

MATHEMATICS ENRICHMENT CLUB.¹
 Solution Sheet 12, August 13, 2013

1. Start with

$$\frac{x + 3y}{2x + 5y} = \frac{4}{7}$$

$$7(x + 3y) = 4(2x + 5y)$$

$$y = x$$

but we mustn't have $2x + 5y = 0$, which means $x \neq -\frac{5}{2}y$. The final solution then is $x = y \neq 0$.

2. Solving simultaneously we get $a = 0$, $b = 6$, $c = 7$, $d = 3$ and $e = 1$, so c is the largest. Can you do this without solving simultaneously?

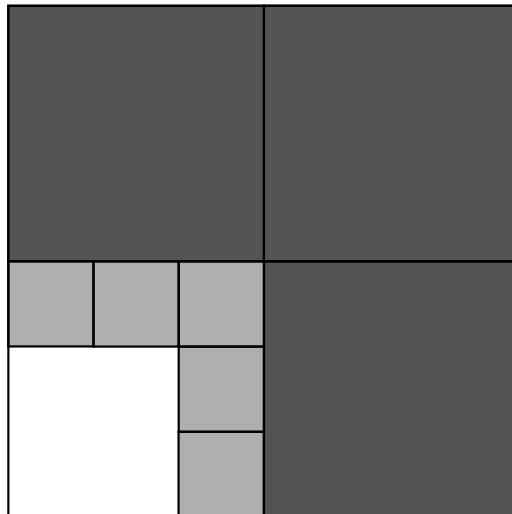


Figure 1: Solution for Question 3

¹Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni. Solution to question 4 provided by G. Liang

3.

4. Begin by constructing the equilateral triangle ADB . Draw the line CD to intersect AB at E . Draw EG parallel to DB and EF parallel to DA . Connect F and G then triangle EFG is equilateral. To prove this is true, construct B^0A^0 parallel to BA and passing through D , and show that BB^0D and AA^0D are similar to GBE and FAE respectively.

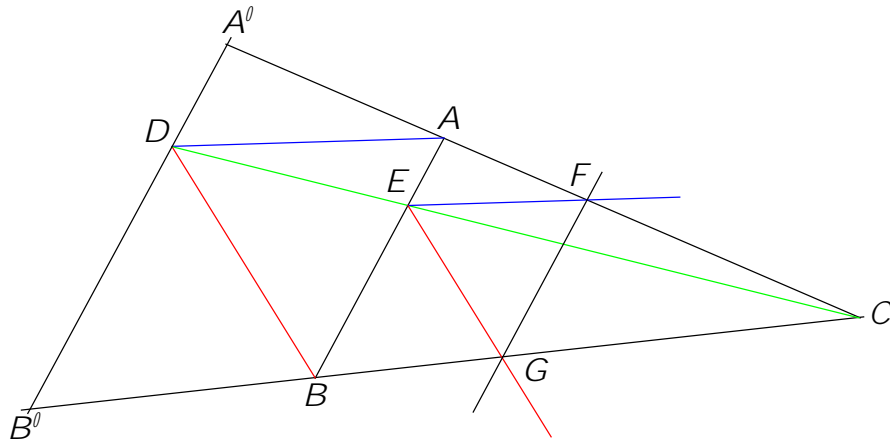


Figure 2: Solution for Question 4

5. Suppose otherwise, i.e., there is a 50×50 corner with no queens on it. Let's suppose the empty corner is the bottom-left (the argument will hold if any corner is chosen). To place 100 queens on the chess board, all not attacking each other, we must at least have 1 queen per row, and 1 queen per column. Since the bottom-left corner is empty, the 50 top rows must have a queen each, and the 50 left columns must also have a queen each. Since none are attacking each other, there can be no queens in the top-right (otherwise not all rows/columns would have a queen).

us the remainder of dividing $2^{15} - 1$ by both $2^5 - 1$ and $2^3 - 1$. So the prime factorisation is $2^{15} - 1 = 31 \cdot 7 \cdot 151$.

(c) We can use

$$\begin{aligned} x^{15} + 1 &= (x^3 + 1)(x^{12} - x^9 + x^6 - x^3 + 1) \\ &= (x^5 + 1)(x^{10} - x^5 + 1): \end{aligned}$$

So we can write

$$\begin{aligned} 2^{15} + 1 &= (2^5 + 1)(2^3 + 1)R \\ &= 33 \cdot 9 \cdot R: \end{aligned}$$

Using long division we get $R = 138 + \frac{1}{3}$ so

$$\begin{aligned} 2^{15} + 1 &= 3^3 \cdot 11 \cdot \frac{3 \cdot 138 + 1}{3} \\ &= 3^2 \cdot 11 \cdot (3 \cdot 110 + 1) \\ &= 3^2 \cdot 11 \cdot 331: \end{aligned}$$