

# **MECH9650**

A ]Wfc UbX 6]cZi ]X]Wg

HYfa 'Hk c''##'&\$&%

# 7ci fgY Cj Yfj ]Yk

## GHJZ:7cbHJWh8YHJ]`g

#### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]m	@cWUhjcb	D\cbY
Gi gUbb 6Y]Yf		D`YUgY'fYUW\'cih j]U'hYUag'cb`m! Y]h\Yf`]b'h\Y U``cWUhYX'hYUa'cf j]U'df]jUhY'aYggU[Y	>%+	

#### GWkcc~7cbhJWh~abZcfa Uhlcb

#### @cWUhjcb

I BGK "A YVX.Ub]WU``UbX"A Ubi ZUWi f]b[ '9b[ ]bYYf]b[

5]bgk cfh\ 'Vi ]`X]b[ '>%+ž'@Yj Y`'%

5Vcj Y '7 c ZZYY 'cb '7 Ua di g

#### < ci fg

-.\$\$-).\$\$da ž'A cbXUm-: f]XUm

ł7`cgYX'cb'di V`]W\c`]XUmgž'GW\cc``gW\YXi `YX'Yj Ybhg'UbX'I b]j Yfg]hm'G\i hXck b

#### K YV

GWXcc``cZA YVXUb]WU``UbX`A Ubi ZUWf f]b[ '9b[ ]bYYf]b[

9b[]bYYf]b[ Gh XYbhGi ddcfhGYfj ]Wg

9b[]bYYf]b['=bXi ghf]U`'HfU]b]b[

LBGK Ghi Xm5VfcUX UbX 91 VX Ub[Y flcf]bVci bX ghi XYbhgŁ

#### I BGK : i h fY Gh XYbhq

#### D\cbY

 $f\check{L}^*\%\&L^-'$  , ) ', ) \$\$ — Bi WYi g Gh XYbh < i V

fŽ\*%%&L'-',) '+\*\*%'-'9b[]bYYf]b['=bXi glf]U`'HfU]b]b[

 $f \c X^* \% \& E-', ) `` \% +- \c -' B G K ``Gh Xm 5 V f c U X ``Ub X ``I B G K ``9 I W X U b [`Y`f \c f`] b V c i b X ``gh X Y b h g k B G K ``Gh Xm 5 V f c U X ``Ub X ``I B G K ``9 I W X U b [`Y`f \c f`] b V c i b X ``gh X Y b h g k B G K ``Gh Xm 5 V f c U X ``Ub X ``I B G K ``9 I W X U b [`Y`f \c f`] b V c i b X ``gh X Y b h g k B G K ``Gh Xm 5 V f c U X ``Ub X ``I B G K ``9 I W X U b [`Y`f \c f`] b V c i b X ``gh X Y b h g k B G K ``Gh Xm 5 V f c U X ``Ub X ``I B G K ``9 I W X U b [`Y`f \c f`] b V c i b X ``gh X Y b h g k B G K ``Gh Xm 5 V f c U X ``Ub X ``I B G K ``9 I W X U b [`Y`f \c f`] b V c i b X ``gh X Y b h g k B G K ``gh X Y b h g k B G K ``gh X Y b h g k B G K ``gh X Y b h g k B G K ``gh X Y b h g k B G K ``gh X Y b h g k B G K ``gh X Y b h g k B G K ``gh X Y b h g k B G K ``gh X Y b h g k B G K ``gh X B G K ``gh$ 

fŽ\*%"&Ł'-',) '(\$-+'-'GW\cc`'CZZW}}

 $\label{thm:condition} $$ \frac{1}{D}YUgY'bchY'h\Uhh\Y'GW\&cc'CZ_WY'k_]^bch_bck'k_Yb\#Zmcif'WcifgY'WcbjYbcf']g'cb'WUadig'cfUjU]^UVY$$ 

#### 9a U]`

9b[]bYYf]b[]Gh XYbhGi ddcfhGYfj ]WYq - W ffYbhgh XYbhYbei ]f]Yg

• Y"[""Ybfc`a Ybhž'dfc[fYgg]cbž'WUg\"fYei Yghgž'Wci fgY']ggi Yg'cf'dfc[fUa!fY`UhYX'ei Yf]Yg

9b[]bYYf]b[] \*\Delta Xi alf]U``HfU]b]b[] -- \*\Delta Xi alf]U``HfU]b]b[] ei Yahaba

