

3. Let R be the radius of the big circle. Draw a triangle that connects the centre of each

x_1 has remainder 1 when divided by 5. then x_1 must be one of 16: 11: :::

4. First we write

$$\frac{n^2 + 11n + 2}{n + 5} = \frac{n^2 + 10n}{n + 5} + 1;$$

and then we complete the square on the nominator of the fraction appearing in the RHS of the above equation,

$$\begin{aligned} \frac{n^2 + 10n}{n + 5} + 1 &= \frac{(n + 5)^2 - 28}{n + 5} + 1 \\ &= n + 5 - \frac{28}{n + 5} + 1: \end{aligned}$$

Now to get an integer on the last line of the above equation, the second term tells us that 28 must be divisible by $n + 5$. Since the factors of 28 are 1; 2; 4; 7; 14; 28, we conclude that $n = 2; 9$ or 23 (here we eliminated the factors that gives a negative n value).

5. The example shows that 4 is in T . We have further that 1 is in T , because $1 = (5 - 4) = (0 + 1)$. Also 3 is in T , because $3 = (4 - 1) = (0 + 3)$. Continuing in this way, we can eventually obtain $5; 4; \dots; 4; 5 \in T$; that is the integers from 5 to 5 are all elements of the set T .

Now to show that every integer is in T , we argue with induction as follows: suppose the set of integers $f; n; \dots; ng$ is in T , since we already know that the case $\emptyset = 5$ is true by the above, it remains to show that $n - 1$ and $n + 1$ is also in T . $n + 1$ is in T , because $n + 1 = [(n - 1) + (n)] = [(n - 2) + (n + 1)]$. It follows that $n - 1$ is also in T , because $n - 1 = [n + 1 + 0] = [(n - 1) + 2]$.

Senior Questions

1].

3. Rewrite the equation in the form $(c - b) = (10b + c)d$, then since a and d are co-prime, we can conclude that $(c - b)$ is positive and divisible by d . Thus $c = b + kd$, where k is a positive integer. Substituting the last equation of c back into the original equation gives (