

9a U]`

9b[]bYYf]b['Gh XYbhGi ddcfhGYfj]Wg - W ffYbhgh XYbhYbei]f]Yg

• Y"[""Ybfc`a Ybhž'dfc[fYgg]cbž'WUg\ fYei Yghgž'Wci fgY]ggi Yg'cf'dfc[fUa!fY`UhYX'ei Yf]Yg

<u>9b[]bYYf]b['±bXi glf]U`'HfU]b]b[</u> '--'±bXi glf]U`'lfU]b]b['ei Ygl]cbg

L BGK "Gh Xm5VfcUX"—gh XmUVfcUX gh XYbhYbei]f]Yg flcf]bVci bX gh XYbhgŁ

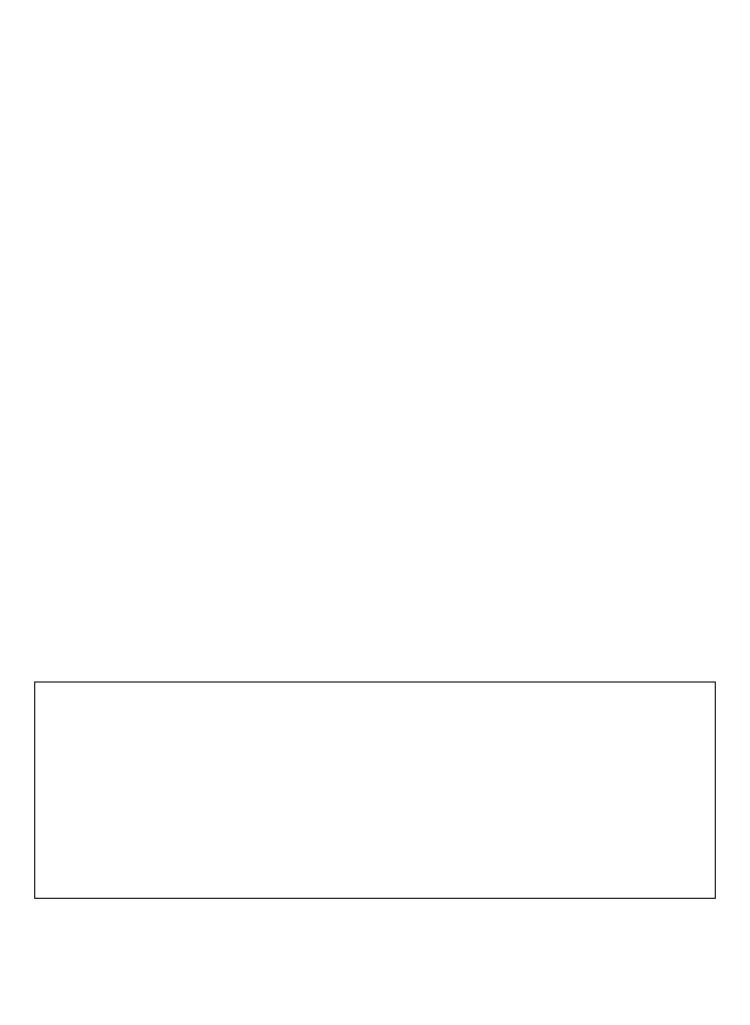
<u>I BGK '91 VX Ub[Y</u>'-'gh XYbhYl VX Ub[Y'Ybei]f]Yg'f7cf']bVci bX'gh XYbhgŁ

I BGK : i h fY Gh XYbhg - dch/bhJU gh XYbhYbei]f]Yg

• Y"["'UXa]gg]cbgž'ZYYgž'dfc[fUa gž'WfYX]hlfUbgZYf

<u>GW.cc``CZZWY</u>'--'GW.cc``[YbYfU``cZZ]WY'UXa]b]glfUljcb'Ybei]f]Yg

B 6. 'h\Y 'fY'Yj UbhihYUa g``]ghYX'UVcj Y'a i ghVY'WcbHUWhYX'Zcf'U``'ghi XYbhYbei]f]Yg"'H\Y'GW\cc``k]``
cb`mVY'UV'Y'hc`fYZYf'ghi XYbhg'cb'hc'h\Y'fY'Yj UbhihYUa ']ZWcbHUWhYX