I BGK 'Gh Xm5VfcUX'-gh XmUVfcUX'gh XYbhYbei ]f]Yg'fZcf']bVci bX'gh XYbhgŁ

I BGK '91 WXUb[Y'-ghi XYbhYl WXUb[Y'Ybei ]f]Yg'fZcf']bVci bX'ghi XYbhgŁ

L B G K :: i h f Y G h XYbhg - dch Ybh U gh XYbh Ybei ]f ] Y g

Y"["UXa]gg]cbgžZYYgždfc[fUagžWfYX]hlfUbgZYf

<u>GWXcc``CZZJWY</u>'--'GWXcc``[YbYfU``cZZJWY'UXa]b]glfUljcb'Ybei]f]Yg

• B6. h\Y fY Yj UbhhYUa g``]ghYX UVcj Y a i ghVY WcbhUWhYX Zcf'U``gh XYbhYbei ]f]Yg"'H\Y GW\cc``k]`` cb`mVY UV`Y hc fYZYf'qh XYbhq'cb'hc h\Y fY`Yj UbhHYUa ']ZWcbhUWhYX

7 ci fgY 8 YHJ]`g 7 fYX]hDc]bhg \*\* Gi a a UfmcZh\Y 7 ci fgY

# 5ggYgga Ybh

 $A \ Uf\_g``UfY``fYh \ fbYX``k \ ]h`\]b``\&``k \ YY\_g``cZh`\Y``Xi \ Y``XUhY``$ 

< ck 'ho'gi Va ]h'Mci fi'gi Va ]h'U``'UggYgga Ybhg'j ]U'a ccX`Y'

,

# 5ggYgga Ybh'HUg\_g

5ggYgga Ybh'HJg_	K Y][ \h	8i Y'8UHY	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX
: UVf]WUH]cb UbX Yj U`i UH]cb cZ U`a ]Wfc!a ]I Yf#Zi ]X]Wg gmghYa	&\$ı	% #\$*#&\$&%'\$).\$\$'DA	%ž'&ž' ž'(ž')
9j U'i UHY'a ]Wfc! UbX V]cZi ]X]WXYj ]Wfg'UbX gmgHYa g'	&\$I	K - ': f]') da '	%ž &ž'  ž'( ž')

cj YfWca Y 'h\Y 'dfcV`Ya "'h\Y 'hYUa g'cZghi XYbhg'WUb'W\ccgY 'Zfca 'U'fUb[ Y'cZ`UhYghfYgYUfW\'hc'dfYgYbhž X]gW gg'UbX'Yj U`i UhY'h\YgY XYj ]WYg'i g]b[ 'h\Y 'hcc`g'h\Ym`YUfbYX"

#### 5XX]h]cbU`XYhJ]`q.

5 Vgc`i hY`ZU]``K %\$': f]XUm') da '

5ggYgga Ybh' . E i ]nnYg

GHJfhXUhY. K ) "UbX"K , "A cbXUmi%"" \$Ua

@Yb[h\.'\*\$a]b

8YHJ]`g.

H\Y`ghi XYbhg`k]``VY`Ug\_YX`hc`WU`W``UhY`fY`Yj UbhidfcV`Ya `gWfbUf]cg`UbX`Ubgk Yf`VUg]Whc`Wca d`Yl ei Ygh]cbg`hc`XYa cbghfUhY`h\Y]f`i bXYfghUbX]b[ `cZh\Y`X]ZYfYbWf`VYhk YYb`a UWfc`UbX`a ]WfcZi ]X]Wg`UbX h\Y]f`fY`Yj UbWf`Zcf`V]c`c[ ]WU``gmghYa g"

#### 5XX]h]cbU`XYhU]`g.

5 Vgc i hY ZU] K) UbX K, : f]XUm) da

Gi Va ]gg]cb bchYg. H\YfY UfY hk c UggYgga Ybhg Zcf h\]g WUhY [cfm

5ggYgga Ybh (. 7 Ugg Wcbhf]Vi h]cb

GHUfhXUhY. K %A cb

@Yb[h\. h\fci [\ci hh\Y Wti fgY

8YHJ]`g.

