

MMAN1130

8 Yg][b'UbX'A Ubi ZUWi f]b[

HYfa 'Hk c''##'&\$&%

7 ci fgY Cj Yfj]Yk

GHJZ:7cbHJWh'8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWJh]cb	D\cbY
8Ub]Y``9[[`Yf	X"Y[[`Yf4 i bgk "YXi "Ui	9a U]`'hc'Vcc_'U Wcbgi `hUh]cb	(\$&<ž 5]bgk cfh\ 6i]`X]b[

GWkcc~7cbhJWh~abZcfa Uh]cb

@c WUhlc b

I BGK "A YVX.Ub]WU``UbX"A Ubi ZUWi f]b['9b[]bYYf]b[

5]bgk cfh\ 'Vi]`X]b['>%+ž'@Yj Y` '%

5Vcj Y 7 c ZZYY cb 7 Ua di g

< ci fg

-.\$\$-).\$\$da ž'A cbXUm-: f]XUm

ł7`cgYX'cb'di V`]W\c`]XUmgž'GW\cc``gW\YXi `YX'Yj Ybhg'UbX'l b]j Yfg]hm'G\i hXck b

K YV

GWXcc``cZA YVXUb]WU``UbX`A Ubi ZUWi f]b['9b[]bYYf]b[

9b[]bYYf]b[]Gh XYbhGi ddcfhGYfi]WYg

9b[]bYYf]b[]=bXi ghf]U``HfU]b]b[

I BGK 'Gh Xm5VfcUX'UbX'9I WXUb[Y'fZcf']bVci bX'gh XYbhgŁ

I BGK : i hi fY Ghi XYbhq

D\cbY

fŽ*%&Ł-',),)\$\$-BiWYigGhXYbh<iV

fŽ*%%&L'-',) '+**%-'9b[]bYYf]b['=bXi glf]U`'HfU]b]b[

fŽ*%%&E-',) '' %+- - I BGK 'Gh Xm5VfcUX'UbX'I BGK '91 WXUb[Y'fZcf']bVci bX'gh XYbhgk

fž*%*&Ł'-',) '(\$-+'-'GW\cc`'CZ]\\\I

 $\label{thm:linear} $$ \frac{1}{D}YUgY'bchY'h\Uhih\Y'GW\&cc''C\Z_WY'k\]^``bch'_bck'k\Yb\#Zmci\ f'Wci\ fgY'Wcbj\ Ybcf']g'cb'WUa\ di\ g'cf'Uj'UV'Y}$

7 fYX]hDc]bhg [·] * ·			
Gi a a UfmcZh\Y 7ci fgY			
< Uj Y`mci `Yj Yf`Wcbg]XYfYX`\ck `k Y`a U_Y`XYj]WYg`UbX`a UWX]bYg3`K o	cbXYfYX [.] \ck	·U·WUf·Yb[]bY·k	Ugʻa UXY

7ci fgY[.]8YHJ]`g

UbX'Uddfcdf]UhY`mZfca 'h\YgY'g]hYg"

 $+ b \cdot WUgg. H YfY \cdot UfY \cdot h \cdot fYY \cdot]b \cdot WUgg \cdot UWhj \cdot [h]Y \cdot g \cdot [b \cdot U \cdot h m d] WU \cdot k \cdot YY _ \cdot k \cdot]WX \cdot k \cdot Y \cdot fYZ \cdot f \cdot h \cdot Y \cdot @YWh \cdot fYZ \cdot H \cdot h \cdot f]U \cdot UbX \cdot 7 \cdot 58 \# 7 \cdot 5A \cdot @UV \cdot V UgYX \cdot cb \cdot h \cdot Y \cdot h a \cdot Y \cdot h y \cdot h \cdot g \cdot Y \cdot h \cdot Y \cdot h \cdot g \cdot Y \cdot g \cdot Y$