WUgg"

 $\label{lem:condition} DfcV`Ya~`gc`j~]b[~`gYgg]cbg`dfcj~]XY`fh`Y`cddcfhi~b]fm'Zcf`ghi~XYbfg`fc`fhYghifh`Y]f`WcbW'dhi~U``ZfUa~Yk~cf_`cb~dfcV`Ya~g''$

H\Y``UVcfUhcf]Yg`ZcW`g`cb`h\Y`<YUhHfUbgZYf`Wca dcbYbhcZh\Y`Wci fgY`UbX`dfcj]XY`ghi XYbhg`k]h\`h\Y cddcfhi b]hmhc`Wca dUfY`gdYVjZJWdUfhg`cZh\Y`h\Ycfmhc`dfUWfJWJ``fYgi `hg`]b`U`Wcblfc``YX`Ybj]fcba Ybh'`H\]g`]g hc`YbWci fU[Y`ghi XYbhg`hc`Wcbg]XYf`h\Y`dfUWfJWJ``]a d`]WUhJcbg`cZh\Y]f`h\YcfYhJWJ```YUfb]b["

5ggYgga Ybh

5ggYgga YbhHUg_g

5ggYgga Ybh'tUg_	K Y][\h	8i Y'8UhY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
: cfa Uhjj Y E i]n ·	B#5	&) #\$*#&\$&%	%
A]X HYfa '91 Ua '	' \$ı	\$%#\$+#&\$&%	%
@UVcfUhcfmiF Ydcfhi	&\$ı	%* #\$+#&\$&%	%
5gg][ba Ybh	%) ၊	' \$ <i>#</i> \$+#&\$&%	&
:]bU``91 Ua '	')।	91 Ua "DYf]cXž'XUhY "H67	&ž¨'

5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh%: cfa Uhjj Y Ei]n

GHJfhXUhY. &%#\$*#&\$&%

@Yb[h\. '&'\ci fg

8 YHJ]`g.

 $\label{thm:converse} H\YfY'k\]^``VY'U'A\ ccX`Y'ei\]n'\YX']b'K\ YY_'(\ 'fc'[\]j\ Y'ghi\ XYbhg'h\Y'cddcfhi\ b]hmhc'j\ Yf]Zmh\Uhih\Ym\Uj\ Y\ i\ bXYfghccX'h\Y'a\ UhYf]U``gc'ZJf'''H\]g'ei\]n'\Ug'U'nYfc'dYfWbhf\$i\ E'k\Y][\ \h]b[\ 'UbX'XcYg'bchWcbhf]Vi\ hY'hc mci f'cj\YfU```Wci\fgY'a\ Uf_/\ck\Yj\YfZ'ghi\XYbhg'UfY'YbWci\fU[\YX'hc'dUff]\W]dUhY'''=i]g']bhYbXYX'h\Uhih\Y'ghmY\UbX'X]ZZJW\'hmcZh\Y'A\ ccX`Y'ei\]n'\k\]```VY'fYdfYgYbhJhj\Y'cZh\Uhi]b'h\Y'A\]X'HYfa\ 'YI\ Ua\ '='U'h\ci\[\`a\Uf_]b[\UbX'ZYYXVUW_'Wca\ a\ Ybhg'\k\]```VY'Ui\ hca\Uh]\W\``mUdd`]YX'Vm\Wca\ di\hYf'a\ Uf_]b[\"$

• ±0X]j]Xi U``gi Va]gg]cb'UbX'a Uf_g'fYh fbYX'UZhYf'Wca d`Yh]cb

• 8 Y U X`]b Y 'Zcf' U V gc`i h Y 'ZU]`. 'B #5

Gi Va]gg]cb bchYg. J]U A ccXY

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh& A]X HYfa '91 Ua

GHJfh:XUhY. \$\%#\$+#&\$&\%

@Yb[h\.'&'<cifg

8YHJ]`g.

