

MATHEMATICS ENRICHMENT CLUB.¹
Problem Sheet 8, June 20, 2012

1. The last digit of 1997^{1997} is
(a) 1 (b) 3 (c) 5 (d) 7 (e) 9:
2. The number of positive integers less than 1000 which are divisible neither by 3 nor 4 is
(a) 582 (b) 499 (c) 500 (d) 7501 (e) None of these:
3. Draw a right triangle ABC with right-angle at C and the sides marked $a; b; c$.
 - (a) Draw the enlargement $A^{\theta}B^{\theta}C^{\theta}$ of ABC by a factor of a .
 - (b) On the same diagram draw the enlargement $A^{\theta\theta}B^{\theta\theta}C^{\theta\theta}$ of ABC by a factor of b , lining up $B^{\theta}C^{\theta}$ with $A^{\theta\theta}C^{\theta\theta}$, so that $A^{\theta\theta} = B^{\theta}$, $C^{\theta\theta} = C^{\theta}$, and $A^{\theta}; C^{\theta}$ and $B^{\theta\theta}$ are collinear, and thus form a new triangle $A^{\theta}A^{\theta\theta}B^{\theta\theta}$.
 - (c) Explain why the angle $A^{\theta}A^{\theta\theta}B^{\theta\theta}$ is a right angle.
 - (d) What theorem have you just proven and why?
4. Find all positive integer solutions to
$$\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$$
5.
 - (a) Find the greatest common divisor of $2^{50} + 1$ and $2^{20} + 1$.
 - (b) Explain why the greatest common divisor of $2^m + 1$ and $2^n + 1$ is at least 3 if m and n are both odd.
6.
 - (a) Prove that the angle in a semicircle is right-angle.
 - (b) Show that if two chords of a circle mutually bisect each other, then they are both diameters.
 - (c) Complete the following statement: If a parallelogram is inscribed in a circle then
7. (a)

Senior Questions

1. Evaluate

$$\lim_{x \nearrow 1} \frac{3x^2 + \sin(2x^2)}{x^2}$$

2. Evaluate

$$\lim_{x \nearrow 0} \frac{3x^2 + \sin(2x^2)}{x^2}$$