H\Y`gh XYbhg`k]``VY`UggYggYX`cb`h\Y]f`WcbhfVi h]cb`h\fci [\ci hh\Y`Wci fgYž'h\]g`k]```]bWi XY`UhhYbXUbW'`

## FYgci fW/g

## DfYgWf]VYX F Ygci fWYg

YHYI Mcc\_

@YWhifY bchYg UbX g`]XYg

J]fhiU`@UV

Ch\YfAccXY fYgci fWg

#### FYWca a YbXYX FYgci fWyg

FYWca a YbXYX`HYI h/cc ...'6]cZi ]X`A YVX.Ub]Wg`! '5b`+bhfcXi Whjcb`hc': `i ]X`A YVX.Ub]WgžA UWfcWjfWi `Uhjcbž'UbX A ]WfcWjfWi `UhjcbI F@

I BGK "@]VfUfmik YVg]hY. "\hdg.#k k k "`]VfUfm'i bgk "YXi "Ui #

A ccX`Y. `\hdg.##a ccX`Y"hY`h'i bgk "YXi "Ui #c[]b#|bXYI "d\d

#### 7ci fgY 9j U'i Uhjcb UbX 8Yj Y'cda Ybh

: YYXVUW\_cb^h\Y`Wci fgY^]g^[ Uh\YfYX'dYf]cX]WU``mi g]b[ 'j Uf]ci g^a YUbgž]bWi X]b[ 'h\Y'I BGK a m9I dYf]YbW'dfcW'ggž]bZcfa U``X]gW gg]cb^]b^h\Y`Z]bU``WUgg^Zcf^h\Y`Wci fgYž'UbX'h\Y`GW\cc^\*g Gh XYbh#GhUZZa YYh]b[ g"'Mci f^ZYYXVUW\_]g^hU\_Yb^gYf]ci g`mž'UbX'Wcbh]bi U``]a dfcj Ya Ybhg`UfY`a UXY`hc^h\Y Wci fgY`VUgYXž]b^dUfhžcb^gi W\ZYYXVUW\_"

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

## 5ggYgga Ybhgi Va ]gg]cb UbX a Uf\_]b[ Wf]hYf]U

G\ci`X'h\Y'WcifgY'\UjY'Ubmbcb!Y`YWfcb]WUggYgga Ybhigi Va ]gg]cbž'h\YgY'g\ci`X'\UjY'U'ghUbXUfX GW\cc`WcjYf'g\YYh'

5```gi Va ]gg]cbg`UfY`YI dYWYX`hc`VY`bYUh`UbX`WYUf`mgYhci h'`Mci f`fYgi `hg`UfY`h\Y`d]bbUWY`cZU```mci f`\UfX k cf\_'UbX`g\ci `X`VY`hfYUhYX`k ]h\`Xi Y`fYgdYWf`DfYgYbh]b[ `fYgi `hg`WYUf`m[ ]j Yg`h\Y`a Uf\_Yf`h\Y`VYghW\UbW\ cZi bXYfghUbX]b[ `mci f`a Yh\cX/Yj Yb`]Zh\Y`bi a Yf]WU``fYgi `hg`UfY`]bWcffYWf

A Uf\_]b[ '[ i ]XY`]bYg`Zcf`Ugg][ ba Ybhgi Va ]gg]cbg`k ]```VY`dfcj ]XYX`Uhh\Y`gUa Y`h]a Y`Ug`Ugg][ ba YbhXYhU]`g hc`Ugg]ghk ]h\`a YYh]b[ 'UggYggUV`Y`fYei ]fYa Ybhg"`Gi Va ]gg]cbg`k ]```VY`a Uf\_YX`UVWcfX]b[ 'hc`h\Y`a Uf\_]b[ [ i ]XY`]bYg`dfcj ]XYX"

#### @UhY 'dc`]Wh

K cf\_'gi Va ]hhYX``UhY`k ]h\ci hUb`Uddfcj YX`YI hYbg]cb`Vmh\Y`Wci fgY`WcfX]bUhcf`cf`XY`Y[ UhYX`Ui h\cf]hm]g gi VYWhhc`U``UhY`dYbU`hmcZ&\$`dYfWYbhf&\$i \ \!cZh\Y`a Ul ]a i a 'a Uf\_'dcgg]V`Y`Zcf`h\UhUggYgga Ybh]hYa ž dYf`WU`YbXUf`XUm'

 $\label{eq:conversion} \text{K cf\_'gi Va ]} \text{httx:'UZhtf:h.Y:'XYUX`]bY:'Zcf:'UVgc`i htt:'ZU]'':]g'bchUWWtdhtx:'UbX:'U'a Uf\_'cZ'nYfc'k ]```VY'Uk UfXYX Zcf:h.Uh:'UgqYgga Ybh']hta " } \\$ 

: cf`gca Y`UggYgga Ybh`]hYa gž`U``UhY`dYbU`hma UmbchVY`Uddfcdf]UhY"`H\YgY`UfY`WYUf`m`]bX]\WUhYX`]b`h\Y Wci fgY`ci h`]bYž`UbX`gi W\`UggYgga Ybhg`fYWY]j Y`U`a Uf\_`cZnYfc`]ZbchWca d`YhYX`Vmh\Y`gdY\\JZ]YX`XUhY" 9I Ua d`Yq`]bWi XY.

