

MATHEMATICS ENRICHMENT CLUB.
Solution Sheet 6, June 2, 2015 ¹

1. Sum of an arithmetic sequence with 11 term is

$$S_{11} = \frac{11}{2}(2a_1 + (11 - 1)d) = 220;$$

and the 6th (middle) term is

$$a_6 = a_1 + (6 - 1)d:$$

A solution to the former equation is $a_1 = 5$ and $d = 3$, hence $a_6 = 5 + 5 \cdot 3 = 20$.

2. (a) Let n

the 12 group case. Thus we conclude the total number of ways Bernard can do this is

23!

By statements (i) and (ii), $2b + 6$ is divisible by b . Therefore b is a divisor of 6; that is b is 1; 2; 3 or 6. By statement (i) and (iv), $9b + 5$ is a prime. Inserting the possible values of b into $9b + 5$, one sees that $b = 2$ is the only solution.

Senior Questions

1. (a) One way to do this is by polynomial long division http://en.wikipedia.org/wiki/Polynomial_long_division, another way is by induction.
- (b) Using part (a), we have $a^n - 1 = (a - 1)(a^{n-1} + a^{n-2} + \dots + a + 1)$. Since $a^n - 1$ is prime, the only factor it can have is 1; we must have $a - 1 = 1$, so $a = 2$.
Suppose n is not prime, then there are positive integers $x > 1$ and $y > 1$ such that $n = xy$. If we write $a^n - 1 = a^{xy} - 1 = (a^x)^y - 1$, then we can use the results of part (a) with a^x instead of a to obtain

$$a^n - 1 = (a^x)^y - 1 = (a^x - 1)[(a^x)^{y-1} + (a^x)^{y-2} + \dots + a^x + 1]:$$

Because the LHS of the above equation is a prime, we can conclude (just as before) that $a^x - 1 = 1$, which means $a^x = 2^x = 2$; $x = 1$, and we have a contradiction.

2. If A shoots C and hits, then B will shoot at C and hit. If A misses C, then B will shoot at C and hit. If A shoots D and hits, then B will shoot at D and hit. If A misses D, then B will shoot at D and hit. If A shoots E and hits, then B will shoot at E and hit. If A misses E, then B will shoot at E and hit. If A shoots F and hits, then B will shoot at F and hit. If A misses F, then B will shoot at F and hit. If A shoots G and hits, then B will shoot at G and hit. If A misses G, then B will shoot at G and hit. If A shoots H and hits, then B will shoot at H and hit. If A misses H, then B will shoot at H and hit. If A shoots I and hits, then B will shoot at I and hit. If A misses I, then B will shoot at I and hit. If A shoots J and hits, then B will shoot at J and hit. If A misses J, then B will shoot at J and hit. If A shoots K and hits, then B will shoot at K and hit. If A misses K, then B will shoot at K and hit. If A shoots L and hits, then B will shoot at L and hit. If A misses L, then B will shoot at L and hit. If A shoots M and hits, then B will shoot at M and hit. If A misses M, then B will shoot at M and hit. If A shoots N and hits, then B will shoot at N and hit. If A misses N, then B will shoot at N and hit. If A shoots O and hits, then B will shoot at O and hit. If A misses O, then B will shoot at O and hit. If A shoots P and hits, then B will shoot at P and hit. If A misses P, then B will shoot at P and hit. If A shoots Q and hits, then B will shoot at Q and hit. If A misses Q, then B will shoot at Q and hit. If A shoots R and hits, then B will shoot at R and hit. If A misses R, then B will shoot at R and hit. If A shoots S and hits, then B will shoot at S and hit. If A misses S, then B will shoot at S and hit. If A shoots T and hits, then B will shoot at T and hit. If A misses T, then B will shoot at T and hit. If A shoots U and hits, then B will shoot at U and hit. If A misses U, then B will shoot at U and hit. If A shoots V and hits, then B will shoot at V and hit. If A misses V, then B will shoot at V and hit. If A shoots W and hits, then B will shoot at W and hit. If A misses W, then B will shoot at W and hit. If A shoots X and hits, then B will shoot at X and hit. If A misses X, then B will shoot at X and hit. If A shoots Y and hits, then B will shoot at Y and hit. If A misses Y, then B will shoot at Y and hit. If A shoots Z and hits, then B will shoot at Z and hit. If A misses Z, then B will shoot at Z and hit.

we have

$$f(x) = 1 - (x)^{2015}$$