

MATHEMATICS ENRICHMENT CLUB.
 Problem Sheet 4, May 28, 2013

1

1. (a) Show that whatever base b is used, the number $(21)_b$ is never equal to twice $(12)_b$.
 (b) Find all the numbers and all bases $b \geq 12$ for which there exists a two digit number $(ac)_b$ which is twice the number obtained by reversing its digits.
 (c) Find all bases b and all numbers $n = (ac)_b$ such that $n = 2 \cdot (ca)_b$.
2. In how many ways is it possible to write 1000 as a sum of consecutive odd integers?
3. Draw a right triangle ABC with right-angle at C and the sides marked $a; b; c$ as usual.
 - (a) Draw the enlargement $A^0B^0C^0$ of ABC by a factor of a .
 - (b) On the same diagram draw the enlargement $A^{00}B^{00}C^{00}$ of $A^0B^0C^0$ with $A^0B^0C^0$ so that A^0, B^0 and A^{00}, B^{00} and C^{00} are collinear. Draw the new triangle $A^{00}B^{00}C^{00}$.
 - (c) Explain why the angle at C^{00} is a right angle.
 - (d)

Senior Questions

1. The hypotenuse of a right-angled triangle is 15 cm and the radius of the inscribed circle is 2cm. Find the perimeter of the triangle.
2. Suppose we place one of the numbers $1, 2, 3, \dots, 2000$ into each of 2000 boxes. Remove the two numbers a and b from any two boxes, chosen at random, and put their difference $a - b$ into one of the two boxes chosen and remove the empty box. Repeat the process until only one box remains. Show that the number in this box must be even.