

7ci fgY[·]Cj Yfj]Yk

GhJZZ 7 cbhJWh 8 YhJ] `g

7cbj Ybcfg

B Ua Y	9a U]`		

7ci fgY 8YHJ] g

$I b h c Z 7 f Y X h^*$

Gi a a UfmcZh\Y'7ci fgY

♣ 'h\]g'Wei fgYz'ghi XYbhg'k]``[U]b'Vch\'UZ bXUa YbhU``UbX'dfUW]WU`i bXYfghUbX]b['cZh\Y'XYg][b'UbX cdYfUh]cb'cZgc`Uf'WV``g'''H\Y'Wei fgY'k]``YbUV`Y'h\Y'ghi XYbhhc'Yl hYbX'h\Y]f'_bck `YX[Y'cZgYa]WebXi Wef XYj]W''UbX'dfcj]XY'U'gci bX'VUg]g'Zcf'U'XYYd'i bXYfghUbX]b['cZW'ffYbhgc`Uf'WV``'hYW\bc`c[]Yg'''? Ym dfUW]WU`'dfcWyggYg'gi W\'Ug'gc`]X'ghUhY'X]Zi g]cbz'[YhYf]b['UbX'XYj]W''WebHUW]b['UfY']bhfcXi WYX''5 'fUb[Y cZ'UVcfUhcfm\VUgYX'UbX'Wea a YfW]U`'gc'Uf'W\``'hYW\bc`c[]Yg'UfY'fyj]Yk YX'hc[Yh\Yf'k]h\'fYW\bh hYW\bc`c[]WU``UXj UbWyg']b'h\Y'Z]Y'X'''H\Y'Wei fgY'a U_Yg'i gY'cZg]a i `Uh]cb'hcc`g'hc'fY]bZcfW''Ub i bXYfghUbX]b['cZXYj]W''d\mg]Wg'UbX'h\Y'X]ZZYfYbhgc`Uf'W\``'hYW\bc`c[]Yg'''H\Y'g]a i `Uh]cb'hcc`g'UfY'U`gc i gYX'hc'XYg][b'UbX'cdh]a]nY'UXj UbWX'gc`Uf'W\``g'WcbW'dhg'''H\Y'Wei fgY']bhfcXi W'g'U'gi]hY'cZfY'Yj Ubh W\UfUWhYf]gUh]cb'hYW\b]ei Yg'h\Uh'UfY'i gYX'hc'i bXYfghUbX'h\Y'W\UfUWhYf]gh]Wg'cZgc`Uf'W\``g']bWi X]b[qdYWfU`'fYqdcbqYz'hYa dYfUhi fY'qYbq]hji]hmz'fYq]qhji Y'`cqgYqz'W' ffYbh[YbYfUh]cb'UbX'cdYb!WfW]hji c'hU[Yg''

7ci fgY'5]a g

- 8Yj Y`cd'k]h\]b'gh XYbhg'U'Z bXUa YbhU`h\YcfYhWU`i bXYfghUbX]b['cZh\Y'cdYfUh]cb'cZgc`Uf'WY``g/
- 91 dcgY'ghi XYbhg'hc'U'k]XY'fUb[Y'cZgc`Uf'WY```HYW\bc`c[]Ygž'k \]W\'UfY'dfUWljgYX']b'``UVcfUhcfm'UbX Wca a YfWJU`'Ybj]fcba Ybhg/UbX
- HYUWX 'ghi XYbhg 'hc 'i gY 'Uj U] 'UV Y 'hcc `g 'UbX 'hY VX b]ei Yg 'hc 'WX UfUWHY f]gY 'gc `Uf 'WY ``g

7ci fgY @YUfb]b['Ci hWca Yg

52hYfigi WWYggZ ``miWca d`Yh]b['h\]g Wci fgYžmci ig\ci `X'VY'UV'Y'hc.

@YUfb]b['Ci hWca Y	95 GHJ[Y %7ca dYhYbW]Yg	
%"'8YgWf]VY'gc`Uf`WY```cdYfUh]cbg'i g]b['d\mg]WU`'UbX'Y`YWYf]WU` a cXY`g	D9%"%ZD9%"&ZD9%" ZD9' "&	
&"'G]a i `UhY'gc`Uf'WY```XYj]WYg'i g]b['bi a Yf]WU`a cXY`g	D9%"&ž`D9%"' ž`D9&"%ž`D9&"'	
' "'8Yg][b'gc`Uf'WY``g'Vmcdh]a]g]b['dUfUaYhYfg'Zcf'aUl]aia YZ]W]YbWn	D9&"%*D9&"	
("'5bU'mgY'a YUgi fYX'W\UfUWNf]gh]Wg'cZgc'Uf'WY``g'hc'XYhYfa]bY gci fWYg'cZ`cgg	D9&"%ŽD9&"	

