MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 1, May 7, 2013

1. Laurie sold two cars for $25 000 each. One he sold at a 20% profit and the other at a 20% loss. How much did he gain or lose ?

2. A number n has exactly 12 divisors. Given that n is divisible by 1,2,3,4,5 and 6 find a possible value of n. Is it the only one? Explain.

3. Without using a calculator, which is larger 31^{24} or 257^{15}.

4. Let $S_n = 2n(2n-1)(2n-2)\cdots(n+1)$. For example, $S_3 = 6 \cdot 5 \cdot 4 = 120$.

   (a) What is the power of 2 in the prime factorisatoin of S_n for $n = 2, 3, 4 \cdots$?

   (b) Make a conjecture based on(i) and prove it.

5. Without using a calculator, show that
$$\sqrt[3]{5\sqrt{13} + 18} - \sqrt[3]{5\sqrt{13} - 18} = 3.$$

   (Hint: Let $x = a - b$ and cube.)

6. Let ABC be a triangle and D, E points on AB, BC respectively, and S be the intersection of AE and CD. If $AD = DB$ and $BE : EC = 2 : 1$, find the ratios $CS : SD$ and $AS : SE$.

7. (a) Let P be an interior point in an equilateral triangle ABC. Prove that we can always form a triangle with sides of length AP, BP, CP. (That is, we have to show that the sum of any two of these lengths is larger than the remaining one.)

   (b) Give an example of a triangle and point inside it for which the above result is not true.