- %" @YUfb]b[.:Ghi XYbh`YUfb]b[:]g`h\Y`Z]fghdf]cf]hm! 'hYUW\]b[:'UbX'UggYgga Ybh'UfY'gYWbXUfm WbWYfbg"'@YUfb]b[:\YfY']g`XYZ]bYX'Ug'[U]b]b[:bYk 'k Umg`cZi bXYfghUbX]b[:h\Y`Z]Y`X'cZXYg][b'UbX a Ubi ZUWh f]b[:]b'a YW\Ub]WU`Yb[]bYYf]b[/bch'Ug'g]a d`ma Ya cf]g]b[:]bZcfa Uh]cb"'K Y'UfY'hfm]b['hc hfUbgZcfa 'mci ']bhc'Yb[]bYYfg'UbX'Wf]h]WU`h\]b_Yfg']b'h\Y'X]gW]d`]bY"
- &" DYYf`+bhYfUWh]cb. '@YUfb]b[']g'U'gcWJU`'UWhjj]hmž'UbX`fYgYUfWX`g\ckg'h\Uh'mci 'k]````YUfb'a cghUbX VYghk\Yb'mci 'UfY'UWhj Y`miHUi [\h'Vmmci f'dYYfg'UbXž']b'hi fbž'k\Yb'mci 'hYUWX'h\Ya "
- ("<][\'ghubxufxg.'K\Y'k]```\uj\Y'\][\'ghubxufxg'Zcf'UW\]YjYa\Ybh']b'h\Y'WcifgYž'UbX'YjYfmcbYf]bWiX]b['ghuZk'k]```VY'UVWcibhuV'Y'Zcf'dih]b[']b'h\Y'YZcfhhc'[Yhmci'hc'h\Y'ghubxUfX"
-) " CdYbbYgg. '5g'a i W. 'Ug'dcgg]V`Yž'h.]g'Wti fgY'k]```VY'WtbXi WhYX']b'h.Y'cdYb'k \YfY'U```dUff]WjdUbhg Wbb'VY'Uk UfY'cZ]hUbX'Wta a Ybhi dcb']h'
- *" **DfcWfgg**. 'H\Y'ZcW g'cZh\Y'Wci fgY'k]``'VY'cb'dfcWfggYgž'bchci hWca Yg"'H\Y'f][\hci hWca Yg'k]``'VY U'VmldfcXi WhcZZc``ck]b['h\Y'WcffYWhdfcWfggYg"

 $\label{thm:local-control} $$HYY`YWi fYg']b'h.]g'Wci fgY'k]``Wcj Yf'WcfY'WcbW'dhg'UbX'VUW_[fci bX'h.Ycfm']b'a Ubi ZUWi f]b['UbX Yb[]bYYf]b['XYg][b"'H\Y''YWi fY'a UhYf]U`']g'Uj U]`UV'Y'hc'ghi XYbhg'Y'YWfcb]WU``mVYZcfY'YUW.'WUgg'j]U HYUa g"$

 $H\Y'758\ "UbX'75A\ "@UVg"UfY"XYg][\ bYX'hc "U`ck 'mci 'hc "dfUWf]gY"Wf]h]WU`g_]``g"]b'h\Y'UfYUg'cZWca di hYf! \\ U]XYX'XYg][\ b'UbX'a Ubi ZUWfi fY"'DfY!`UV'k cf_k]``VY'Uj U]`UV'Y'VYZcfY'h\Y'ghUfficZWUgg'UbX']g'UV'Y'hc 'VY k cf_YX'h\fci [\']b'mci f'ck b'h]a Y"'8i f]b['h\Y'`UVgž'h\Y'dfY!`UV'k cf_k]``VY'YI dUbXYX'i dcb'k]h\ cddcfhi b]mhc 'qYY_'Ugg]qhUbW']b'UfYUg'cZX]ZZW `hm'$

 $H Y'Hi hcf]U'g'k]'``dfcj]XY']a dcffUbhWcbhYl hi U']gUf]cb'VYfk YYb'h\Y'dfUWf]WU''g_]``g'VY]b[''h\Y'`UVgžh\Y'WcbWfdhg'VY]b['']bHcXi WfX']b'h\Y''YWhi fYg'UbX'\][\`][\h\ck''h\YgY'UfY'fY`UfYX'k\Yb'XYg][b]b[''UbX a Ubi ZJWhi f]b[']b'h\Y'fYU''k cfX''$

5XX]hjcbU`7ci fgY ±bZcfa Uhjcb

 $H\setminus]g^*]g^*U^{**}ib]hcZWfYX]hflc7 \& WcifgY^UbX^*]bjc^*jYg^{**}\setminus cifg^*dYf^*kYY_^*fh \#\&cZgWkYXi^*YX^*cb^*]bY^WcbhUWf$

 $\label{thm:condition} $$HY'bcfa\ U''k\ cf_cux'YI\ dYWfuhcbg'cZU'ghi\ XYbh'ufY'UddfcI\]a\ UhY'm'&)\ '\ci\ fg'dYf'hYfa\ 'Zcf'YUW'I\ C7ž]bWi\ X]b[\ 'WUgg'Wcbh'uWi\ci\ fgž'ch\Yf''YUfb]b[\ 'UWhjj\]h]Ygž'dfYdUfUhjcb''UbX'hJa\ Y'gdYbh'cb''U'''UggYggUV'Yk\ cf "$

