MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 3, May 20, 2014 [1]

1. Suppose the two numbers were $a < b$ and the incorrect result c. Then $ab - 70 = c$ and $\frac{c}{a} = 48 + \frac{17}{a}$. So $c = 48a + 17$ which means $ab = 48a + 17 + 70$ or $a(b - 48) = 87$. The only two factors of 87 (that aren't 1 or 87) are 3 and 29, so a ... y

$$\frac{1}{x+y+z} \quad \frac{1}{z}$$

or

$$\frac{x+y}{xy} = \frac{(x+y)}{z(x+y+z)}:$$

So we'd have to satisfy

$$(x+y)(z(x+y+z)) = xy(x+y)$$
$$(x+y)(zx+zy+z^2+xy) = 0$$
$$(x+y)(x(z+y)+z(y+z)) = 0$$
$$(x+y)(z+y)(x+z) = 0$$

So we'd need to have either $x = y$, $z = y$ or $x = z$ provided none are zero. In each case, the number not speci ed is free to be whatever it pleases, i.e. solutions are $x = ; y = ; z =$, $x = ; y = ; z =$ or $x = ; y = ; z =$ for ; real numbers.

3. Let's wite $N = 100a + 10b + c$ or $[abc]$ for short. The ve numbers that can be obtained by permuting the digits are $[acb]; [bac]; [bca]; [cab]$ and $[cba]$. We know that

$$\frac{1}{5}([acb] + [bac] + [bca] + [cab] + [cba]) = N;$$

and further, inlcuding the mean in a set of data doesn't change the mean, so we can expand the above to

$$\frac{1}{6}([abc] + [acb] + [bac] + [bca] + [cab] + [cba]) = N:$$

[1] Some problems from UNSW's publication *Parabola*, and the Tournament of Towns in Toronto

By adding up the left hand side here we obtain

$$\frac{1}{6}(2(a+b+c) \quad 100+2(a+b+c) \quad 10+2(a+b+c)) = N$$

or

$$111(a+b+c) = 3N \quad \text{or } N = 37(a+b+c):$$

Since $N < 500, a+b+c$

Solving the quadratic using the quadratic formula, we get that $x = \frac{1 \quad \sqrt{5}}{4}$. From here we can use calculators, guess and check or plain ingenuity to nd that the ns that satisfy

$$\sin\frac{\ }{n} = \frac{1}{2}; \quad \text{or} \quad \frac{1 \quad \sqrt{5}}{4}$$

are 6 or 10.

6. Wikipedia has the best possible answer for this one:

> The original (333) Rubik's Cube has eight corners and twelve edges. There are 8! (40,320) ways to arrange the corner cubes. Seven can be oriented independently, and the orientation of the eighth depends on the preceding seven, giving 37 (2,187) possibilities. There are 12!/2 (239,500,800) ways to arrange the edges, since an even permutation of the corners implies an even permutation of the edges as well. (When arrangements of centres are also permitted, as described below, the rule is that the combined arrangement

as $k \to 0$, where $y = g(x_0)$. Lets' rewrite

$$g(x_0 + h) = h(\quad + g'(x_0)) + g(x_0)$$

then

$$f(g(x_0 + h)) = f(h(\quad + g'(x_0)) + g(x_0)):$$

Call $k = h(\quad + g'(x_0))$ then the above is just $f(k + y)$. So

$$\frac{f(k + y) \quad f(y)}{h} = \frac{h(\quad + g'(x_0))(\quad + f'(y))}{h}$$

using the previous expressions. As $h \to 0$, $k \to 0$ so $\to 0$ too. Further, $\to 0$ so the above tends to $g'(x_0)f'(y) = g'(x_0)f'(g(x_0))$ which is the product rule, but more importantly, the limit of the above as $h \to 0$ exists.

3. Since fg and f/g are differentiable, then $fg \quad f/g$ is differentiable by question 1 above. So $f(x)^2$ is differentiable at x_0. The function $h(x) = \sqrt{x}$ for $x > 0$ is differentiable, so $h(f(x)^2) = f(x)$ is differentiable provided $f(x_0) \neq 0$.

   If $f(x_0) = 0$ then consider

$$\frac{f(x_0 + h)}{h} = \frac{f(x_0 + h)g(x_0 + h)}{g(x_0 + h)h}$$

   as $h \to 0$, $g(x_0 + h) \to g(x_0)$ and $\lim_{h \to 0}(fg)(x_0 + h)/h$ exists, so $\lim_{h \to} f(x_0 + h)/h$ does too.