ba Ybh

@Yb[h\. (\$\$\$ k cfXg

8YHJ]`g.

H\Y`5gg][ba Ybh`]g`U`[fci d`Ugg][ba Ybh`]bj c`j]b[`(!) `dYcd`Y"`9UW\`[fci d`k]```VY`fYei]fYX`hc`W\ccgY`Ub cf[Ub]gUh]cb`UbX`WcbXi Wh`]b!XYdh\`fYgYUfW\`]bhc`h\Y`a UbU[Ya YbhcZh\Y`cf[Ub]gUh]cb`UbX`dfcj]XY`U dfYgYbhUh]cb`cZh\Y]f`Z|bX]b[g"

5XX]h]cbU`XYhJ]`g.

Bia VYfcZUggYgga Ybhg.cbY WUgY ghi Xmi

8 YUX`]bY Zcf UVgc`i hY ZJ]`. %&#\$, #&\$&%

A Uf_g'UfY'fYh fbYX. 'k c'k YY_g'UZhYf'gi Va]gg]cb'

Gi Va]gg]cb bchYg. J]U A ccXY

 $\label{eq:higher} \begin{tabular}{ll} High \$

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@Yb[h\.'%\$'ai`hjd`Y'W\c]WY

8YHJ]`g.

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7ci fgY'GWXYXi `Y

u	

FYgci fWyg

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HYI Mcc_

FcVV]bgžGDž6Yf[a UbžFžGHJ[jž-žUbX.7ci `hYfžA žA UbU[Ya YbhžDfYbh]WY < U``ž&\$% ž, h\ 'YX]h]cb"

 $\label{eq:conditional} H\Y'hYIhVcc_']g'Uj\ U]`UV`Y'Zcf'di\ fW\UgY'Uh'h\Y'I\ B\ G\ K\ 'Vcc_g\cd''$

FYZYfYbWyg

- 6Uffc`ž? ž'HY]b" A ž'A Uh\Yk gž; ž'A Uff]bž'8 ž'A UbU[Ya Ybh-5 DUW]Z]WF]a : cW gž'A W fUk <]``ž'&\$\$, "
- '7 Ua d`]b[ž'>žDcc`Yž'8 ž'K]gbYfž'F ž'GW\Yfa Yf\cfbž'>F ž'A UbU[Ya Ybhž'>c\b'K]`YmUbX'Gcbgž'5i ghfU`]Už @hXž'&\$\$*"

5bch\Yf`Uj Ybi Y`cZgYUfW\`Wci`X`VY`XcbY`h\fci [\`UWW'gg]b[`5i glfU`]Ub`:]bUbWJU``F Yj]Yk 'g`6cgg`k YVg]hY Vcqg"UZ"Wca "Ui ž`k\YfY`j Uf]ci q`k YVq]hYq`Wb`VY`Zci bX"

I BGK '@]VfUfmik YVg]hY. `\hhdg.#k k k "`]VfUfm'i bgk "YXi "Ui #A ccX`Y. \hhdg.#k ccX`Y"hY`h'i bgk "YXi "Ui #c[]b#bXYI "d\d

7 ci fgY 9j U`i Uhcb UbX 8Yj Y`cda Ybh

: YYXVUW_cb^h\Y`Wci fgY`]g`[Uh\YfYX`dYf]cX]WD``mi g]b['j Uf]ci g`a YUbgž`]bWi X]b['h\Y' BGK a m9I dYf]YbWf`dfcWfggž]bZcfa U``X]gWi gg]cb`]b`h\Y`Z]bU``WUgg`Zcf'h\Y`Wci fgYž'UbX'h\Y GW\cc`'g`Ghi XYbh#GhUZa YYh]b[g"'Mci f'ZYYXVUW_']g`HU_Yb`gYf]ci g`mž'UbX`Wcbh]bi U``]a dfcj Ya Ybhg`UfY a UXY`hc`h\Y`Wci fgY`VUgYXž]b`dUfhžcb`gi W\`ZYYXVUW_"

±b 'h\]g 'Wci fgYz fYW bh]a dfcj Ya Ybhg 'fYgi 'h]b['Zfca 'gh XYbhZYYXVUW_']bWi XY 'dfcj]X]b['a cfY 'fYW bhWJgY gh X]Yg 'UbX]bWi X]b['XYgWf]dh] Y 'ei Ygh]cbg 'U`cb['k]h\ 'a i 'h]d 'Y 'W\c]WY 'ei Ygh]cbg ']b 'h\ Y 'Z]bU` 'YI Ua "

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

5ggYgga Ybhgi Va]gg]cb UbX a Uf_]b[Wf]hYf]U

G\ci`X'h\Y'WcifgY'\UjY'Ubmbcb!Y`YWfcb]WUggYgga Ybhigi Va]gg]cbžh\YgY'g\ci`X'\UjY'U'ghUbXUfX GW\cc``WcjYf'g\YYh'

UggYgga YbhidYfZcfa UbWYžmci 'UfY'Y`][]V`Y`hc'Udd`m'Zcf'GdYWJU``7cbg]XYfUhJcb'df]cf'hc'gi Va]htjb['UbUggYgga Ybhicf'g]htjb['Ub'Yl Ua"