HYUW]b['GhfUhY[]Yg'

5ggYgga Ybh

5ggYgga YbhhUg_	K Y][\h	8iY'8UhY	7 ci fgY`@YUfb]b[Ci hWca Yg`5ggYggYX
%" ⁻ 91 YfW]gYg	' \$I	K YY_'' ž'+	%ž &ž'' ž'(
&"'Ei]n	%) ।	K YY_'% %\$	%ž'&ž''
' "'Dfc YWh	&) ı	K YY_"%\$	%ž'&ž''
(":]bU``91 Ua	' \$1	Bch5dd`]WUV`Y	&ž¨'

5ggYgga Ybh'% '9l YfW]gYg'

8i Y"XUhY."K YY_" ž"+

 $5\,b\,U^*mg]g\check{z}^*9j\,\,U^*i\,\,Uh]c\,b^*Ub\,X^*8\,Yg][\,\,b^*c\,Zgc^*Uf^*WY^*g$

5ggYgga Ybh'& Ei]n

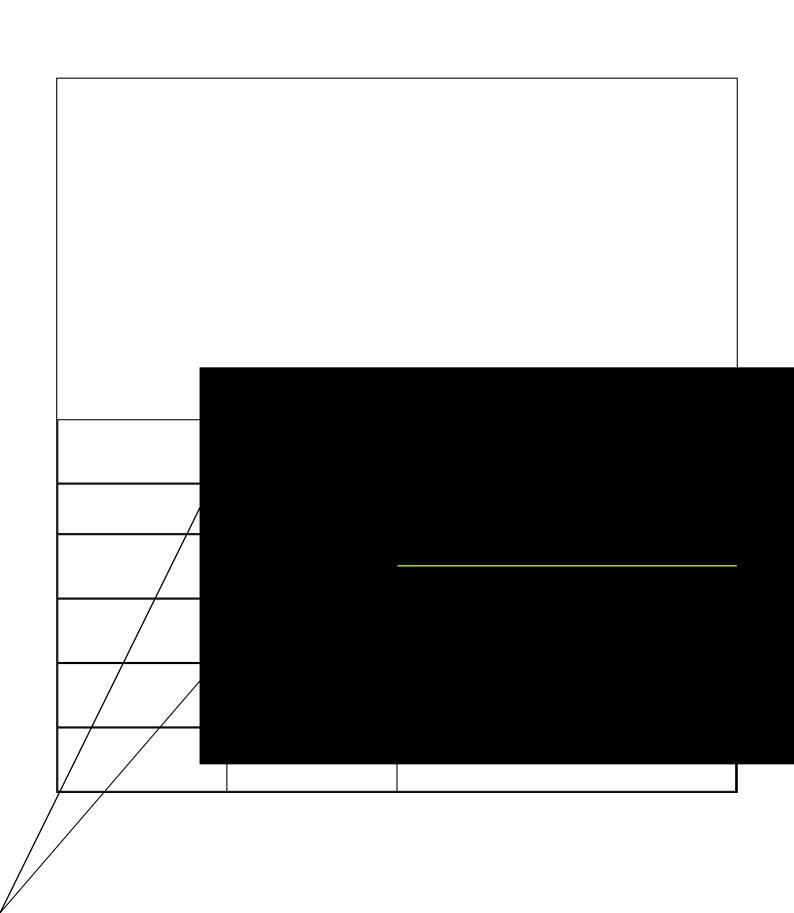
8i Y'XUhY.'K YY_'%%

@YWhifY'a UhYf]U``Zfca 'fYgdYWhjjY'kYY_"

5ggYgga Ybh' Dfc YWh

8i Y'XUhY.'K YY_'%\$

5bU'mg]gž'9jU'iUh]cb'UbX'8Yg][b'cZgc'Uf'WY'`g



FYgci fW/g

DfYgWf]VYX F Ygci fWYg

A "'5"; fYYb'f%, &Ł'"Gc`Uf'7Y``g""FYX'Vcc_"

\hdg.#kkk"djYXiWUhjcb"cf[#

FYWca a YbXYX FYgci fW/g

@YUfb]b[fYgci fW/g Zcf'h\]g Wti fgY]bWi XY.

