MATHEMATICS ENRICHMENT CLUB.[1]
Solution Sheet 3, May 21, 2013

1. The dimensions of the brick are integers

$$a_0 = c_0 \mod 10$$

$$a_1 = \left(\left\lfloor \frac{c_0}{10} \right\rfloor + c_1\right) \mod 10$$

$$a_2 = \left(\left\lfloor \frac{c_1 + \left\lfloor \frac{c_0}{10} \right\rfloor}{10} \right\rfloor + c_2\right) \mod 10$$

$$a_3 = \left(\left\lfloor \frac{c_2 + \left\lfloor \frac{c_1 + \left\lfloor \frac{c_0}{10} \right\rfloor}{10} \right\rfloor}{10} \right\rfloor + c_3\right) \mod 10$$

Solving in order from b_0 to b_3 one finds $b_0 = 3$, $b_1 = 5$ or 0. Then if $b_1 = 5$ we find no solution for b_3, so $b_1 = 0$. Then $b_2 = 0$ or 5, but this time if b_2 , so

Senior Questions

1. Solve using induction, or visit http://en.wikipedia.org/wiki/Squared_triangular_number#Proofs for a cute geometrical representation.
2. $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}(2n^3 + 3n^2 + n)$, so $\lim_{n \to \infty} \frac{1^2+2^2+\cdots+n^2}{n^3} = \frac{1}{3}$.
3. (I could be wrong) Choose one of 13 values for the triplet and one of 4 suits to exclude and there are 13×4 possible triplets, then $\binom{12}{5}$ combinations of the remaining suit are left. So there are $13 \times 4 \times \binom{12}{5}$ possible hands.