HYghWcj Yf]b['\YUh'hfUbgZYf'Wca dcbYbhcZh\Y'Wci fgY"

8YHJ]`g.

- HYghWcj Yf]b['Zi]Xgža]I hi fYg UbX Wca Vi gh]cb Wca dcbYbhcZh\Y Wci fgY"
- 5ggYgga Ybh'Wf]hYf]U.':]bU`Yl Ua 'hYgh]b['a UhYf]U`'Zfca 'k YY_g') 'hc'%\$
- ±bX]j]Xi U``gi Va]gg]cb`UbX`A Uf_g`fYh fbYX`i dcb`fY`YUgY`cZZ]bU``fYgi `hg
- 8 YUX`]bY'Zcf'UVgc`i hY'ZJ]`. 'B#5

Gi Va]gg]cb bchYg. J]UA ccXY

Hi fb]h]b gYh]b[. H\]g]g bchU Hi fb]h]b Ugg][ba Ybh

1	

FYgci fWyg

FYWca a YbXYX FYgci fWYg

£; .: WSfS`V? See FdS`eXWd 8g`VS_W fS&S`V 3bb/[USf[a`e

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

5ggYgga Ybhgi Va]gg]cb UbX a Uf_]b[Wf]hYf]U

G\ci`X'h\Y'WcifgY'\UjY'Ubmbcb!Y`YWfcb]WUggYgga Ybhigi Va]gg]cbž'h\YgY'g\ci`X'\UjY'U'ghUbXUfX GW\cc`WcjYf'g\YYh'

5```gi Va]gg]cbg`UfY`YI dYWYX`hc`VY`bYUh`UbX`WYUf`mgYhci h'`Mci f`fYgi `hg`UfY`h\Y`d]bbUWY`cZU```mci f`\UfX k cf_'UbX`g\ci `X`VY`hfYUhYX`k]h\`Xi Y`fYgdYWf`DfYgYbh]b[`fYgi `hg`WYUf`m[]j Yg`h\Y`a Uf_Yf`h\Y`VYghW\UbW\ cZi bXYfghUbX]b[`mci f`a Yh\cX/Yj Yb`]Zh\Y`bi a Yf]WU``fYgi `hg`UfY`]bWcffYWf

A Uf_]b['[i]XY`]bYg`Zcf`Ugg][ba Ybhgi Va]gg]cbg`k]```VY`dfcj]XYX`Uhh\Y`gUa Y`h]a Y`Ug`Ugg][ba YbhXYhU]`g hc`Ugg]ghk]h\`a YYh]b['UggYggUV`Y`fYei]fYa Ybhg"`Gi Va]gg]cbg`k]```VY`a Uf_YX`UVWcfX]b['hc`h\Y`a Uf_]b[[i]XY`]bYg`dfcj]XYX"

@UhY 'dc`]Wh

K cf_'gi Va]hhYX``UhY`k]h\ci hUb`Uddfcj YX`YI hYbg]cb`Vmh\Y`Wci fgY`WcfX]bUhcf`cf`XY`Y[UhYX`Ui h\cf]hm]g gi VYWhhc`U``UhY`dYbU`hmcZ&\$`dYfWYbhf&\$i \ \!cZh\Y`a Ul]a i a 'a Uf_'dcgg]V`Y`Zcf`h\UhUggYgga Ybh]hYa ž dYf`WU`YbXUf`XUm'