Mci 'g\ci `X'U]a 'hc'gdYbX'U\ci h'\&'\#k 'cb'h\]g'Wci fgY"'H\Y'UXX]h]cbU``h]a Y'g\ci `X'VY'gdYbh]b'a U_]b[gi fY'h\Uhimci 'i bXYfghUbX'h\Y'`YWh fY'a UhYf]U'ž'Wca d`Yh]b['h\Y'gYhUgg][ba Ybhgž'\I fh\Yf'fYUX]b[ž'UbXfYj]q]b['\Icf'UbmYl Ua]bUh]cbg"

5ggYgga Ybh

8 YHJ] YX]bZcfa Uh]cb cb U``UggYgga Ybhg]g Uj U] UV Y h\fci [\'HYUa g"

5ggYgga Ybh'HUg_g

5ggYgga Ybh'HJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
7 ca di hYf!U]XYX'8Yg][b G_]``g`	(\$1	K YY_'(': f]XUm#K YY_'+ : f]XUm	(ž') ž'* ž'+
7B7'A UWX]b]b['5ggYgga Ybh	&) ı	:]`Y`Gi Va]gg]cb`!`K YY_`, ž HYgh]b['!`9 Ua 'DYf]cX	' ž'(ž') ž'*
5gg][ba Ybh	')	K YY_`%\$`!`: f]XUm	%ž' &ž' * ž' +

5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% 7ca di hYf!U]XYX 8Yg][bG_]``g

 $\textbf{GhUfhXUhY}. \ \texttt{Bch5dd`]} \textbf{WUV`Y}$

8YHJ]`g.

5XX]h]cbU`XYhJ]`g.

5ggYgga Ybh& 7B7 A UW\]b]b[5ggYgga Ybh

GHUfhXUhY. BNsho5tddAJVbJVpYlicable

8 YHJ] g.

8Yg][bž758ž75A 'UbX'a Ubi ZUWh fY Wca dcbYbhg'i g]b['7B7'a UWX]bYfm'

5XX]h]cbU`XYhJ]`g.

k YY_

@Yb[h\.'I d'hc'&\$'dU[Yg

8YHJ]`g.

7 UgY ghi X]Yg UbX [i]XYX ei Ygh]cbg UVci hih Y ``]b_'VYhk YYb XYg][b UbX a Ubi ZUWhi fY "

5XX]h]cbU`XYhJ]`g.

 $\label{thm:convergence} Gi\ Va\]gg]cb\ 'hc\ 'Hi\ fb]h]b\ 'VcI\ ']b\ 'A\ ccX\ 'Y\ 'Ug\ 'dYf\ 'Ugg\ 'gga\ Ybh'[\ i\]XY\]b\ Yg''\ 'A\ Uf_g\ 'hc\ 'VY\ 'fYhi\ fb\ YX\ 'i\ dcb\ 'fY\ 'YUg\ 'c\ Z\ Z]b\ U\ 'a\ Uf_g''$

5 HYY6 BUXUUB VVVY ET YFEA YF6YAA Y bhg

	1	

K YY_"%\$."&"5i [i gh'!"*	Hi M@UV	758°@UVg.°CdYb°7cbgi`HJHjcb
5i [igh		

FYgci fWyg

DfYgWf]VYX F Ygci fWYg

7`Y[`WWd[`Y6dSi [`Yž5"'K "'6ci bXmž'AW fUk'<]```f\+h\'9X]h]cbŁ"

FYWca a YbXYX FYgci fWfg

? S` g\S\Ufg\d[` Y 7` Y[` \W\d[` Y S` \V F\\UZ` a\\a'\k\z'\G'''? \U`\d__\]\Ub\`\Ub\X`\G\`F\`\G\\X\a\]\X'''\`D\f\Y\b\\]\\W'\'< \U``

7ci fgY 9j U'i Uhjcb UbX 8Yj Y'cda Ybh

: YYXVUW_cb^h\Y`Wci fgY`]g`[Uh\YfYX`dYf]cX]WU``mi g]b['j Uf]ci g`a YUbgž]bWi X]b['h\Y`I BGK a m9I dYf]YbW'`dfcW'ggž]bZcfa U``X]gW gg]cb^j]b^h\Y`Z]bU``WUgg`Zcf`h\Y`Wci fgYžUbX`h\Y`GW\cc`'g Gh XYbh#GhUZZa YYh]b[g"'Mci f`ZYYXVUW_`]g`hU_Yb`gYf]ci g`mžUbX`Wcbh]bi U``]a dfcj Ya Ybhg`UfY`a UXY`hc`h\Y Wci fgY`VUgYXž]b`dUfhžcb`gi W\`ZYYXVUW_"