%" K YY\_`mcb`]bY`hYghg`cf``UVcfUhcfmk cf\_`k cfh\`U`ga U```dfcdcfh]cb`cZh\Y`gi VYWha Uf\_žcf &" C b`]bY`ei ]nnYg`k \YfY`Ubgk Yfg`UfY`fY`YUgYX`hc`gh XYbhg`cb`Wca d`Yh]cbžcf

- "DfcZYgg]cbU``UggYgga YbhihUg\_gž'k\YfY`h\Y`]bhYbh]cb`]g`hc`WfYUhY`Ub`Ui h\Ybh]WUggYgga Ybhih\Uh\Ug`Ub`UVgc`i hY`gi Va ]gg]cb`XUhYž'cf
- ("DUgg# U]`UggYgga YbhHJg\_g"

## 91 Ua ]bUh]cbg

UggYgga YbhidYfZcfa UbWYžmci 'UfY'Y`][]V`Y`hc'Udd`m'Zcf'GdYWJU``7cbg]XYfUhJcb'df]cf'hc'gi Va]htjb['UbUggYgga Ybhicf'g]htjb['Ub'Yl Ua"

I BGK 'bck '\Ug'U': <u>|hhc'G|h#Gi Va |hfi `Y</u>ž'k \]W\ 'a YUbg't\\Uh']Zmci 'UthYa dh'Ub'Yl Ua 'cf'gi Va ]h'U'd]YW'cZ UggYgga Ybhž'mci 'UfY'XYWUf]b[ 'mci fgY`ZZ]h'Ybci [\'hc'Xc'gc'UbX'WUbbch`UhYf'Udd`mZcf'GdYW]U` 7 cbg]XYfUh]cb"

: cf`XYhJ]`g`cZUdd`m]b[ 'Zcf`GdYW]U``7cbg]XYfUh]cb`UbX`WcbX]h]cbg`Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhUfm UggYgga Ybhž'd`YUgY`gYY`h\Y`]bZcfa Uh]cb`cb`I BGK 'g`<u>GdYW]U``7cbg]XYfUh]cb`dU[Y</u>"

D`YUgY'bchY'h\Uhghi XYbhg'k]``bch'VY'fYei]fYX'hc'dfcj]XY'Ubm'XcW'a YbhUfm'Yj]XYbWY'hc gi ddcfh'UVgYbWg'Zfca 'Ubm'WUggYg'a]ggYX'**VYWUi gY'cZ'7CJ=8!%** 'di V`]W\YU'h\ 'a YUgi fYg'gi W\'Ug]gc`Uh]cb"'| BGK 'k]``bch'VY']bg]gh]b[ 'cb'a YX]WU`'VW'fh]ZJWUh'Yg'Zfca 'UbmcbY'XYYa YX'hc'VY'U'dcg]h]j Y'WUgYZ cf'k\Yb'h\Ym\Uj Y'fYWcj YfYX"'Gi W\'WYfh]ZJWUh'Yg'UfY'X]ZZJW 'hhc'cVhU]b'UbX'di h'Ub'i bbYWYggUfmghfU]b'cb ghi XYbhg'UbX'a YX]WU`'ghUZZ''

 $5\,dd^{3}WUhlcbg^{3}Zcf^{3}gdYW_{U}^{3}W^{3}bg^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}xYfUh$ 

# 5 WUXYa ]W<cbYghmUbX D`U[ ]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[`U`W`hifY`cZ`YUfb]b[`]bZcfaYX`VmUWUXYa]W]bHY[f]hm'`5``IBGK ``ghiXYbhg`\UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'`D`U[]Uf]gaibXYfa]bYg`UWUXYa]W]bHY[f]hmUbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVW{\WSeaXafZWbES`VbSee[YfZW_aX\Sekagdai`ž$ 

Mci 'UfY'U'gc'fYa ]bXYX'h\UhWUfYZ ``h]a Y'a UbU[ Ya Ybh]g'Ub']a dcftUbhdUfhcZgh XmUbX'cbY'cZh\Y ]XYbh]Z]YX'WUi gYg'cZd`U[ ]Uf]ga ']g'dccf'h]a Y'a UbU[ Ya Ybh'Gh XYbhg'g\ci `X'U``ck 'gi Z]W]Ybh'h]a Y'Zcf fYgYUfW\žXfUZ\]b[ 'UbX'h\Y'dfcdYf'fYZYfYbW]b[ 'cZgci fWYg']b'dfYdUf]b[ 'U``'UggYgga Ybh'lUg\_g''