- 'H\Y'D7 %8 'gcZk UfY'dUW_U[Y']g']bWi XYX'cb'U``'f\Y'Wca di hYfg']b'(%* 'UbX'())
- `A " `5" ; fYYb `f% , &Ł"Gc`Uf `7 Y``g"" F YX 'Vcc_"
- >YbbmBY`gcbž"H\Y`D\mg]Wg`cZGc`Uf`7Y``g"
- 'D\chcjc'\U]WGc'\Uf'9bYf[m': fca ': i bXUa YbHJ'g'hc'5dd']\Wh]cbg
- \bullet DYHYf'K i fZY` \check{z} 'D\mg]Wg'cZGc`Uf'7Y``g

5XX]h]cbU``fYZYfYbW'`a UhYf]U``a UXY`Uj U]`UV`Y`hc`gh XYbhg`j]U`A ccX`Y

I BGK "@]VfUfmik YVg]hY. \hdg.#k k k "`]VfUfm'i bgk "YXi "Ui #

 $A \ ccX`Y. `\ hdg. \#a \ ccX`Y"hY`h'i \ bgk "YXi "Ui \ \#c[\]b\#bXYI "d\ d$

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

K cf_'gi Va]httX``Uht'`k]h\ci hUb`Uddfcj YX`YI htbg]cb`Vmh\Y`Wci fgY`WccfX]bUhcf`cf`XY`Y[UhYX`Ui h\cf]hm]g gi VYWhhc`U``UhY`dYbU`hmcZ)ı 'a Uf_'fYXi Wf]cb`dYf`XUmž`Wcbg]ghYbhk]h\`ch\Yf`GDF 99`Wci fgYgž'UbX WUddYX`Uh)`XUmg`f\%\$`\ci fgtž'UZhYf`k\]W\`U`ghi XYbhWUbbchgi Va]h\Ub`UggYgga Ybh'

 $H\Y``UhY`dYbU`hm]g`Udd`]YX`dYf`WU`YbXUf`XUmf]bWi~X]b[~k~YY_YbXg`UbX`di~V`]W\c`]XUmgŁ'h\Uhih\Y$

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa]haYbhhc`ZcghYf]b[``U`W``hifY`cZ`YUfb]b[`]bZcfaYX`VmUWUXYa]W]bHY[f]hm'`5``IBGK `ghiXYbhg`\UjY`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'D`U[]Uf]gaibXYfa]bYg`UWUXYa]W]bHY[f]hmUbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[eVW{`WSege[YfZW_aXSekagdai`ž$

Mci 'UfY'U'gc'fYa]bXYX'h\UhWUfYZ ``h]a Y'a UbU[Ya Ybh]g'Ub']a dcftUbhdUfhcZgh XmUbX'cbY'cZh\Y]XYbh]Z]YX'WUi gYg'cZd`U[]Uf]ga ']g'dccf'h]a Y'a UbU[Ya Ybh'Gh XYbhg'g\ci `X'U``ck 'gi Z]W]Ybh'h]a Y'Zcf fYgYUfW\žXfUZ\]b['UbX'h\Y'dfcdYf'fYZYfYbW]b['cZgci fWYg']b'dfYdUf]b['U``'UggYgga Ybh'lUg_g''

₹d`U[]Uf]ga i]gʻZci bXi]bimci fik cf_ik \Ybimci 'UfYi]bʻZ[fghmYUfžimci fi`YWi fYfik]``cZYfimci 'Ugg]ghUbWYihc
]a dfcj Yimci fi'UWUXYa]Wg_]``g'''H\Yma UmUg_imci ihci`cc_'Uhigca Yicb`]bYifYgci fWygžiUhhYbXih\Yi@YUfb]b[
7 YbhfYžcfigca Yh]a YgifYgi Va]himci fik cf_ik]h\h\YidfcV`Ya iZ[l YX'''< ck Yj Yfia cfYigYf]ci gi]bghUbWygi]biZ[fgh
mYUfžigi W\ugighYU`]b[iUbch\Yfighi XYbhgik cf_icfidUm]b[igca YcbYihciXcimci fik cf_žia UmVYi]bj Ygh][UhYX
i bXYfih\YiGhi XYbhA]gWcbXi WhDfcWYXi fYg''