 $\label{eq:conversion} \text{K cf_'gi Va]} \text{httx:'UZhtf:h.Y:'XYUX`]bY:'Zcf:'UVgc`i htt:'ZU]'':]g'bchUWWtdhtx:'UbX:'U'a Uf_'cZ'nYfc'k]```VY'Uk UfXYX Zcf:h.Uh:'UgqYgga Ybh']hta " } \\$

: cf`gca Y`UggYgga Ybh`]hYa gž`U``UhY`dYbU`hma UmbchVY`Uddfcdf]UhY"`H\YgY`UfY`WYUf`m`]bX]\WUhYX`]b`h\Y Wci fgY`ci h`]bYž`UbX`gi W\`UggYgga Ybhg`fYWY]j Y`U`a Uf_`cZnYfc`]ZbchWca d`YhYX`Vmh\Y`gdY\\JZ]YX`XUhY" 9I Ua d`Yq`]bWi XY.

%" K YY_`mcb`]bY`hYghg`cf``UVcfUhcfmk cf_`k cfh\`U`ga U```dfcdcfh]cb`cZh\Y`gi VYWha Uf_žcf &" C b`]bY`ei]nnYg`k \YfY`Ubgk Yfg`UfY`fY`YUgYX`hc`gh XYbhg`cb`Wca d`Yh]cbžcf

- "DfcZYgg]cbU``UggYgga YbhihUg_gž'k\YfY`h\Y`]bhYbh]cb`]g`hc`WfYUhY`Ub`Ui h\Ybh]WUggYgga Ybhih\Uh\Ug`Ub`UVgc`i hY`gi Va]gg]cb`XUhYž'cf
- ("DUgg# U]`UggYgga YbhHJg_g"

91 Ua]bUh]cbg

UggYgga YbhidYfZcfa UbWYžmci 'UfY'Y`][]V`Y`hc'Udd`m'Zcf'GdYWJU``7cbg]XYfUhJcb'df]cf'hc'gi Va]htjb['UbUggYgga Ybhicf'g]htjb['Ub'Yl Ua"

: cf`XYhJ]`g`cZUdd`m]b['Zcf`GdYW]U``7cbg]XYfUh]cb`UbX`WcbX]h]cbg`Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhUfm UggYgga Ybhž'd`YUgY`gYY`h\Y`]bZcfa Uh]cb`cb`I BGK 'g`<u>GdYW]U``7cbg]XYfUh]cb`dU[Y</u>"

D`YUgY'bchY'h\Uhghi XYbhg'k]``bch'VY'fYei]fYX'hc'dfcj]XY'Ubm'XcW'a YbhUfm'Yj]XYbWY'hc gi ddcfh'UVgYbWg'Zfca 'Ubm'WUggYg'a]ggYX'**VYWUi gY'cZ'7CJ=8!%** 'di V`]W\YU'h\ 'a YUgi fYg'gi W\'Ug]gc`Uh]cb"'| BGK 'k]``bch'VY']bg]gh]b['cb'a YX]WU`'VW'fh]ZJWUh'Yg'Zfca 'UbmcbY'XYYa YX'hc'VY'U'dcg]h]j Y'WUgYZ cf'k\Yb'h\Ym\Uj Y'fYWcj YfYX"'Gi W\'WYfh]ZJWUh'Yg'UfY'X]ZZJW 'hhc'cVhU]b'UbX'di h'Ub'i bbYWYggUfmghfU]b'cb ghi XYbhg'UbX'a YX]WU`'ghUZZ''

 $5\,dd^{3}WUhlcbg^{3}Zcf^{3}gdYW_{U}^{3}W^{3}bg^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}xYfUh$

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[`U`W`hifY`cZ`YUfb]b[`]bZcfaYX`VmUWUXYa]W]bHY[f]hm'`5``IBGK ``ghiXYbhg`\UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'`D`U[]Uf]gaibXYfa]bYg`UWUXYa]W]bHY[f]hmUbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVW{\WSeaXafZWbES`VbSee[YfZW_aX\Sekagdai`ž$