₹d`U[]Uf]ga i]gʻZci bXi]bimci fik cf\_ik \Ybimci 'UfYi]bʻZ[fghmYUfžimci fi`YWi fYfik]``cZYfimci 'Ugg]ghUbWYihc
]a dfcj Yimci fi'UWUXYa]Wg\_]``g'''H\Yma UmUg\_imci ihci`cc\_'Uhigca Yicb`]bYifYgci fWygžiUhhYbXih\Yi@YUfb]b[
7 YbhfYžcfigca Yh]a YgifYgi Va]himci fik cf\_ik]h\h\YidfcV`Ya iZ[l YX'''< ck Yj Yfia cfYigYf]ci gi]bghUbWygi]biZ[fgh
mYUfžigi W\ugighYU`]b[iUbch\Yfighi XYbhgik cf\_icfidUm]b[igca YcbYihciXcimci fik cf\_žia UmVYi]bj Ygh][UhYX
i bXYfih\YiGhi XYbhA]gWcbXi WhDfcWYXi fYg''

FYdYUHYX'd`U[]Uf]ga 'fYj Yb']b'Z|fghmYUfłź'd`U[]Uf]ga 'UZHYf'Z|fghmYUfźcf'gYf]ci g']bghUbWYgža Um'U`gc'VY ]bj Ygh][UhYX'i bXYf'h\Y'Gh XYbhA ]gWcbXi WhDfcWYXi fYg"'H\Y'dYbU'h]Yg'i bXYf'h\Y'dfcWYXi fYg'WUb']bWi XY U'fYXi Whcb']b'a Uf\_gž'ZU]`]b['U'Wci fgY'cf'Zcf'h\Y'a cghgYf]ci g'a UhHYfg'fi]\_Y'd`U[]Uf]ga ']b'Ub'\cbci fg h\Yg]gŁYj Yb'gi gdYbg]cb'Zfca 'h\Y'i b]j Yfg]mm''H\Y'Gh XYbhA ]gWcbXi WhDfcWYXi fYg'UfY'Uj U]`UV'Y'\YfY.

kkk"[g"i bgk"YXi"Ui #dc]Wn#XcWa Ybhg#ghi XYbha ]gWcbXi WhdfcWYXi fYg"dXZ

# 5 WUXYa] W = b Zcfa Uh]cb

# 7 fYX]hdc]bhg

 $7\,ci\,fgY\,'WfYX]hi]g\,'WU\,'W'\,'Uh'X\,']b\,'I\,\,b]hg!\,C\,Z\,7\,fYX]hfl\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf\_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,C\,(A)$ 

- A ccX`Y
- <u>@UV'5WWYgg</u> 7 ca di hb[': UW]hYg

# 5ddYbX]l. '9b[]bYYfg'5i glfU`]U'f95Ł'DfcZYgg]cbU`'9b[]bYYf'7ca dYhYbWm GHJbXUfX

Dfc[fUa '±ohYbXYX'@YUfb]b['CihWca Yg				
? bck `YX[ Y`UbX`g_]```VUgY				
D9%"%7cadfY\Ybg]jYžh\YcfmVUgYX`ibXYfghUbX]b[`cZh\Y`ibXYfd]bb]b[`bUhifU``UbXd\mg]\W'`g\\Yb\W`g`UbX`h\Y`Yb[]bYYf]b[`ZbXUaYbhU`g'Udd`]\WV`Y`hc'h\Y`Yb[]bYYf]b[`X]g\\]d`]bY				
D9%"&"7cbWYdhi U``i bXYfgHJbX]b[ 'cZh\Y'a Uh\Ya Uh]Wgž'bi a Yf]WJ``UbU`mg]gž'gHJh]gh]Wgž'UbX Wca di hYf'UbX']bZcfa Uh]cb'gW]YbWYg'k\]W\`i bXYfd]b'h\Y'Yb[ ]bYYf]b[ 'X]gW]d`]bY				
D9%" '±0!XYdh\'i bXYfghUbX]b[ 'cZgdYW]U']ghVcX]Yg'cZ_bck 'YX[ Y'k ]h\]b'h\Y'Yb[ ]bYYf]b[ X]gW]d`]bY				