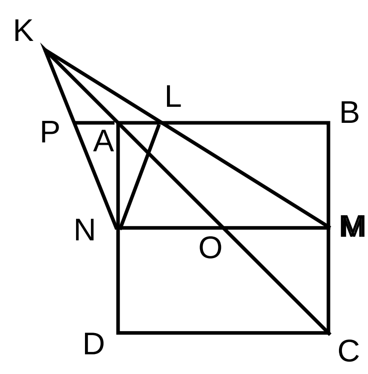
# MATHEMATICS ENRICHMENT CLUB.
## Solution Sheet 4, May 19, 2015[1]

1. First write $2016 = 2^5 3^2 7$, then divide both sides by $2^b$ we get

$$
\begin{aligned}
2^{a \;\; b} \quad 1 &= 2^{5 \;\; b} 3^2 7 \\
2^{a \;\; b} &= 2^{5 \;\; b} 3^2 7 + 1. \qquad (1)
\end{aligned}
$$

   Since $2^a \quad 2^b = 2016 > 0$, $a > b$, which implies the LHS of equation (1) is an even number. For the RHS of (1) to be even, we must have $b = 5$. Substituting $b = 5$ into (1), then $2^{a \;\; 5} = 64$, solving to obtain $a = 11$.

2. Let $O$ be the midpoint of $NM$, extend the line $AB$ so that it intercepts $KN$ at the point $P$; see below. Since $NM$ and $PL$ are parallel and $O$ is the mid point of $NM$, $A$ is the midpoint of $PL$ (this is a special case of the intercept theorem http://en.wikipedia.org/wiki/Intercept_theorem ). Therefore the triangles $PNA$ and $ANL$ are congruent to each other, hence \ $PNA$ = \ $ANL$.




3. We can write $n$ as $n = 3^a 5^b 7^c \quad N$, where the number $N$ has no factors of 3, 5 or 7. Then $\frac{1}{3}n = 3^{a \;\; 1} 5^b 7^c \quad N$, $\frac{1}{5}n = 3^a 5^{b \;\; 1} 7^c \quad N$ and $\frac{1}{7}n = 3^a 5^b 7^{c \;\; 1} \quad N$. Because we are looking minimal $N$, we may as well set $N = 1$. So for $\frac{1}{3}n$ to be a perfect cube, $\frac{1}{5}n$ to be a perfect  fth power and $\frac{1}{7}$ to be a perfect seventh power, we must have $a \quad 1$ a multiple of 3 and $a$ a multiplied of $5; 7$; the smallest such $a$ is 70. To  nd $n$, repeat this argument to obtain $b$ and $c$.

[1]Some problems from UNSW's publication Parabola, and the Tournament of Towns in Toronto

4. We have

$$k^3 \quad 1 = (k \quad 1)(k^2 + k + 1) = (k \quad 1)(k(k + 1) + 1)$$

and

$$k^3 + 1 = (k + 1)(k^2 \quad k + 1) = (k + 1)(k(k$$

Suppose we have $n$ integers, $x_1; : : : x_n$ from the list $0; 1; 2$ such that their sum is even. We know there is $f(n)$ ways to choose these $n$ numbers, and we can either pick $x_{n+1}$ to be 0 or 2 so that the sum of $x_1; : : : ; x_{n+1}$ is even; the total number of ways we can pick these $n + 1$ integers is $2f(n)$.

On the other hand, if the initial $n$ integers, $x_1; : : : x_n$

(b) $(n+1)! < n(1! + 2! + \dots + n!)$ because

$$(n+1)! = (n+1)n$$