Mci 'UfY'U'gc'fYa]bXYX'h\UhWUfYZ ``h]a Y'a UbU[Ya Ybh]g'Ub']a dcftUbhdUfhcZgh XmUbX'cbY'cZh\Y]XYbh]Z]YX'WUi gYg'cZd`U[]Uf]ga ']g'dccf'h]a Y'a UbU[Ya Ybh'Gh XYbhg'g\ci `X'U``ck 'gi Z]W]Ybh'h]a Y'Zcf fYgYUfW\žXfUZ\]b['UbX'h\Y'dfcdYf'fYZYfYbW]b['cZgci fWYg']b'dfYdUf]b['U``'UggYgga Ybh'lUg_g''

₹d`U[]Uf]ga i]gʻZci bXi]bimci fik cf_ik \Ybimci 'UfYi]bʻZ[fghmYUfžimci fi`YWi fYfik]``cZYfimci 'Ugg]ghUbWYihc
]a dfcj Yimci fi'UWUXYa]Wg_]``g'''H\Yma UmUg_imci ihci`cc_'Uhigca Yicb`]bYifYgci fWygžiUhhYbXih\Yi@YUfb]b[
7 YbhfYžcfigca Yh]a YgifYgi Va]himci fik cf_ik]h\h\YidfcV`Ya iZ[l YX'''< ck Yj Yfia cfYigYf]ci gi]bghUbWygi]biZ[fgh
mYUfžigi W\ugighYU`]b[iUbch\Yfighi XYbhgik cf_icfidUm]b[igca YcbYihciXcimci fik cf_žia UmVYi]bj Ygh][UhYX
i bXYfih\YiGhi XYbhA]gWcbXi WhDfcWYXi fYg''

FYdYUHYX'd`U[]Uf]ga 'fYj Yb']b'Z|fghmYUfłź'd`U[]Uf]ga 'UZHYf'Z|fghmYUfźcf'gYf]ci g']bghUbWYgža Um'U`gc'VY]bj Ygh][UhYX'i bXYf'h\Y'Gh XYbhA]gWcbXi WhDfcWYXi fYg"'H\Y'dYbU'h]Yg'i bXYf'h\Y'dfcWYXi fYg'WUb']bWi XY U'fYXi Whcb']b'a Uf_gž'ZU]`]b['U'Wci fgY'cf'Zcf'h\Y'a cghgYf]ci g'a UhHYfg'fi]_Y'd`U[]Uf]ga ']b'Ub'\cbci fg h\Yg]gŁYj Yb'gi gdYbg]cb'Zfca 'h\Y'i b]j Yfg]mm''H\Y'Gh XYbhA]gWcbXi WhDfcWYXi fYg'UfY'Uj U]`UV'Y'\YfY.

kkk"[g"i bgk"YXi"Ui #dc]Wn#XcWa Ybhg#ghi XYbha]gWcbXi WhdfcWYXi fYg"dXZ

5WUXYa]W±bZcfaUhjcb

7 fYX]hdc]bhg

 $7\,ci\,fgY\,'WfYX]hi]g\,'WU\,'W'\,'Uh'X\,']b\,'I\,\,b]hg!\,C\,Z\,7\,fYX]hfl\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,C\,(A)$

- A ccX`Y
- <u>@UV'5WWYgg</u> 7 ca di hb[': UW]hYg

5ddYbX]l. '9b[]bYYfg'5i glfU`]U'f95Ł'DfcZYgg]cbU`'9b[]bYYf'7ca dYhYbWm GHJbXUfX

Dfc[fUa '±bhYbXYX'@YUfb]b['Ci hWca Yg	
?bck `YX[Y`UbX`g_]```VUgY	
D9%'%7ca dfY\Ybg]j Yž'h\YcfmVUgYX'i bXYfgHJbX]b['cZh\Y'i bXYfd]bb]b['bUhi fU`'UbX	