FYdYUHYX'd`U[]Uf]ga 'fYj Yb']b'Z|fghmYUfłź'd`U[]Uf]ga 'UZHYf'Z|fghmYUfźcf'gYf]ci g']bghUbWYgža Um'U`gc'VY]bj Ygh][UhYX'i bXYf'h\Y'Gh XYbhA]gWcbXi WhDfcWYXi fYg"'H\Y'dYbU'h]Yg'i bXYf'h\Y'dfcWYXi fYg'WUb']bWi XY U'fYXi Whcb']b'a Uf_gž'ZU]`]b['U'Wci fgY'cf'Zcf'h\Y'a cghgYf]ci g'a UhHYfg'fi]_Y'd`U[]Uf]ga ']b'Ub'\cbci fg h\Yg]gŁYj Yb'gi gdYbg]cb'Zfca 'h\Y'i b]j Yfg]mm''H\Y'Gh XYbhA]gWcbXi WhDfcWYXi fYg'UfY'Uj U]`UV'Y'\YfY.

kkk"[g"i bgk"YXi"Ui #dc]Wn#XcWa Ybhg#ghi XYbha]gWcbXi WhdfcWYXi fYg"dXZ

5 WUXYa]W = DZcfa Uhjcb

7ci fgY '9j U'i Uh]cb 'UbX '8Yj Y`cda Ybh'

: YYXVUW_cb'h\Y'Wci fgY']g'[Uh\YfYX'dYf]cX]WU``mi g]b['j Uf]ci g'a YUbgž]bWi X]b['h\Y'I BGK a m9I dYf]YbW''dfcWYggž]bZcfa U``X]gW gg]cb']b'h\Y'Z]bU``WUgg'Zcf'h\Y'Wci fgYž'UbX'h\Y'GW\cc''g Gh XYbh#GHJZZa YYh]b[g"'Mci f'ZYYXVUW_']g'HU_Yb'gYf]ci g`mž'UbX'Wcbh]bi U``]a dfcj Ya Ybhg'UfY'a UXY'hc'h\Y Wci fqY'VUqYXž']b'dUfhž'cb'qi W\'ZYYXVUW"

 $5 ```ghi XYbhg`UfY`YI dYWhYX`hc`fYUX`UbX`VY`ZJa]`]Uf`k]h\`I BGK `[i]XY`]bYg`UbX`dc`]WYg''` \pm 0`dUfh]Wi`Ufž ghi XYbhg`g\ci`X`VY`ZJa]`]Uf`k]h\`h\Y`Zc``ck]b[.$

(E5 HY b X Ub W)

OH BGK '9a U] '5XXfYgg

(FGdYW)U`7cbg]XYfUh]cb

(191 Uag

(E) ddfci YX 7 U'W 'Uhcfa

(E5WUXYa]W<cbYghmUbX D`U[]Uf]ga

(Bei]hUV'Y'@YUfb]b['GYfj]WYa

7F = 7CG

7F = 7CG Dfcj]XYf 7cXY. \$\$\$-,;

5W_bck `YX[Ya YbhcZ7ci blfm

 $K Y : UW\underline{b} ck `YX[Y : h, Y : 6YXY[U : dY cd `Y : k \ c : UfY : h, Y : hfUX]h\underline{b} cb : `W : ghcX]Ubg : cZh, Y ``UbXg : cb : k \]WX : I BGK ? Ybg]b[hcb : WUa di g :]g ``cWUhYX"$

5ddYbX]l. '9b[]bYYfg'5i glfU`]U'f95Ł'DfcZYgg]cbU`'9b[]bYYf'7ca dYhYbWm GHJbXUfX

Dfc[fUa '±ohYbXYX'@YUfb]b['CihWca Yg				
?bck `YX[Y`UbX`g_]```VUgY				
D9%"%7cadfY\Ybg]j Yž'h\Ycfm'VUgYX'i bXYfgHJbX]b['cZh\Y'i bXYfd]bb]b['bUhi fU`'UbXd\mg]\WJ`'g\JYb\W'g'UbX'h\Y'Yb[]bYYf]b['Z bXUaYbHJ`g'Udd`]\WJV`Y'hc'h\Y'Yb[]bYYf]b['X]g\Jd`]bY				
D9%"&"7 cbWYdh U``i bXYfghJbX]b['cZh\Y'a Uh\Ya Uh]Wgž'bi a Yf]WJ``UbU`mg]gž'ghJh]gh]Wgž'UbX Wca di hYf'UbX']bZcfa Uh]cb'gW]YbWYg'k \]W\'i bXYfd]b'h\Y'Yb[]bYYf]b['X]gW]d`]bY				
D9%" =: #e!XYdh_i bXYfghUbX]b['cZgdYW]U`]ghVcX]Yg`cZ_bck `YX[Y`k]h\]b`h\Y`Yb[]bYYf]b[X]gW]d`]bY				