

MATHEMATICS ENRICHMENT CLUB.<sup>1</sup>  
Problem Sheet 5, May 28, 2012

1. Two classes of 20 and 30 students average 66% and 56% respectively on an examination. What is the average for all the students on the exam?
2. A mathematics test has 5 questions on each of which people can score 0,1,2 or 3 marks.

Senior Questions.

1. Let  $f(x) = \left(1 + \frac{1}{x}\right)^x$ .

(a) Prove that  $\frac{f'(x)}{f(x)} = \log\left(1 + \frac{1}{x}\right) - \frac{1}{1+x}$ :

(b) By considering the area under the curve  $y = \frac{1}{t}$  for  $t$  from 1 to  $1 + \frac{1}{x}$ , show that  $\log\left(1 + \frac{1}{x}\right) > \frac{1}{1+x}$  and deduce that  $f(x)$  is increasing.

2. Suppose  $a > b > 0$ . Find  $\lim_{n \rightarrow \infty} (a^n + b^n)^{\frac{1}{n}}$ .

3. By considering  $\cos(A + B) + \sin(A - B) = 0$  find the general solution (for  $\theta$ ) of  $\cos n\theta + \sin m\theta = 0$ .