: cf`XYhJ]`g`cZUdd`m]b['Zcf`GdYW]U``7 cbg]XYfUh]cb`UbX`WcbX]h]cbg`Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhUfm UggYgga Ybhž'd`YUgY`gYY`h\Y`]bZcfa Uh]cb`cb`I BGK 'g`<u>GdYWU``7 cbg]XYfUh]cb`dU[Y</u>"

D`YUgY'bchY'h\Uhghi XYbhg'k]``bch'VY'fYei]fYX'hc'dfcj]XY'Ubm'XcW'a YbhUfm'Yj]XYbWY'hc gi ddcfh'UVgYbWg'Zfca 'Ubm'WUggYg'a]ggYX'**VYWUi gY'cZ'7CJ=8!%** 'di V`]W\YU'h\ 'a YUgi fYg'gi W\'Ug]gc`Uh]cb"'| BGK 'k]``bch'VY']bg]gh]b['cb'a YX]WU`'VW'fh]ZJWUh'Yg'Zfca 'UbmcbY'XYYa YX'hc'VY'U'dcg]h]j Y'WUgYZ cf'k\Yb'h\Ym\Uj Y'fYWcj YfYX"'Gi W\'WYfh]ZJWUh'Yg'UfY'X]ZZJW 'hhc'cVhU]b'UbX'di h'Ub'i bbYWYggUfmghfU]b'cb ghi XYbhg'UbX'a YX]WU`'ghUZZ''

 $5\,dd^{3}WUhlcbg^{3}Zcf^{3}gdYW_{U}^{3}W^{3}bg^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}xYfUhlcb^{$

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[`U`W`hifY`cZ`YUfb]b[`]bZcfaYX`VmUWUXYa]W]bHY[f]hm'`5``IBGK ``ghiXYbhg`\UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'`D`U[]Uf]gaibXYfa]bYg`UWUXYa]W]bHY[f]hmUbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVW{\WSeaXafZWbES`VbSee[YfZW_aX\Sekagdai`ž$

Mci 'UfY'U'gc'fYa]bXYX'h\UhWUfYZ ``Hja Y'a UbU[Ya Ybh']g'Ub']a dcffUbhdUfhcZgh XmUbX'cbY'cZh\Y]XYbhJZYX'WUi gYg'cZd`U[]Uf]ga ']g'dccf'hja Y'a UbU[Ya Ybh''Gh XYbhg'g\ci `X'U``ck 'gi ZZWYbh'hja Y'Zcf fYgYUfW\žXfUZjb['UbX'h\Y'dfcdYf'fYZYfYbW]b['cZgci fWYg']b'dfYdUf]b['U```UggYgga Ybh'hUg_g''

₹d`U[]Uf]ga i]gʻZci bXi]bimci fik cf_ik \Ybimci 'UfYi]bʻZ[fghmYUfžimci fi`YWi fYfik]``cZYfimci 'Ugg]ghUbWYihc
]a dfcj Yimci fi'UWUXYa]Wg_]``g'''H\Yma UmUg_imci ihci`cc_'Uhigca Yicb`]bYifYgci fWygžiUhhYbXih\Yi@YUfb]b[
7 YbhfYžcfigca Yh]a YgifYgi Va]himci fik cf_ik]h\h\YidfcV`Ya iZ[l YX'''< ck Yj Yfia cfYigYf]ci gi]bghUbWygi]biZ[fgh
mYUfžigi W\ugighYU`]b[iUbch\Yfighi XYbhgik cf_icfidUm]b[igca YcbYihciXcimci fik cf_žia UmVYi]bj Ygh][UhYX
i bXYfih\YiGhi XYbhA]gWcbXi WhDfcWYXi fYg''

FYdYUHYX'd`U[]Uf]ga 'fYj Yb']b'Z|fghmYUfłź'd`U[]Uf]ga 'UZHYf'Z|fghmYUfźcf'gYf]ci g']bghUbWYgža Um'U`gc'VY]bj Ygh][UhYX'i bXYf'h\Y'Gh XYbhA]gWcbXi WhDfcWYXi fYg"'H\Y'dYbU'h]Yg'i bXYf'h\Y'dfcWYXi fYg'WUb']bWi XY U'fYXi Whcb']b'a Uf_gž'ZU]`]b['U'Wci fgY'cf'Zcf'h\Y'a cghgYf]ci g'a UhHYfg'fi]_Y'd`U[]Uf]ga ']b'Ub'\cbci fg h\Yg]gŁYj Yb'gi gdYbg]cb'Zfca 'h\Y'i b]j Yfg]mm''H\Y'Gh XYbhA]gWcbXi WhDfcWYXi fYg'UfY'Uj U]`UV'Y'\YfY.

kkk"[g"i bgk"YXi"Ui #dc]Wn#XcWa Ybhg#ghi XYbha]gWcbXi WhdfcWYXi fYg"dXZ

5WUXYa]W±bZcfaUhjcb

7 fYX]hdc]bhg

 $7\,ci\,fgY\,'WfYX]hi]g\,'WU\,'W'\,'Uh'X\,']b\,'I\,\,b]hg!\,C\,Z\,7\,fYX]hfl\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,C\,(A)$