± 'h\]g'Wci fgYžfYWYbh]a dfcj Ya Ybhg'fYgi `h]b['Zfca 'gh XYbhZYYXVUW_']bWi XY'fYXYg][b]b['a Uf_]b['Wf]hYf]U Zcf'758'WUggYg']b'gi W. 'U'k Umh\UhZcW gYg'cb'dfcWygg'fUh\Yf'h\Ub'ci hWca Y"'@YWi fY 'WcbhYbh\Ug'VYYb fYU`][bYX'hc 'Ybgi fY 'fY`Yj UbWY'hc 'UggYgga Ybh' DYfgcbU``Yb[fUj]b[g'UfY'bck ']a d`Ya YbhYX'Zcf'h\Y'7B7 a UV\X]b]b['UggYgga Ybh'

UggYgga YbhidYfZcfa UbWYžmci 'UfY'Y`][]V`Y`hc'Udd`m'Zcf'GdYWJU``7cbg]XYfUhJcb'df]cf'hc'gi Va]htjb['UbUggYgga Ybhicf'g]htjb['Ub'Yl Ua"

: cf`XYhJ]`g`cZUdd`m]b['Zcf`GdYW]U``7cbg]XYfUh]cb`UbX`WcbX]h]cbg`Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhUfm UggYgga Ybhž'd`YUgY`gYY`h\Y`]bZcfa Uh]cb`cb`I BGK 'g`<u>GdYW]U``7cbg]XYfUh]cb`dU[Y</u>"

D`YUgY'bchY'h\Uhghi XYbhg'k]``bch'VY'fYei]fYX'hc'dfcj]XY'Ubm'XcW'a YbhUfm'Yj]XYbWY'hc gi ddcfh'UVgYbWg'Zfca 'Ubm'WUggYg'a]ggYX'**VYWUi gY'cZ'7CJ=8!%** 'di V`]W\YU'h\ 'a YUgi fYg'gi W\'Ug]gc`Uh]cb"'| BGK 'k]``bch'VY']bg]gh]b['cb'a YX]WU`'VW'fh]ZJWUh'Yg'Zfca 'UbmcbY'XYYa YX'hc'VY'U'dcg]h]j Y'WUgYZ cf'k\Yb'h\Ym\Uj Y'fYWcj YfYX"'Gi W\'WYfh]ZJWUh'Yg'UfY'X]ZZJW 'hhc'cVhU]b'UbX'di h'Ub'i bbYWYggUfmghfU]b'cb ghi XYbhg'UbX'a YX]WU`'ghUZZ''

 $5\,dd^{3}WUhlcbg^{3}Zcf^{3}gdYW_{U}^{3}W^{3}bg^{3}XYfUhlcb^{3}k^{3}XYfUhlcb^{3}xYfUhlcb^{$

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[``U`W``hifY`cZ`YUfb]b[`]bZcfaYX`VmUWUXYa]W]bHY[f]hm'`5``IBGK ``ghiXYbhg``UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'`D`U[]Uf]gaibXYfa]bYg`UWUXYa]W]bHY[f]hmUbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVWY] WW Sege[YfZWiadVead[VWSeaXafZWdeS`VbSee[YfZW_aXXSekagdai`ž$

‰

5WUXYa]W±bZcfaUhjcb

7 fYX]hdc]bhg

 $7\,ci\,fgY\,'WfYX]hi]g\,'WU\,'W'\,'Uh'X\,']b\,'I\,\,b]hg!\,C\,Z\,7\,fYX]hfl\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,cf_`cUX\,'YI\,\,dYWfUh]cb\,'Zcf'cbY\,'I\,\,C\,7\,L''\,'H\,\backslash\,Y\,'bcfa\,\,U\,''k\,\,C\,(A)$

- <u>A ccX`Y</u>
- <u>@UV 5 WW gg</u> <u>7 ca di hlb[: UW] hly g</u>

5ddYbX]l. '9b[]bYYfg'5i glfU`]U'f95Ł'DfcZYgg]cbU`'9b[]bYYf'7ca dYhYbWm GHJbXUfX

Dfc[fUa '±bhYbXYX'@YUfb]b['CihWca Yg	
?bck `YX[Y`UbX`g_]```VUgY	
D9%"%7cadfY\Ybg]jYžh\YcfmVUgYX`ibXYfgHJbX]b[`cZh\Y`ibXYfd]bb]b[`bUhifU``UbXd\mg]WJ``gW]YbWYg`UbX`h\Y`Yb[]bYYf]b[`Z bXUaYbHJ`g`Udd`]WJV`Y`hc`h\Y`Yb[]bYYf]b[`X]gW]d`]bY	
D9%"&"7cbWYdhi U``i bXYfghUbX]b['cZh\Y'a Uh\Ya Uh]Wgžbi a Yf]WU``UbU`mg]gžghUh]gh]WgžUbX Wca di hYf'UbX']bZcfa Uh]cb'gW]YbWYg'k\]W\)i bXYMHJb'h\Y'Yb[]bYYf]b[' %J gW]d`]bY	