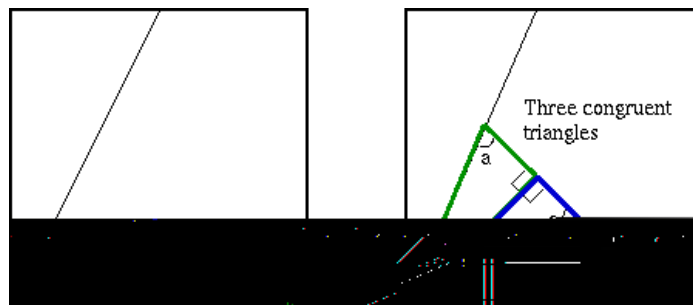


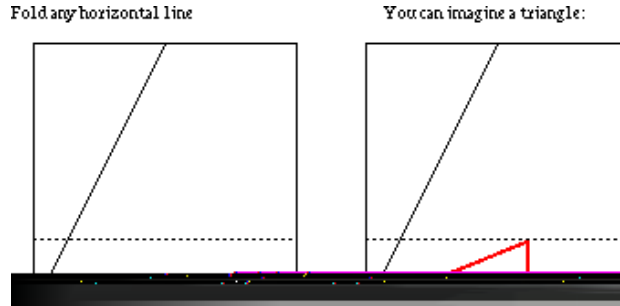
MATHEMATICS ENRICHMENT CLUB.¹
Problem Sheet 2, May 7, 2012

1. Find all pairs of primes $(p; q)$ such that p divides $q^2 - q$ and q divides $p^2 + p$.
2. If x is a number between 4 and 8 and y is a number between 20 and 40, what are the smallest and largest possible values of $\frac{y}{x}$?
3. Write the quartic $x^4 + 4$ as the product of two quadratics. What about $x^4 + 1$?
4. Find all positive integers $x; y; z$ such that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{5}{8}$:
 (Hint: Suppose $x \leq y \leq z$ and hence find the possible values of x .)
5. Suppose that a set S contains the numbers 1,2,3,4 and that the sum of any four different numbers in S is also a number in S . Show that S contains every positive integer greater than 21.
6. Let ABC be a triangle whose incircle has radius r . Let s equal to half the perimeter of the triangle. Show that the area of the triangle is rs :
7. Let S be the centroid of the triangle ABC and let $A_1; B_1; C_1$ be the midpoints of the sides, (with A_1 opposite A and so on). If AA_1 meets B_1C_1 at G , find the ratio of the areas of B_1GS to the area of ABC .
8. Tricsecting the angle (taken from www.math.lsu.edu/~verrill/origami/trisect/)
 Suppose we could put three congruent triangles in the picture as shown:

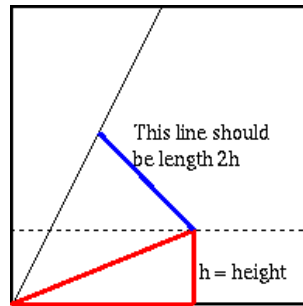


¹Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.

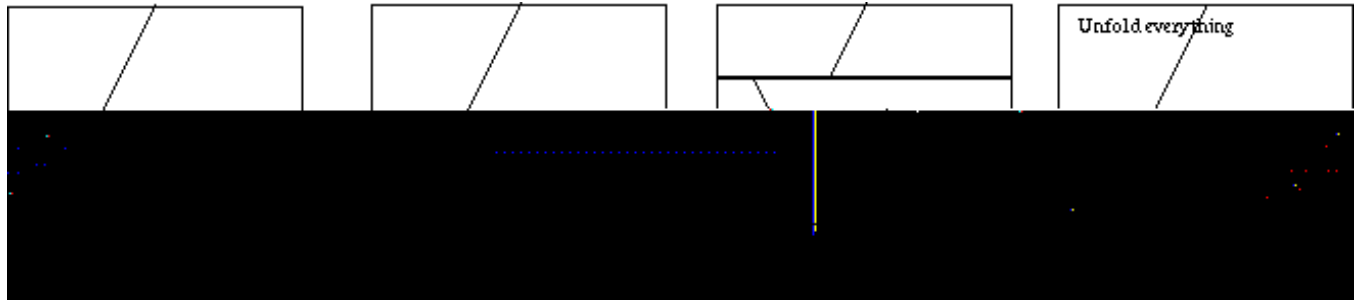
These triangles trisect the angle. So we need to know how to get them there.
 Choose some height for the lower triangle, any height, and crease a horizontal line at this height; ie, just crease any horizontal line you want:



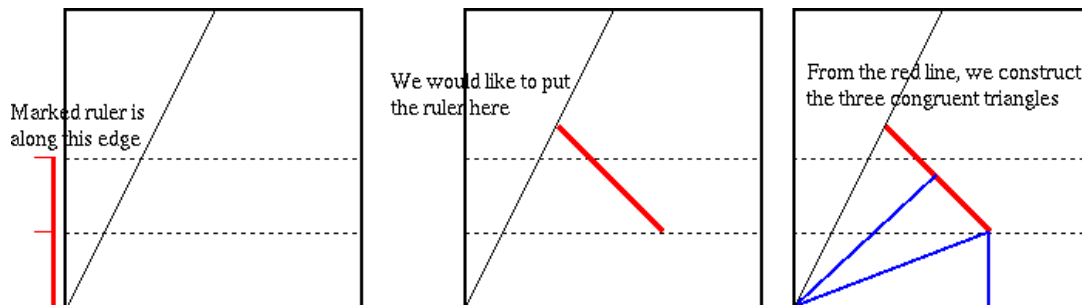
We need to get the blue line of the following picture somehow:



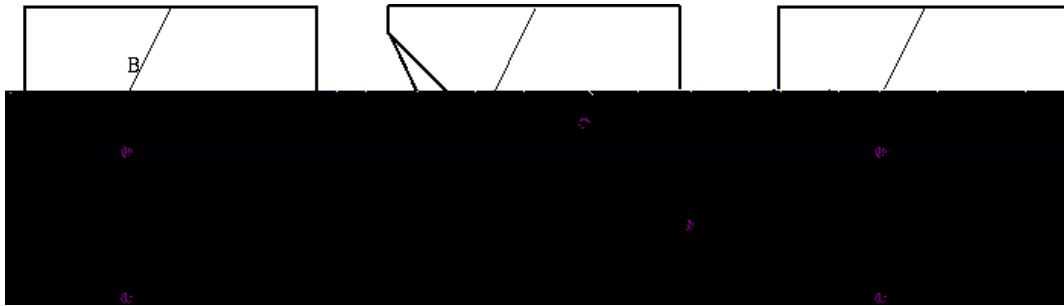
We can make a kind of "marked ruler" in the side of the paper, by folding over the paper again:



Now this "marked ruler" is used to find the bold line we needed:



To do this, fold the paper so point b touches line B, and point d touches line D:



Senior Questions.

1. If x is positive, how large must x be so that $\frac{10}{x^2 + x} - x$ differs from $\frac{1}{2}$ by less than 0.02?
2. Find the largest integer that exactly divides $11^{k+2} + 12^{2k+1}$ for all positive integers k .
3. Solve the equation $\cot^{-1} x - \cot^{-1}(x + 2) = \frac{\pi}{12}$.