

**MATHEMATICS ENRICHMENT CLUB.<sup>1</sup>**  
**Solution Sheet 1, May 7, 2012**

## Answers

1. Either  $p|q$  or  $p|q-1$  (1st condition), and either  $q|p$  or  $q|p+1$  (2nd condition). We have the following scenarios:

$nq = p$	$pm = q$	$pm = q - 1$
$nq = p + 1$	$p = q$	$mng = q - 1$ implies $p = 1 = q$
	$pmn = p + 1$ implies $p = 1 = q$	$pm + nq = p + q$ implies $p = 1, q = 2$

2. Easy

3. Complete the square, then take difference of two squares. Answers are  $(x^2 - 2x + 2)(x^2 + 2x + 2)$  and  $(x^2 - \sqrt{2}x + 1)(x^2 + \sqrt{2}x + 1)$ .

4. Suppose  $x \leq y \leq z$ . Then  $5/8 = 1/x + 1/y + 1/z \leq 3/x$ , so  $x < 5$ . This means there are only 4 possible values for  $x$ .

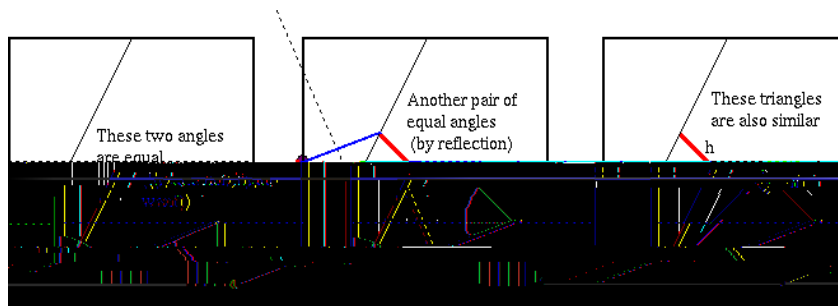
$x = 1$  No solutions.

$x = 2$  Solve  $1/y + 1/z = 1/8$ . So  $8 \leq y \leq 2 * 8$ . Testing  $y$  values in this range gives: (9, 72), (10, 40).

$x = 3$  Solve  $1/y + 1/z = 7/24$ . Since  $1/4 < 1/y + 1/z < 1/3$ . So  $3 \leq y \leq 2 * 4$ , Answers (4, 24), (6, 8).

$x = 4$  Solve  $1/y + 1/z = 3/8$ . Answers: (3, 24), (4, 8).

8. You can check that the angles are the same for these triangles:



<sup>1</sup>Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.